AJHSME SOLUTIONS PAMPHLET

FOR STUDENTS AND TEACHERS

1st ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
1985

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AMERICAN MATHEMATICS COMPETITIONS

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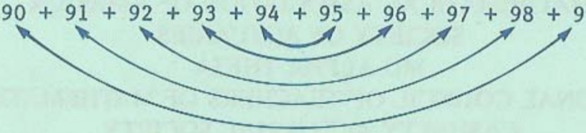
1. (A) $\frac{3 \times 5}{9 \times 11} \times \frac{7 \times 9 \times 11}{3 \times 5 \times 7} = \frac{3 \times 5 \times 7 \times 9 \times 11}{3 \times 5 \times 7 \times 9 \times 11} = 1.$

2. (B) By estimating, we see that the desired sum is between $10 \times 90 = 900$ and $10 \times 100 = 1000$, so it must be 945.

OR

Pair the numbers as shown. The sum of each pair is 189, so the desired sum is $5 \times 189 = 945$.

$90 + 91 + 92 + 93 + 94 + 95 + 96 + 97 + 98 + 99$

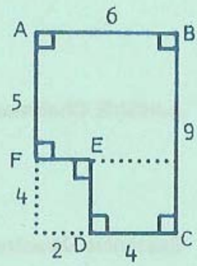


3. (D) $\frac{10^7}{5 \times 10^4} = \frac{10 \times 10^6}{5 \times 10^4} = 2 \times 10^2 = 200.$

OR

$\frac{10^7}{5 \times 10^4} = \frac{10^3}{5} = \frac{1000}{5} = 200$

4. (C) The area is greater than $6 \times 5 = 30$ and less than $6 \times 9 = 54$ so (C) must be correct.



OR

Extending FE partitions the polygon into a rectangle and a square whose areas are 30 and 16 respectively.

OR

Extending AF and DC to form the large rectangle shows that area is $(6 \times 9) - (4 \times 2) = 54 - 8 = 46$.

5. (C) By reading the graph, there are 5 A's, 4 B's, 3 C's, 3 D's, and 5 F's. Thus the fraction of satisfactory grades is $\frac{5+4+3+3}{20} = \frac{15}{20} = \frac{3}{4}$

OR

By reading the graph, $\frac{5}{20} = \frac{1}{4}$ of the grades are not satisfactory so $1 - \frac{1}{4} = \frac{3}{4}$ of the grades are satisfactory.

6. (D) The 7.5 cm stack is "half again" as tall as the 5 cm stack, so it will contain $500 + \frac{1}{2}(500) = 500 + 250 = 750$ sheets.

OR

If n is the number of sheets of paper in the 7.5 cm stack, then $\frac{5}{500} = \frac{7.5}{n}$. Thus n = 750 sheets.

7. (C) The number of black squares is one less than the number of the row, so the 37th row contains 36 black squares.

8. (A) If a = −2, the set is {6, −8, −12, 4, 1} so 6 = 3a is the largest. Notice that 4a and $\frac{24}{a}$ could be eliminated immediately since they are negative if a is negative.

9. (A) The desired product equals $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \ldots \times \frac{8}{9} \times \frac{9}{10} = \frac{1}{10}$ Notice that since $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$, the product is less than $\frac{1}{3}$ so (C), (D) and (E) are easily eliminated.

10. (C) $\dfrac{\frac{1}{3} + \frac{1}{5}}{2} = \dfrac{\frac{8}{15}}{2} = \dfrac{4}{15}$

11. (E) If face X is placed on the bottom of the cube, then faces U, V, W and Z are the sides and face Y is the top.

12. (B) The perimeter of the triangle and the square is $6.2 + 8.3 + 9.5 = 24$ cm. Thus the length of the side of the square is 6 cm and the area is 36 cm^2.

13. (B) To keep the units of miles and hours, first note 45 minutes $= \frac{45}{60} = \frac{3}{4}$ hour and 30 minutes $= \frac{1}{2}$ hour. Since distance = rate × time, your total distance is

$$4 \times \frac{3}{4} + 10 \times \frac{1}{2} = 3 + 5 = 8 \text{ miles.}$$

The distance is less than $4 + 10 = 14$ miles, so (D) and (E) can be easily eliminated.

14. (B) The difference is .5% of \$20 = .005 × \$20 = \$.10.

15. (C) In addition to the 100 numbers from 200-299, there are 20 numbers ending in 2 (e.g., 112, 342) and 20 numbers with a ten's digit of 2 (e.g., 127, 325). But the numbers 122 and 322 are counted twice in this process, so there are a total of $100 + 20 + 20 - 2 = 138$.

16. (D) Since the ratio is 2:3, $\frac{2}{5}$ of the students are boys and $\frac{3}{5}$ of them are girls. Thus there are $\frac{1}{5}$ more girls than boys and $\frac{1}{5} \times 30 = 6$.

17. (D) To get an average of 85 on 7 tests, you needed a total of $7 \times 85 = 595$ points. After 6 tests, you had a total of $6 \times 84 = 504$ points. Thus you needed $595 - 504 = 91$ points on the seventh test.

OR

If n was your score on the seventh test, then $\frac{6(84) + n}{7} = 85$ so $n = 91$.

OR

To raise your average by one point, you needed seven additional points on the seventh test, so your score was $84 + 7 = 91$.

18. (E) Only (E) satisfies the hypothesis that ten copies of the pamphlet cost more that \$11.00.

OR

If P is the price of the pamphlet, then $9P < 10$ and $10P > 11$ or $1.10 < P < 1.1111...$. Thus P = \$1.11.

19. (B) If $2(\ell + w)$ is the original perimeter, then the new perimeter is $2(1.1\ell + 1.1w) = 2.2(\ell + w)$ which is 10% more than $2(\ell + w)$.

20. (C) January has 31 days. Had January 1 fallen on a Monday or Tuesday, then there would have been five Tuesdays - 2, 9, 16, 23, 30 or 1, 8, 15, 22, 29. Likewise, had January 1 fallen on a Friday or Saturday, there would have been five Saturdays. Thus (C) is correct.

21. (E) If the initial salary is thought of as \$100 then the first 10% increase gives \$110. The second 10% increase gives \$110 + \$11 = \$121. The third increase gives \$121 + \$12.10 = \$133.10 and the fourth increase gives \$133.10 + \$13.31 = \$146.41 for an increase of 46.41%

OR

If S is the initial salary then the salary after four 10% increases is $(1.1)(1.1)(1.1)(1.1)S = 1.4641S$ for an increase of 46.41%.

22. (B) There are 10 digits. Excluding 0 and 1 leaves 8 digits. Thus $\frac{1}{8}$ of all telephone numbers begin with 9. Of these, $\frac{1}{10}$ end with 0 giving $\frac{1}{8} \times \frac{1}{10} = \frac{1}{80}$ which begin with 9 and end with 0.

23. (E) There are $1200 \times 5 = 6000$ times a day a student attends a class. Thus there are $\frac{6000}{30} = 200$ times a day a teacher teaches a class, so there must be $\frac{200}{4} = 50$ teachers.

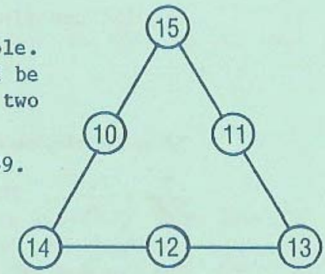
OR

If each student took 4 classes and each teacher taught 4 classes, then $\frac{1200}{30} = 40$ teachers would be required. But each student takes 5 classes, so $\frac{5}{4} \times 40 = 50$ teachers are needed.

24. (D) If the sum S is as large as possible. then the three largest numbers should be at the vertices so they each occur in two sums S. Thus

$$S = \frac{2(13 + 14 + 15) + 10 + 11 + 12}{3} = 39.$$

One such triangle is indicated.



25. (A) If Jane is wrong, then there is a card with a vowel on one side and an odd number on the other side. Such a card cannot have a consonant or an even number on either side. Thus the only card which could prove Jane wrong is the one with a "3" on one side. The other side would have to be a vowel.

OR

The easiest way for Mary to show Jane was wrong is for her to turn over a card showing a vowel, but there is no such card. But if Jane were correct, then so is the contrapositive: "If an odd number is on one side of a card, then a consonant is on the other side." Thus, Mary showed Jane was wrong by turning over the card marked with a 3.

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1. (A) The average rainfall per hour equals the total rainfall divided by the total number of hours. Since July has 31 days, the average is $\frac{366}{31\times 24}$ inches per hour.

2. (A) A large positive number has a small reciprocal and vice-versa. The smallest positive number $\frac{1}{3}$ has the largest reciprocal, $\frac{1}{\frac{1}{3}} = 3$.

3. (C) The three smallest numbers in the set are -3, -1, 7. Their sum is $-3 + -1 + 7 = 3$.

4. (C) The desired product is about $2(40) - .2(40) = 80 - 8 = 72$, so (C) is correct.

5. (D) Since 1000 minutes = $\frac{1000}{60}$ hours = $16\frac{2}{3}$ hours = 16 hours, 40 minutes, the contest ended 16 hours 40 minutes past noon or at 4:40 a.m.

6. (E) $\dfrac{2}{1-\frac{2}{3}} = \dfrac{2}{\frac{1}{3}} = 2\times 3 = 6.$

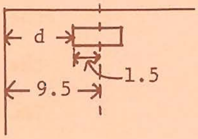
7. (B) Since $\sqrt{8} < \sqrt{9} = 3$, and $\sqrt{80} < \sqrt{81} = 9$, the desired whole numbers are 3,4,5,6,7,8; there are six of them.

8. (E) If B×2 ends in 6, then B is 3 or 8. Since the product exceeds 6000, B must be 8.

9. (E) The possible routes are MADCN, MACN, MBADCN, MBACN, MBCN, and MBN.

Query: Can you think of a systematic method to count the routes for more complicated figures?

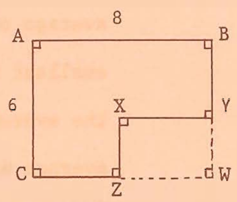
10. (B) Using the diagram, we see that the distance d = 9.5 feet - 1.5 feet = 8 feet.



11. (A) $(3*5)*8 = (\frac{3+5}{2})*8 = 4*8 = \frac{4+8}{2} = 6.$

12. (D) A student received the same grade on both tests if s/he is counted on the main diagonal (from the top left to the bottom right) of the table. Thus the number of students receiving the same grade on both tests is $2 + 4 + 5 + 1 + 0 = 12$. Consequently $\frac{12}{30} = \frac{4}{10} .= 40\%$ of the students received the same grade on both tests.

13. (C) Since XY = ZW and XZ = YW, the perimeter of polygon ABYXZC is equal to the perimeter of rectangle ABWC, or $2(8 + 6) = 28$. Note that the solution to the problem does not depend on the position of the point X inside the rectangle.



14. (C) The maximum value for the quotient $\frac{b}{a}$ is formed by choosing the largest possible value for b and the smallest possible value for a, or $\frac{1200}{200} = 6$.

15. (B) The sale price of the coat is 50% of \$180 or \$90. The additional discount is 20% of \$90 or \$18, so the Saturday price is \$90 - \$18 = \$72.

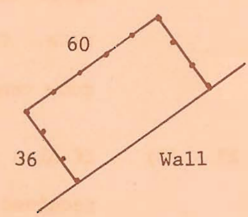
OR

The Saturday price is 80% of the sale price and the sale price is 50% of the original price, so the Saturday price is .8(.5(\$180)) = .8(\$90) = \$72.

16. (A) If the fall sales of 4 million hamburgers are 25% of the yearly sales, then the yearly sales are 16 million hamburgers. Thus the winter sales are $16 - (4.5 + 5 + 4) = 2.5$ million.

17. (E) The number o^2 is always odd. Now (no) is odd if n is odd and even if n is even. Thus the sum $(o^2 + no)$ is even if n is odd, since the sum of two odd numbers is even; the sum is odd if n is even, since the sum of an odd number and an even number is odd.

18. (B) The fewest number of posts is used if the wall serves as the longer side of the rectangular grazing area. Thus there are 6 posts on the 60 meter side (including the corners) and 3 more posts on each 36 meter side for a total of 12 posts.



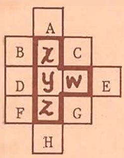
19. (D) The trip was 57,060 - 56,200 = 860 miles long and 12 + 20 = 32 gallons of gasoline were used during the trip. Thus the average number of miles per gallon was $\frac{860}{32} = 26.9$. Note that the 6 gallons needed to fill the tank at the start of the trip had no effect on the answer. No matter how many gallons were needed to fill the tank at the start, the average miles per gallon for the trip would be 26.9.

20. (D) Estimate each of the quantities to obtain the approximation:

$$\frac{(300)^5}{30(400)^4} = \frac{300}{30}\left(\frac{300}{400}\right)^4 = 10\left(\frac{3}{4}\right)^4 = 10\left(\frac{81}{256}\right)$$

$$\approx 10\left(\frac{1}{3}\right) \approx 3.$$

21. (E) Label the four squares in the T-shaped figure X,Y,Z,W as shown. First think of W as the base of the cube. Then X,Y,Z will be three of the "sides" and the fourth side could be A, E, or H. Now think of Y as the base. Then X,W,Z are three sides and B, D, or F could be the fourth side. C and G are not possible because four sides of a cube cannot come together at a point.



22. (C) If Alan received an "A", then all the others would have received an "A". If Beth received an "A", then Carlos and Diana would also have received an "A". Thus only Carlos and Diana received an "A".

23. (B) The radius of the large circle is 2 since it is a diameter of a small circle. The area of the large circle is $\pi(2)^2 = 4\pi$ and the area of each small circle is π. The shaded area is (by symmetry) half the difference of the areas of the large circle and the two small circles, i.e. $\frac{1}{2}(4\pi - 2\pi) = \pi$. Thus the desired ratio is 1.

24. (B) Al must be assigned to one of the lunch groups. The probability that Bob is assigned to the same lunch group is approximately $\frac{1}{3}$ ($\frac{199}{599}$ exactly) and the probability that Carol is assigned to that same group is also approximately $\frac{1}{3}$ ($\frac{198}{598}$). Thus the probability that all three are assigned to the same group is approximately $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

25. (D) In a set of whole numbers which are equally spaced, the average of the numbers in the set is the average of the smallest number and the largest number. For example, the average of $\{1,3,5,7,9\} = \frac{1+9}{2} = 5$, and the average of $\{2,5,8,11,14,17\}$ is $\frac{2+17}{2} = 9.5$.

In this problem, then, the averages are:

A: $\frac{2+100}{2} = 51$, B: $\frac{3+99}{2} = 51$,

C: $\frac{4+100}{2} = 52$, D: $\frac{5+100}{2} = 52.5$,

E: $\frac{6+96}{2} = 51$.

One could "guesstimate" that the set with the "largest" numbers should have the largest average. The numbers 5 and 100 are (overall) larger than the corresponding numbers in the other sets.

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SOLUTIONS — 1987 AJHSME

1. E The sum is .426.

2. B $\frac{2}{25} = \frac{8}{100} = .08.$

3. E Pairing the addends as shown, we see the desired product is $(2)(5)(180) = 1800.$

$$2(81 + 83 + 85 + 87 + 89 + 91 + 93 + 95 + 97 + 99)$$

4. C A right angle is 1/4 of a full circle, so there are $\frac{1}{4}(500) = 125$ clerts in a right angle.

5. A The area is $(.4\text{ m})(.22\text{ m}) = .088\text{ m}^2$.

6. B A negative product occurs when multiplying a positive number by a negative number. The minimum product will occur, then, when multiplying the smallest negative number by the largest positive number. In this case, that product is $(-7)(3) = -21$.

7. C The three faces shown of the larger cube must contain half the smaller shaded cubes, so there are $2(4 + 5 + 1) = 20$ shaded smaller cubes. By carefully looking at the large cube, we see that each smaller cube has at most one face (contained in a face of the large cube) shaded.

8. B $7 + 3 + B > 9$, so we "carry 1" from the ten's column to the hundred's column. Similarly $1 + 8 + A > 9$ since $A > 0$, so we "carry 1" from the hundred's place to the thousand's place. $1 + 9 = 10$, so the sum is a 5-digit number of the form 10CD9.

OR

Since A is a digit from 1 to 9, the sum 9876 + A32 must be between 10,000 and 100,000. Thus the sum of the three whole numbers must have 5 digits.

9. C The least common denominator(LCD) is the least common multiple of the denominators 2,3,4,5,6,7 or $2\cdot2\cdot3\cdot5\cdot7 = 420$.

10. B The desired sum may be written as $299(4 + 3 + 2 + 1) - 1 = 299(10) - 1 = 2990 - 1 = 2989$.

11. B The desired sum can be rewritten as $2 + 3 + 5 + \frac{1}{7} + \frac{1}{2} + \frac{1}{19}$ which equals $10 + \frac{1}{2} + \left(\text{a number less than } \frac{1}{2}\right)$ so B is correct.

12. C The large rectangular region can be subdivided into 24 congruent rectangular regions of which 2 are shaded.

OR

$\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$ of the rectangular region is shaded.

13. E If the numerator of a fraction is less than half its denominator, then the value of the fraction is less than $\frac{1}{2}$. Consequently all the fractions other than (E) are less than $\frac{1}{2}$ while $\frac{151}{301} > \frac{1}{2}$.

14. B There are $60\cdot60 = 3600$ seconds in an hour. Thus the computer does $3600\cdot10{,}000 = 36{,}000{,}000$ or 36 million additions in an hour.

15. D The regular price for three tires is \$240 - \$3 = \$237. Thus the regular price of one tire is $\frac{\$237}{3} = \$79$.

16. E In order to have a seasonal shooting average of 50% when having attempted 30 + 10 = 40 shots, Joyce must have made 20 of them. Thus she made 8 of her 10 shots in the next game.

17. D Since Bret is in seat #3 and statement (1) is false, Carl must be in seat #1. Then, since statement (2) is false, Abby must be in seat #4 leaving Dana in seat #2.

18. C The 12 people not dancing are $\frac{2}{3}$ of the people remaining, so 18 people remained. Thus there were 2(18) = 36 people in the room originally.

19. A The next four numbers displayed are 4, 16, 256, and $256^2 \approx 60{,}000$. Thus 500 is exceeded on the fourth depression of the $\boxed{x^2}$ key.

20. A To show the statement is false, we must find a value of n so that n is not prime and n - 2 is prime. Such a value is n = 9.

21. C Statements i and ii are false; iii and iv are true.

i. $3^* + 6^* = \frac{1}{3} + \frac{1}{6} \neq \frac{1}{9}$ iii. $2^* \cdot 6^* = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12} = 12^*$

ii. $6^* - 4^* = \frac{1}{6} - \frac{1}{4} \neq \frac{1}{2}$ iv. $10^* \div 2^* = \frac{1}{10} \cdot \frac{2}{1} = \frac{1}{5} = 5^*$

22. D By the Pythagorean Theorem, AC = 5. Since AC = BD, the radius of the circle is 5. The area of the shaded region, then, is $\frac{1}{4}\cdot\pi\cdot 5^2 - 3\cdot 4$ (a quarter circle with a rectangle deleted) $= \frac{25\pi}{4} - 12$. This quantity is clearly greater than $\frac{76}{4} - 12 = 7$ and less than $\frac{25(3\frac{1}{5})}{4} - 12 = 8$ since $3 < \pi < 3\frac{1}{5}$.

23. D The total Black population is the sum of the Black populations in each of the four regions or $5 + 5 + 15 + 2 = 27$ million. Thus $\frac{15}{27} = \frac{5}{9} \approx 55.56\%$ lived in the South. (The population figures were taken from the World Almanac and adjusted slightly for convenience.)

24. D If John answered 13 or more questions correctly, then his score would have been at least $13(5) - 7(2) = 51$ (13 correct, 7 incorrect). Checking the other cases, we find that John could have answered 12 correctly, 6 incorrectly, and left 2 unanswered for a score of $12(5) - 6(2) = 48$. Note that 10 correct, 1 incorrect, 9 unanswered also give a score of 48.

25. A Since Jack and Jill cannot remove the same number, there are $10 \cdot 9 = 90$ ways they can remove the two balls from the jar as shown by the unshaded squares on the grid. Those squares representing an even sum are labeled "E". There are 40 such squares - 4 in each column (or row) since the two numbers must both be odd or both be even. The probability is $\frac{40}{90} = \frac{4}{9}$.

Jill \ Jack	1	2	3	4	5	6	7	8	9	10
1			E		E		E		E	
2				E		E		E		E
3	E				E		E		E	
4		E				E		E		E
5	E		E				E		E	
6		E		E				E		E
7	E		E		E				E	
8		E		E		E				E
9	E		E		E		E			
10		E		E		E		E		

OR

There are 10 ways to select the first number but only 4 ways to select the second since it must have the same parity (both odd or both even) as the first. Thus the probability is $\frac{10 \cdot 4}{10 \cdot 9} = \frac{4}{9}$.

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4th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, DECEMBER 1, 1988

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SOLUTIONS - 1988 AJHSME

1. D — The scale is divided into fourths and the needle is just past the $\frac{1}{4}$ mark, so the reading must be between 10.25 and 10.5.

2. C — The product can be rewritten as $8 \times \frac{1}{4} \times 2 \times \frac{1}{8} =$
$8 \times \frac{1}{8} \times \frac{1}{4} \times 2 = \frac{1}{2}$

3. D — Since $\frac{2}{20} = \frac{3}{30} = \frac{1}{10}$, the desired sum is $.1 + .1 + .1 = .3$.

4. B — In each row, including the first, there is one more dark square than light square. Since there are 8 rows, there must be 8 more dark squares than light squares.

5. C — Since $\angle CBD$ is a right angle, side BC must cross the protractor at 70°. Thus the measure of $\angle ABC$ is $70° - 20° = 50°$.

OR

$\angle ABC = \angle ABD - \angle CBD = (\angle OBD - \angle OBA) - \angle CBD =$
$(160° - 20°) - 90° = 50°$.

6. E — $\frac{(.2)^3}{(.02)^2} = \frac{.008}{.0004} = \frac{80}{4} = 20$

OR

$\frac{.2}{.02} \times \frac{.2}{.02} \times .2 = 10 \times 10 \times .2 = 20.$

7. B — The product is approximately $(2.5)(8)(10) = (20)(10) = 200$.

8. B — Although one could solve this problem by counting decimal places in the product, it is more more insightful to realize that the answer is approximately $.1(2) = .2$, so (B) is correct.

9. D — All but the triangle in the upper right are isosceles.

10. A 60 days is 8 weeks and 4 days, so the desired day is four days after a Thursday , that is, a Monday.

OR

60 days is 9 weeks less 3 days, so the desired day is three days before a Thursday , also a Monday.

11. E $10^2 = 100$, $11^2 = 121$, $12^2 = 144$, $13^2 = 169$ so (E) is correct.

12. C The cost per person $= \dfrac{\text{total cost}}{\text{number of people}}$.

Thus $\dfrac{\$20 \text{ billion}}{250 \text{ million}} = \dfrac{2 \times 10^{10}}{2.5 \times 10^{8}} = .8 \times 10^2 = \$80.$

OR

Since $\dfrac{1000000000}{250000000} = 4$, the cost per person is ($4)(20) or $80.

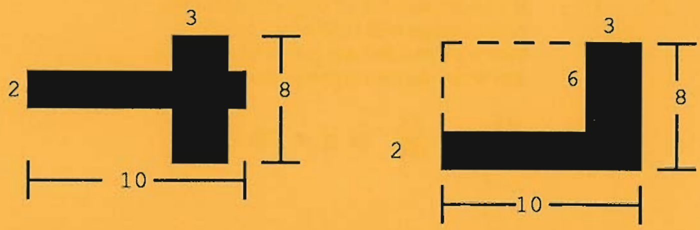
13. D The circumference of the circular patio is $2\pi(12) \approx (2)(3.14)(12) \approx 75$ feet, thus it would take about 75 bushes to surround the patio.

14. E The factor pairs for 36 are 1x36, 2x18, 3x12, 4x9, and 6x6. The largest sum of such a pair is 37.

15. C $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ and its reciprocal is $\frac{6}{5}$.

16. E The arrangement pictured shows 6 X's is possible. If there were 7 X's on a 3x3 board, then one row must contain 3 X's.

17. B



The total shaded area is the sum of the areas of the "horizontal" rectangle and the "vertical" rectangle minus the area of the "overlapping" rectangle that is part of both of the other rectangles. Thus the desired area is $2(10) + 3(8) - 2(3) = 38$.

OR

"Slide" the rectangles as shown in the figure on the right so they form an L-shaped figure. We see that the shaded area is $(10)(2) + (6)(3) = 38$ or $(7)(2) + (8)(3) = 38$ or $(10)(8) - (7)(6) = 38$.

18. C The total weight of the ten children is $6(150) + 4(120) = 1380$, so the average weight is $\frac{1380}{10}$ or 138 pounds.

OR

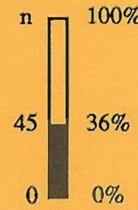
The average weight of 4 boys and 4 girls is 135 pounds. The other two boys would raise this average by $\frac{30}{10}$ or 3 pounds.

19. A Adding 3 to each term in the original arithmetic sequence yields the sequence 4,8,12,16,20, ... in which the one-hundredth term is 400. Subtracting 3 from each term shows that 397 is the one-hundredth term of the original sequence.

OR

We may obtain the 100th term in the sequence by adding 4 to the first term 1 a total of 99 times. Thus the 100th term is $1 + 99(4) = 397$.

20. C If n is the number of cups of coffee the coffeemaker will hold when it is full, then we can label the gauge as shown and write the two equivalent ratios:



$$\frac{45}{n} = \frac{36}{100} \text{ or } n = 125 \text{ cups.}$$

OR

If 36% is 45 cups, then 4% is $\frac{1}{9}(45)$ or 5 cups so that 100% is 25(5) or 125 cups.

21. C There are three possible values for the new number, n. It is either less than 6, between 6 and 9, or greater than 9. If the five elements are listed in increasing order, these three possibilities in the table below are: one of the first two, the middle one, and one of the last two. Since the median is the middle number in a set of five elements, there are three values for the median: 6, n, or 9.

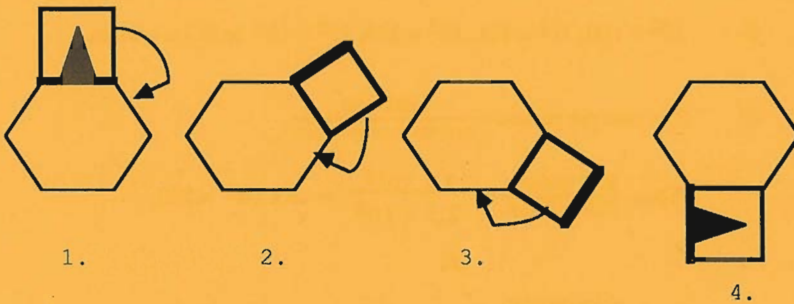
					Median
n	3	6	9	10	6
3	n	6	9	10	6
3	6	n	9	10	n
3	6	9	n	10	9
3	6	9	10	n	9

Query: Can you find the values of n and which of the two alternatives occurs in the first and third cases ?

22. E An item whose original cost was \$100, for example, will cost \$25 more or \$125. The sale price of a \$125 item will be 80% of its current price or .8(\$125) = \$100 - - the original cost. The same kind of comparison can be made for any original cost.

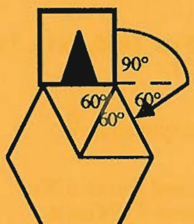
23. D Since the computer disks are bought in groups of 4 and sold in groups of 3, it is easier to consider them in groups of 12 or dozens. Each dozen costs \$15 and sells for \$20 giving a profit of \$5. Thus to get a profit of \$100, she must sell 20 dozen or 240 computer disks.

24. A



Keep track of the "bottom" side of the square in the first figure. In the fourth figure, it will appear on the left, so the solid triangle will be in the position shown in (A).

OR



Each time the square "rolls" to the next edge of the hexagon, it turns through an angle of 150°. In going from the top to the bottom of the hexagon, the square makes three such turns for a total of $3(150) = 450°$. This 450° represents one complete revolution and 1/4 of a second revolution.

25. A In each hour from 1:00 through 9:59, there are six such times. From 3:00 to 3:59, for example, these times are 3:03, 3:13, 3:23, 3:33, 3:43, 3:53. From 10:00 to 12:59, there is one such time in each hour: 10:01, 11:11,and 12:21. Thus there are a total of $9(6) + 3 = 57$ such times.

AMERICAN MATHEMATICS COMPETITIONS

AJHSME SOLUTIONS PAMPHLET
FOR STUDENTS AND TEACHERS

5th ANNUAL

AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION

(AJHSME)

THURSDAY, NOVEMBER 30, 1989

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Richardson, TX 75083-0688

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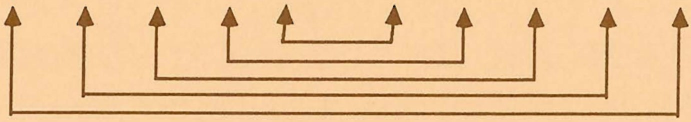
Department of Mathematics and Statistics
University of Nebraska
Lincoln, NE 68588-0322

SOLUTIONS 1989 AJHSME

1. E The sum is $5(50) = 250$ as can be seen by grouping the sum as shown.

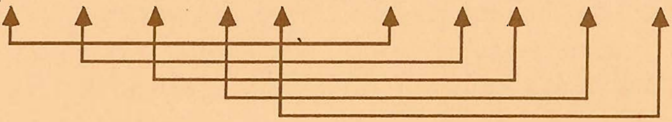
$$(1 + 11 + 21 + 31 + 41) + (9 + 19 + 29 + 39 + 49)$$



OR

The sum is $10 + 30 + 50 + 70 + 90 = 250$ as can be seen by grouping the sum as shown.

$$(1 + 11 + 21 + 31 + 41) + (9 + 19 + 29 + 39 + 49)$$



2. D The sum is $.2 + .04 + .006 = .246$.

3. A We can compare the five numbers in the set by annexing zeroes so each one has four decimal places. The numbers, then, are .9900, .9099, .9000, .9090, .9009 so that $.99 = .9900$ is the largest.

4. E $\frac{401}{.205} \approx \frac{400}{.2} = \frac{4000}{2} = 2000$ where $\approx$ means "is approximately equal to"

OR

$$\frac{401}{.205} = \frac{1}{.205} \times 401 \approx 5 \times 400 = 2000$$

5. D Using the standard order of operations,

$-15 + 9 \times (6 \div 3) = -15 + 9 \times 2$ [operate inside parentheses]

$= -15 + 18$ [multiplication precedes addition]

$= 3.$

6. C Since the distance between two consecutive marks is $\frac{1}{5}$ of the total distance,

$y = 3 \times (\frac{1}{5} \times 20) = 3 \times 4 = 12.$

OR

Use a proportion: $\frac{y}{20} = \frac{3}{5}$, so $y = 12$.

7. D The value of 20 quarters and 10 dimes is \$5.00 + \$1.00 = \$6.00. Since the value of 10 quarters is \$2.50, the remaining \$3.50 must consist of 35 dimes.

OR

We can think of the $20 - 10 = 10$ quarters as being "traded in" for dimes. The value of 10 quarters, \$2.50, is 25 dimes. Combining them with the original 10 dimes gives a total of 35 dimes.

8. E Using the distributive law, the product equals

$\{2 \times 3 \times 4\} \times (\frac{1}{2}) + \{2 \times 3 \times 4\} \times (\frac{1}{3}) + \{2 \times 3 \times 4\} \times (\frac{1}{4}) =$

$\{3 \times 4\} + \{2 \times 4\} + \{2 \times 3\} = 12 + 8 + 6 = 26.$

Although it is more tedious, one can perform the operations within each set of parentheses to obtain:

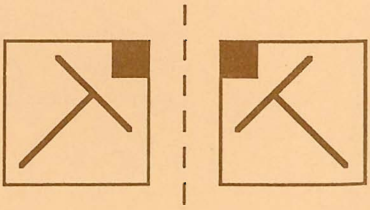
$24\,(\frac{6}{12} + \frac{4}{12} + \frac{3}{12}) = 24\,(\frac{13}{12}) = 26.$

9. C We see that 2 out of every 5 students or $\frac{2}{5}$ or 40% are boys.

Note: The number of students in the class is not needed to solve the problem.

10. D The numerals on a clock face divide it into twelve equal sections of 30°. The smaller of the two regions determined by the hands at 7:00 p.m. covers five of these sections, so the desired angle is 150°.

11. B Two figures are symmetric with respect to a line if the figures coincide when the paper is folded along that line.



12. E $$\frac{1-\frac{1}{3}}{1-\frac{1}{2}} = \frac{\frac{2}{3}}{\frac{1}{2}} = \frac{2}{3} \times \frac{2}{1} = \frac{4}{3}.$$

OR

Multiplying both numerator and denominator by 6 yields

$$\frac{6(1-\frac{1}{3})}{6(1-\frac{1}{2})} = \frac{6-2}{6-3} = \frac{4}{3}.$$

13. A To get .9 in the numerator, we must divide it by 10. To maintain equality, we must also divide the denominator by 10. Thus

$$\frac{\frac{9}{10}}{\frac{7 \times 53}{10}} = \frac{\frac{9}{10}}{\frac{7}{10} \times 53} = \frac{.9}{.7 \times 53}.$$

In all other cases, the resulting fraction is $\frac{1}{10}$ or $\frac{1}{100}$ of the original fraction.

14. C The smallest difference occurs when the minuend is as small as possible and the subtrahend is as large as possible. Thus $245 - 96 = 149$ is the smallest difference.

15. D The shaded area is the difference of the area of the parallelogram and the area of the unshaded triangle. The area of the parallelogram is $10 \times 8 = 80$. The base of the triangle is $10 - 6 = 4$ and its height is 8, so its area is $\frac{1}{2} \times 4 \times 8 = 16$. Thus the shaded area is $80 - 16 = 64$.

OR

The shaded region is a trapezoid with bases 10 and 6, and height 8. Its area is $\frac{1}{2}(8)(10+6) = 64$.

16. A It is not possible to write 47 as the sum of two primes. When 47 is written as the sum of two whole numbers, one must be even and the other odd. Since 2 is the only even prime, there is only one case to consider. In that case, the other summand must be 45 which is not a prime since it has a factor of 5.

SOLUTIONS 1989 AJHSME

17. B Since N is between 9 and 17, the average $\frac{6+10+N}{3}$ must lie between $\frac{6+10+9}{3} = 8\frac{1}{3}$ and $\frac{6+10+17}{3} = 11$. Only 10 lies in this range.

OR

The average of $6 = 10 - 4$, 10, and $14 = 10 + 4$ is clearly 10. Since $9 < 14 < 17$, a possible average is 10. Since the question has only one answer, it must be 10.

18. B Depressing the key one time yields the reciprocal of 32 or $\frac{1}{32}$. Thus depressing the key a second time yields the reciprocal of $\frac{1}{32}$ or $\frac{1}{\frac{1}{32}} = 32$.

19. B The graph shows expenditures of a bit more than \$2 million by the beginning of June and a bit more than \$4.5 million by the end of August. Thus about $4.5 - 2.0 = \$2.5$ million was spent during the summer months.

20. D Each number will share a corner with every number on the cube except the one on the opposite face. Thus the combinations involving 1-3, 2-5, and 4-6 cannot occur. Consequently the largest sum is $6 + 5 + 3 = 14$.

21. D Since $25\% = .25 = \frac{1}{4}$, Jill bought $\frac{1}{4} \times 128 = 32$ apples. June bought $\frac{1}{4}$ of the remaining $128 - 32 = 96$ apples or 24 apples. This leaves 72 apples -- one for the teacher and 71 for Jack.

OR

Keep track of the number of apples that Jack has at the end of each transaction:

Jill: $\frac{3}{4} \times 128 = 96$

June: $\frac{3}{4} \times 96 = 72$

Teacher: $72 - 1 = 71$.

22. C The 6 letters will go through a complete cycle every 6 lines and the 4 numbers every 4 lines. The letters and numbers together will go through a complete cycle every LCM (6,4) = 12 times. Thus AJHSME 1989 occurs for the first time on the 12th line.

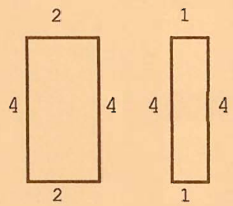
23. C There are 6 faces exposed on each of the four sides for a total of 6 x 4 = 24 square meters. From above, there are a total of 9 square meters exposed since the cubes in the second and third layers essentially cover the unexposed portion of the bottom layer. So there are a total of 24 + 9 = 33 square meters to paint.

OR

The bottom layer has 12 complete faces, four "$\frac{1}{2}$ faces", and four "$\frac{3}{4}$ faces". The second layer has 8 complete faces and four "$\frac{3}{4}$ faces". The top layer has 5 complete faces. The total, then, is

$$12 + 4\left(\frac{1}{2}\right) + 4\left(\frac{3}{4}\right) + 8 + 4\left(\frac{3}{4}\right) + 5 = 33$$ square meters.

24. E



If we let the side of the original square be 4 units, then the large rectangle and one of the small rectangles formed have the dimensions shown. Therefore the ratio of the perimeters is

$$\frac{1+4+4+1}{2+4+4+2} = \frac{10}{12} = \frac{5}{6}.$$

25. C The set of all the possible outcomes of spinning each wheel is
{ (5, 6) , (5, 7), (5, 9), (8,6), (8, 7), (8,9), (4,6), (4,7), (4,9), (3,6), (3,7), (3,9) }
There are six even sums among the 12 outcomes for a probability of $\frac{6}{12} = \frac{1}{2}$.

OR

Consider the outcome on the right wheel first. Whatever number occurs, there are two out of four chances for the number on the left wheel to give an even sum. Thus the desired probability is $\frac{2}{4} = \frac{1}{2}$.

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6th ANNUAL
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THURSDAY, NOVEMBER 29, 1990

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SOLUTIONS 1990 AJHSME

1. C In the smallest such sum, the two smallest digits are in the hundred's places, the next two digits in the ten's places and the two largest digits are in the one's places. One example is 468 + 579 = 1047.

   Query: In how many ways can this sum of 1047 be achieved?

2. A An increase in the tenth's place gives a larger value than an increase in any of the other decimal places. Since 1 is in the tenth's place of .12345, (A) is correct.

3. E Each shaded piece above or below the diagonal is matched by an identical unshaded piece meaning $\frac{1}{2}$ of the total area is shaded.

4. E From the way we multiply whole numbers, the unit's digit of the square of a whole number is determined by the square of the unit's digit of that whole number. The possible squares of unit's digits are: $0^2 = \boxed{0}$, $1^2 = \boxed{1}$, $2^2 = \boxed{4}$, $3^2 = \boxed{9}$, $4^2 = 1\boxed{6}$, $5^2 = 2\boxed{5}$, $6^2 = 3\boxed{6}$, $7^2 = 4\boxed{9}$, $8^2 = 6\boxed{4}$, $9^2 = 8\boxed{1}$.

   Note that 2,3,7, or 8 will never occur as the unit's digit of a square.

5. B The desired product is about $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} = 0.125$.

   OR

   The desired product is about $(.5)(.5)(.5) = 0.125$.

6. D All of the choices,except (C) and (D), are near 13,579. In (D), the result is the product (13579)(2468) while in (C) the result is much less than 13,579.

7. C For the product of three numbers to be positive, either all three of the numbers must be positive or one must be positive and two must be negative. Since there are only two positive numbers, only the latter case is possible. Thus the largest such product is $(-3)(-2)(5) = 30$.

8. D The sale price was $\frac{3}{4}(\$80) = \$60$. Thus the tax was \$6 and the total selling price was \$66.

9. D Five of the fifteen scores [77, 75, 84, 78, 80] are in the "C range", so the desired percent is $\frac{5}{15} = \frac{1}{3} = 33\frac{1}{3}\%$

10. A Since the date behind C is one less than that behind A, the date behind the desired letter must be one more than that behind B. This date is behind P.

11. E Since 11, 12, 13, 14, and 15 are on five of the faces, the number on the remaining face must be 10 or 16. In the first case, 10 must be on the face opposite the face with 15 which is impossible since $15 + 10 = 25$ would force 14 and 11 to be on opposite faces. Thus 16 is opposite 11, 12 is opposite 15, and 14 is opposite 13. The sum on each pair of opposite faces is 27, and the desired sum, is $3 \times 27 = 81$.

12. B One-fourth of the 24 numbers in the list begin with each of the four given digits. Those in positions 1 - 6 begin with 2; those in positions 7 - 12 begin with 4; those in positions 13 - 18 begin with 5. Thus the desired number is the fifth one beginning with 5: 5247, 5274, 5427, 5472, $\boxed{5724}$, 5742.

13. C The first ounce costs \$ .30. The additional 3.5 ounces would cost 4(\$ .22) = \$ .88. Thus the postage was \$ .30 + \$ .88 = \$1.18.

14. B Since one quarter of the balls are blue and there are 6 blue balls, there must be 24 balls in the bag. Thus there are $24 - 6 = 18$ green balls.

OR

Since one quarter of the balls are blue, three quarters of them must be green. Thus there are three times as many green balls as blue balls, so there are $3 \times 6 = 18$ green balls.

15. E The total area of the four squares is 100 cm^2, so the area of each square is 25 cm^2. Thus the side of each square is 5 cm and the perimeter of the figure is 10(5 cm) = 50 cm.

16. D By grouping as shown below, there are $\frac{199+1}{2} = 100$ groups of 10 for a sum of 1000:

$[1990 - 1980] + [1970 - 1960] + \ldots + [30 - 20] + 10$

17. A The number of cubic feet is $3 \times 60 \times \frac{1}{4} = 45$. Since there are 27 cubic feet in 1 cubic yard, there are $\frac{45}{27} = 1\frac{2}{3}$ cubic yards of concrete required. Thus 2 cubic yards must be ordered.

OR

Since 3 feet = 1 yard, 60 feet = 20 yards. Also 3 inches = $\frac{3}{36} = \frac{1}{12}$ yard. Thus $1 \times 20 \times \frac{1}{12} = 1\frac{2}{3}$ cubic yards of concrete are needed, so 2 cubic yards must be ordered.

18. C The original prism had 12 edges. Each "cut-off" corner yields 3 additional edges, so the new figure has a total of $12 + 8 \times 3 = 36$ edges.

19. B In order for the fewest number of seats to be occupied, there must be someone in every third seat, beginning with #2 and ending with #119. There are a total of $\frac{120}{3} = 40$ occupied seats.

OR

Consider some simpler cases and make a table:

Number of seats in the row:	3	6	9	12
Number of occupied seats in the row:	1	2	3	4

In each case, the middle seat in every group of three seats must be occupied, so the desired number of occupied seats in a row of 120 seats is $\frac{120}{3} = 40$.

20. A The difference between the incorrect sum and the actual sum is \$980,000 – \$98,000 = \$882,000. Since this difference is equally shared by all 1000 families, the difference between the means is $\frac{\$882000}{1000} = \$882$.

SOLUTIONS 1990 AJHSME

21. B Working backward from 1024, divide each number [e.g. 1024] by the preceding number [e.g. 64] to get the previous number [e.g. 16] in the list. Thus $64 \div 16 = 4$, $4 \div 4 = 1$, and so on:

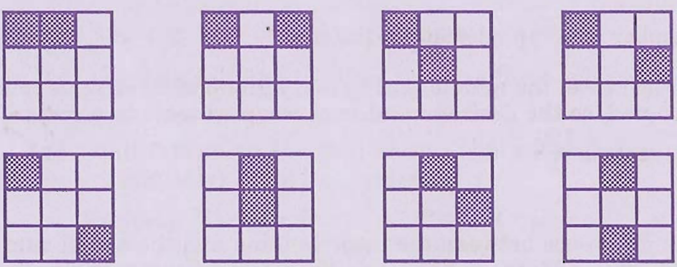
$\frac{1}{4}$, 4 , 1 , 4 , 4 , 16 , 64 , 1024

22. B Since Chris takes the last piece of candy, each person receives the same number of the other 99 pieces of candy. Thus the number of students at the table must be a factor of 99. Only (B) fulfills this condition.

23. B The rate will the largest when the graph is the "steepest". During the second hour the distance traveled is about 500 miles, so the average speed during that hour is about 500 mph. For the other hours, the speeds are less than 350 mph.

24. C The second balance shows each Δ balances ◊ •. Replace each Δ on the first balance with ◊ •. Then after removing three •'s from each side, the balance has ◊◊◊◊ on the left and •••••• on the right. Thus ◊◊ will be balanced by •••.

25. C There are 8. Be systematic. You can begin with the five cases that have one corner square. Then consider the other three cases that do not have a corner square.



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THURSDAY, NOVEMBER 21, 1991

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Upper Arlington, OH 43221

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1. **(B)** Write the problem vertically and compute the difference:

$$\begin{array}{r} 1,000,000,000,000 \\ -\ \ 777,777,777,777 \\ \hline 222,222,222,223 \end{array}$$

OR

What must be added to 777,777,777,777 to get 1,000,000,000,000? From the right, one adds a final digit of 3 and then eleven 2's.

2. **(C)** Using the standard order of operations, first simplify the numerator and then the denominator. Finally compute the quotient:

$$\frac{16+8}{4-2} = \frac{24}{2} = 12.$$

Note. Keying $16 + 8 \div 4 - 2$ on the calculator will give an incorrect answer for this problem. The problem means $(16+8) \div (4-2) = 24 \div 2 = 12$.

3. **(E)** Using arithmetic notation

$$\begin{array}{r} 200,000 \\ \times\ 200,000 \\ \hline 40,000,000,000 \end{array}$$

OR

Using scientific notation $(2 \times 10^5)(2 \times 10^5) = 4 \times 10^{10} = 40 \times 10^9 = 40$ billion.

OR

Two hundred times two hundred is forty thousand. A thousand thousands is a million. The answer is forty thousand millions, or forty billion.

4. **(E)** Each of the five numbers on the left side of the equation is approximately equal to 1,000. Thus N can be found by computing the difference between 1,000 and each number, so $N = 9 + 7 + 5 + 3 + 1 = 25$.

OR

Since

$$\begin{aligned} & 991 + 993 + 995 + 997 + 999 \\ &= (1000-9) + (1000-7) + (1000-5) + (1000-3) + (1000-1) \\ &= 5000 - (9+7+5+3+1) = 5000 - 25, \end{aligned}$$

it follows that $N = 25$.

5. **(B)** A collection of non-overlapping dominoes must cover an even number of squares. Since checkerboard **(B)** has an odd number of squares, it follows that it cannot be covered as required. A little experimentation shows how the other checkerboards can be covered.

6. **(C)** First mark the largest number in each column.

10	6	4	3	2
11	**|7|**	14	10	8
8	3	4	5	**|9|**
|13|	4	**|15|**	**|12|**	1
8	2	5	9	3

Determine if any of the marked numbers is the smallest in its row. Only 7 is.

Note. One could also begin by finding the smallest number in each row. Then a check of the columns yields the answer 7.

7. **(D)** Rounding each number to one significant digit (highest place value) yields

$$\frac{(500,000)(10,000,000)+(10,000,000)(500,000)}{(20,000)(.05)}$$

which equals $\dfrac{(500,000)(10,000,000+10,000,000)}{1,000}$

which equals $(500)(20,000,000) = 10,000,000,000.$

8. **(D)** The largest quotient would be a positive number. To obtain a positive quotient either both numbers must be positive or both must be negative. Using two positive numbers, the largest quotient is $\dfrac{8}{1} = 8$. Using two negative numbers, the largest quotient is $\dfrac{-24}{-2} = 12$.

9. **(B)** A number is divisible by 3 if it is a multiple of 3, and it is divisible by 5 if it is a multiple of 5. There are 15 multiples of 3, and 9 multiples of 5 which are whole numbers less than 46. However, 3 numbers (15, 30 and 45) which are divisible by both 3 and 5 have been counted twice. Thus the total number which are divisible by either 3 or 5 or both is $15 + 9 - 3 = 21$.

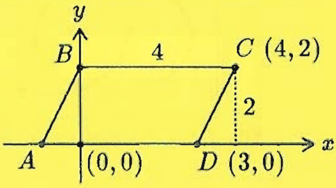
OR

Using the Sieve of Eratosthenes and marking each 3rd number, multiples of 3, and each 5th number, multiples of 5, yields

$$1, 2, \boxed{3}, 4, \boxed{5}, \boxed{6}, 7, 8, \boxed{9}, \boxed{10}, 11, \ldots, \boxed{45}, 46.$$

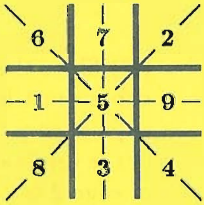
Thus, the total number which are divisible by either 3 or 5 or both is 21.

10. **(B)** The parallelogram rests on the horizontal axis. Since the coordinates of point C are $(4,2)$, it follows that the height of the parallelogram is 2. Since point B is $(0,2)$, it follows that the length of the base $\overline{BC}$ is 4. The area of a parallelogram is base times height. Thus the area is $4 \times 2 = 8$.



11. **(B)** After the 5 is selected, a sum of 10 is needed. There are four pairs that yield 10: 9+1, 8+2, 7+3, 6+4. Thus there are four 3-element subsets which include 5 and whose sum is 15.

Note. In the classic 3×3 "magic square" there are 4 lines through the middle of the square. The sum of the numbers along each line equals 15 and includes the number 5.



12. **(D)** Any fraction of the form $\dfrac{(k-1)+k+(k+1)}{k}$ equals 3, since $(k-1) + k + (k+1) = 3k$ and $\dfrac{3k}{k} = 3$. The denominator of the fraction must equal the middle term of the numerator. Thus $N = 1991$.

OR

Note that $\dfrac{2+3+4}{3} = \dfrac{9}{3} = \dfrac{3 \times 3}{3} = 3 \times \dfrac{3}{3} = 3 \times 1 = 3$.

Also, $\dfrac{1990 + 1991 + 1992}{N} = \dfrac{3 \times 1991}{N} = 3 \times \dfrac{1991}{N} = 3 \times 1 = 3$.

Thus N must equal 1991.

13. **(C)** Since $2 \times 5 = 10$, each zero at the end of the product comes from a product of 2 and 5 in the prime factorization of the number. Since $25 = 5 \times 5$ and $8 = 2 \times 2 \times 2$, it follows that there are fourteen factors of 5 and 9 factors of 2. This yields 9 pairs of 2×5 and results in 9 zeros at the end of the product.

OR

Multiplying the given numbers using a calculator gives an answer equivalent to 3.125×10^{12} which equals $3,125,000,000,000$. This results in 9 zeros at the end of the product.

OR

Factoring each number yields

$$\overbrace{2 \times 2 \times 2}^{8} \times \overbrace{2 \times 2 \times 2}^{8} \times \overbrace{2 \times 2 \times 2}^{8}$$
$$\times \underbrace{5 \times 5}_{25} \times \underbrace{5 \times 5}_{25} \times \underbrace{5 \times 5}_{25} \times \underbrace{5 \times 5}_{25} \times \underbrace{5 \times 5}_{25} \times \underbrace{5 \times 5}_{25} \times \underbrace{5 \times 5}_{25}.$$

Pairing each factor of 2 with a factor of 5 yields $(2 \times 5)^9 \times 5^5 = 10^9 \times$ (an odd number). Thus the product ends in nine zeros.

OR

Since $25 \times 25 \times 8 = 25 \times 200 = 5000$, regrouping yields

$$(25 \times 25 \times 8) \times (25 \times 25 \times 8) \times (25 \times 25 \times 8) \times 25$$
$$= 5000 \times 5000 \times 5000 \times 25 = 125,000,000,000 \times 25 = 3,125,000,000,000.$$

14. **(D)** If one student earns $5 + 5 + 5 = 15$ points, no other student can earn more than $3 + 3 + 3 = 9$ points.
If one student earns $5 + 5 + 3 = 13$ points, no other student can earn more than $3 + 3 + 5 = 11$ points.
However, if one student earns $5 + 3 + 3 = 11$ or $5 + 5 + 1 = 11$ points, some other student can earn $3 + 5 + 5 = 13$ or $3 + 3 + 5 = 11$ points.
Thus 13 points is the smallest number of points a student must earn to be guaranteed of earning more points than any other student.

15. **(C)** When the one-foot cube is removed, three square feet of surface area are "removed", but three new square feet of surface area are "uncovered". Thus, the original surface area is unchanged.

16. **(B)** A fold from the top leaves #9-16 on the bottom.
A fold from the bottom leaves #9-12 on the bottom.
A fold from the right leaves #9 and #10 on the bottom.
A fold from the left leaves #10 on the bottom with number #9 moving to the top.

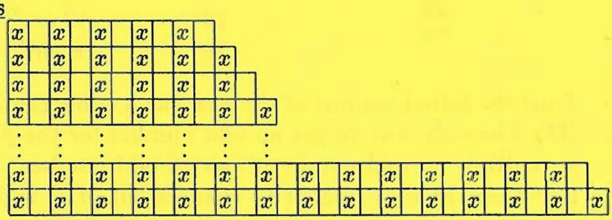
17. **(C)** The first row has 10 seats, so 5 students can sit in row 1. The second row has 11 seats, so 6 students can sit in row 2. The third row has 12 seats, so 6 students can sit in row 3. ... The last (20^{th}) row has 29 seats, so 15 students can sit in row 20. The sum is $5+6+6+7+7+\cdots+14+14+15$. Regrouping yields

$$\begin{array}{rl} & 5+\ 6+\ 6+\ 7+\ 7+\ 8+\ 8+\ 9+\ 9+10 \\ & +\ 15+14+14+13+13+12+12+11+11+10 \\ \hline = & 20+20+20+20+20+20+20+20+20+20 \ = 10(20) = 200. \end{array}$$

OR

Draw a diagram:

Row	Seats	Students
1	10	5
2	11	6
3	12	6
4	13	7
⋮	⋮	⋮
19	28	14
20	29	15



Thus, the sum is $5+6+6+7+7+\cdots+14+14+15 = 200$.

18. **(C)** Regardless of the scale on the vertical axis, 9 **X**'s out of 30 **X**'s represent employees who have worked 5 years or more. This is $\frac{9}{30}$ or 30%.

19. **(C)** Since the mean is 10, it follows that the sum of the numbers is $10 \times 10 = 100$. Taking the smallest possible values for the 9 smaller numbers would give the largest possible value of the tenth number. Thus, the largest possible number is $100-(1+2+3+4+5+6+7+8+9) = 100-45 = 55$.

20. **(A)** For the sum to be 300, $A = 2$ in the hundreds' place, since $A = 1$ gives numbers too small and $A = 3, 4, \ldots$ makes the sum too large. If $A = 2$ then $B = 7$, since $A = 2$ and $B = 8$ or 9 would be too large for the ones' place and $A = 2$ and $B = 6, 5, \ldots$ would not be enough to carry a 1 from the tens' to the hundreds' place. Thus, if $A = 2$ and $B = 7$ and $A + B + C = 10$ in the ones' place, then $C = 1$.

OR

Since the second column carry is 1, $A = 2$. The first column carry is also 1 since the sum of B and C is less than or equal to 17 and $A = 2$. In the second column, this makes $A + B + (\text{first column carry}) = 2 + B + 1 = 10$, so $B = 7$. Then from the first column $A + B + C = 2 + 7 + C = 10$, so $C = 1$.

21. **(A)** The rate of change is 4 cubic centimeters per 3°. The temperature change is $32° - 20° = 12°$. The corresponding change in volume is $12 \times \frac{4}{3} = 16$ cubic centimeters. Thus the initial volume of the gas was $24 - 16 = 8$.

OR

Work backwards, changing the volume of the gas by 4 cubic cm each time the temperature changes by 3°:

Temperature	Volume
32°	24 cm^3
29°	20 cm^3
26°	16 cm^3
23°	12 cm^3
20°	8 cm^3

Thus the initial volume of the gas was 8 cubic centimeters.

22. **(D)** The only way to get an odd number for the product of two numbers is to multiply an odd number times an odd number. This happens if one spins 1 or 3 on the first spinner (2 chances out of 3) and 5 on the second spinner (1 chance out of 3). Thus, the probability of an odd product is $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$. If one does not get an odd product, then the product is even. Hence the probability of an even product is $1 - \frac{2}{9} = \frac{7}{9}$.

OR

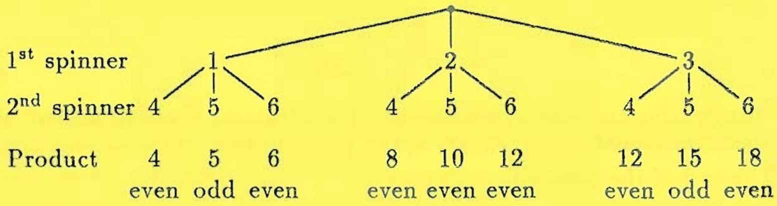
The sample space of pairs to be multiplied is:

$\boxed{1 \times 4}$	$\boxed{2 \times 4}$	$\boxed{3 \times 4}$
1×5	$\boxed{2 \times 5}$	3×5
$\boxed{1 \times 6}$	$\boxed{2 \times 6}$	$\boxed{3 \times 6}$

Successful pairs are marked. The probability is $\frac{7}{9}$.

OR

Use a tree diagram:



Thus the probability that the product is even is $\frac{7}{9}$.

23. (A) There are 100 females in the band, 80 in the orchestra, and 60 in both. Thus, there are $(100 + 80) - 60 = 120$ females in at least one of the groups. Since the total is 230, then there are $230 - 120 = 110$ males in at least one of the groups. There are 80 males in band and 100 males in orchestra, thus to find the number of males in both, $(80 + 100) - ? = 110$. There are 70 in both. Finally, the number of males in band who are not in orchestra is $80 - 70 = 10$.

OR

Make a chart for the given information:

	# in band	#in orchestra	# in both
Male :	80	100	?
Female :	100	80	60
Totals :	180	180	60+?

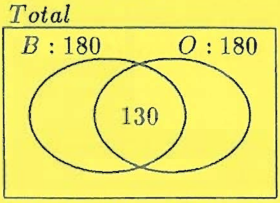
The total number of students is $180 + 180 - (60+?) = 230.$

Solving the equation we obtain $180 + 180 - 60 - ? = 230,$

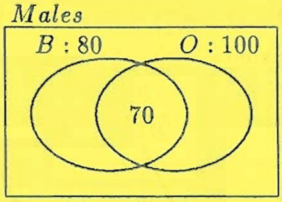
$? = 70.$

Since 70 males are in both band and orchestra, it follows that $80 - 70 = 10$ males are in band who are not in orchestra.

OR

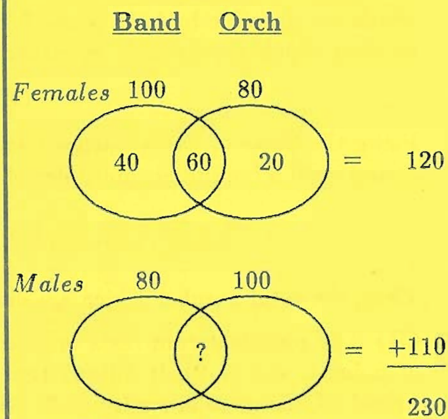


$180+180-230 = 130$ *in both*
$130-60 = 70$ *males in both*



$80-70 = 10$ *males in band only*

OR



There are 130 in both, thus ? equals 70. It follows that 10 males are in band only.

24. **(E)** Since the edge of each smaller cube must be a whole number, the smaller cubes must be $1 \times 1 \times 1$ or $2 \times 2 \times 2$ cubes. There can be only one smaller cube of edge 2, so the rest of the smaller cubes have edge 1. Since the volume of the original cube was $3 \times 3 \times 3 = 27$ cubic cm, and the volume of the cube of edge 2 is $2 \times 2 \times 2 = 8$ cubic cm, then there must be $27 - 8 = 19$ cubes of edge 1 (volume = 1 cubic cm). There are a total of 20 cubes so $N = 20$.

25. **(C)** Since $1/4$ turns white, it follows that $3/4$ remains black after the first change. After the second change, $3/4$ of the remaining $3/4$ stays black, or $\frac{3}{4} \times \frac{3}{4}$ remains black. Thus, after the fifth change, the amount remaining black is

$$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{243}{1024}$$

of the original area.

AMERICAN MATHEMATICS COMPETITIONS

AJHSME SOLUTIONS PAMPHLET
FOR STUDENTS AND TEACHERS

8th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 19, 1992

Sponsored by

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This Solutions Pamphlet gives at least one solution for each problem on this year's Examination and shows that all the problems can be solved using material normally associated with the mathematics curriculum for the eighth grade or lower. These solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

We hope that teachers will share these solutions with their students.

Questions and comments about the problems and solutions (but **not** requests for the Solutions Pamphlet) shoula be addressed to:

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1. **(B)** Group the numerator in pairs from the left, and group the denominator in pairs from the left:

$$\frac{\overbrace{10-9}^{1} + \overbrace{8-7}^{1} + \overbrace{6-5}^{1} + \overbrace{4-3}^{1} + \overbrace{2-1}^{1}}{\underbrace{1-2}_{-1} + \underbrace{3-4}_{-1} + \underbrace{5-6}_{-1} + \underbrace{7-8}_{-1} + 9}$$

Hence, the answer is $\dfrac{5(1)}{4(-1)+9} = \dfrac{5}{5} = 1.$

OR

Regroup the numerator and denominator into positive and negative terms,

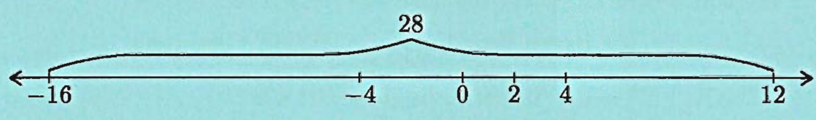
$$\frac{(10+8+6+4+2)-(9+7+5+3+1)}{(1+3+5+7+9)-(2+4+6+8)} = \frac{30-25}{25-20} = \frac{5}{5} = 1.$$

2. **(D)** $1\frac{1}{5} = \frac{6}{5} \neq \frac{5}{4}$.

3. **(D)** To obtain the largest difference, subtract the smallest number, -16, from the largest number, 12. Thus $12-(-16)=28$.

OR

Graphing the numbers on the number line, the difference is represented by the distance between two points. The largest difference would be represented by the longest distance between numbers, which is the distance between -16 and 12, a distance of 28.



4. **(E)** Judy had a total of 35 hits, of which $35-(1+1+5)=28$ were singles. Thus $\dfrac{28}{35} = \dfrac{4}{5}$ or 80% were singles.

OR

Out of 35 hits, $1+1+5=7$ were not singles, so $\dfrac{7}{35} = \dfrac{1}{5}$ or 20% were not singles. Thus $100\% - 20\% = 80\%$ were singles.

5. **(E)** The area of the circle is between 1/2 and 1. To see this, draw squares around and inside the circle. The area of the large square is 1, the area of the small square is 1/2, and the circle fits between the two squares. The area of the rectangle with the circle removed is therefore between 5 and 5.5, so the whole number closest to this area is 5.

OR

The area of the rectangle is $2 \times 3 = 6$, and the area of the circle with radius $\frac{1}{2}$ is $\pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}$, which is slightly larger than $\frac{3}{4}$. Thus the area of the resulting figure is slightly smaller than $6 - \frac{3}{4}$, so it is closest to 5.

6. **(D)** $(1 + 3 - 4) + (2 + 5 - 6) = 0 + 1 = 1$.

OR

Note that 

where $x = a + d$, $y = b + e$ and $z = c + f$; i.e., the sum of two 'triangular expressions' is the value of the 'triangular expression' obtained by summing the respective components. It follows that the required sum is $(1 + 2) + (3 + 5) - (4 + 6) = 3 + 8 - 10 = 1$.

7. **(A)** The only 3-digit whole numbers with a digit-sum of 26 are 899, 989 and 998. Of these, only 998 is even. Thus there is only one such number.

8. **(C)** Since he bought 1500 pencils at \$0.10 each, he paid $1500 \times \$0.10 = \$150$. To make \$100 profit he must take in $\$150 + \$100 = \$250$. Therefore, selling the pencils for \$0.25 each, he must sell $\$250 \div \$0.25 = 1000$ pencils.

9. **(B)** The ratio of males to the total population is 1 to 3. Thus, there are 1/3 of 480, or 160 males in the town.

10. **(B)** The area of each of the small shaded triangles is $\frac{1}{2} \times 2 \times 2 = 2$. There are ten of these, so the shaded area is $2 \times 10 = 20$.

OR

The area of the large triangle is $\frac{1}{2} \times 8 \times 8 = 32$. Only 10 of the 16 small triangles are shaded. Thus the shaded area is $\frac{10}{16}$ of 32, or 20.

11. **(B)** The total frequency for all colors is $50 + 60 + 40 + 60 + 40 = 250$. The frequency for blue is 60. Thus the percent that preferred blue is $60/250$, or 24%.

12. **(C)** The total number of miles of wear is $30,000 \times 4 = 120,000$. Since this wear is shared equally by each of the 5 tires, each tire traveled $120,000 \div 5 = 24,000$ miles.

OR

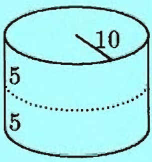
Since each of the tires was on for $\frac{4}{5}$ of the driving, it follows that each was used $\frac{4}{5} \times 30,000 = 24,000$ miles.

13. **(B)** If the mean is 90, then the sum of all five scores is $5 \times 90 = 450$. Since the median of the five scores is 91, at least one score must be 91 and two other scores must be greater than or equal to 91. Since 94 is the mode, there are two scores of 94. The sum of the remaining scores must equal $450 - (94 + 94 + 91) = 171$.

14. **(D)** Since 4 gallons is the difference between being $1/3$ full and $1/2$ full, it follows that 4 gallons is $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ of the capacity of the tank. Thus the capacity of the tank must be 24 gallons.

15. **(C)** The pattern repeats every 9 letters. Dividing 1992 by 9 yields a remainder of 3. Therefore, the 1992^{nd} letter corresponds to the third letter in the sequence, which is C.

16. **(B)** Cylinder **(B)** can be obtained by stacking one copy of the given cylinder on top of another. The formula for the volume of a cylinder with radius r and height h is $V = \pi r^2 h$. Use this to show that none of the other cylinders has twice the volume of the given cylinder:



Cylinder	Volume
Given :	$\pi \times 10^2 \times 5 = 500\pi$
(A) :	$\pi \times 20^2 \times 5 = 2000\pi$
(C) :	$\pi \times 5^2 \times 20 = 500\pi$
(D) :	$\pi \times 20^2 \times 10 = 4000\pi$

Note. If the radius remains the same and the height is doubled, then the volume will double, as in **(B)**. Doubling the radius while the height remains the same will multiply the volume by 4, as in **(A)**.

17. **(B)** For any triangle, the sum of the lengths of any two sides must be greater than the length of the third side. Thus $6.5 + s$ must be greater than 10. The smallest such whole number for s is 4.

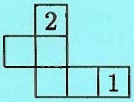
18. **(A)** During the 4 hours, the car traveled a total of $80 + 0 + 100 = 180$ miles for an average speed of $180/4 = 45$ miles per hour.

19. **(C)** There are 21 segments between the 5^{th} and 26^{th} exits. Using the minimum length of 5 miles, 20 segments would yield $20 \times 5 = 100$ miles. This leaves $118 - 100 = 18$ miles for the other segment.

OR

There are 21 segments between the 5^{th} and 26^{th} exits. If each segment were its minimal 5 miles length, then the total distance between the 5^{th} and 26^{th} exits would be 105 miles. Since $118 - 105 = 13$, all 13 additional miles could occur between one pair of consecutive exits. Such a pair would be $5 + 13 = 18$ miles apart.

20. **(D)** Any attempt to fold the squares would result in square 1 being superimposed on square 2. Have students cut and fold the other four patterns into cubes.



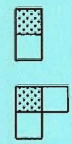
21. **(B)** Compute the ratios for each month:

Month	Drums	Bugles	Diff.	Diff. : Lower	% exc.
Jan :	7	9	2	2 : 7	29%
Feb :	5	3	2	2 : 3	67%
Mar :	9	6	3	3 : 6	50%
Apr :	9	12	3	3 : 9	33%
May :	8	10	2	2 : 8	25%

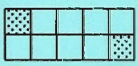
Thus the percent is greatest in February.

Note. Students can estimate the required ratio by visually comparing the difference between the columns to the shorter column.

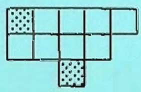
22. **(C)** When a new tile is added to the original figure, it may have one or two sides in common with the given tiles, as shown. When a tile shares one side, the original perimeter is increased by 2. When a tile shares two sides, there is no change in the perimeter. By adding two tiles, the only possible changes to the perimeter are increases of 0, 2 or 4. Hence, the possible values of the perimeter are 14, 16 or 18.



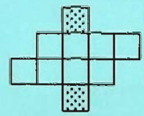
Note. Examples of the three possibilities are shown.



original = 14
new perimeter = 14



original = 14
new perimeter = 16



original = 14
new perimeter = 18

23. **(B)** Make a table and fill in the products greater than 10.

×	**1**	**2**	**3**	**4**	**5**	**6**
1 :	1	2	3	4	5	6
2 :	2	4	6	8	10	**12**
3 :	3	6	9	**12**	**15**	**18**
4 :	4	8	**12**	**16**	**20**	**24**
5 :	5	10	**15**	**20**	**25**	**30**
6 :	6	**12**	**18**	**24**	**30**	**36**

Since there are 17 such products out of a possible 36 products, the probability is 17/36.

24. **(A)** The four quarter-circles that lie inside the square have a total area equal to the area of one of the circles, 9π. The length of a side of the square is equal to two radii, 6, and thus the square has area 36. The difference is $36 - 9\pi < 36 - 9(3) = 9$, so it is closest to 7.7. (The area, to one decimal place, is 7.7.)

25. **(D)** After the first pouring, $\frac{1}{2}$ remains. After the second pouring $\frac{1}{2} \times \frac{2}{3}$ remains. After the third pouring $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$ remains. How many pourings until $\frac{1}{10}$ remains?

$$\frac{1}{\not 2} \times \frac{\not 2}{\not 3} \times \frac{\not 3}{\not 4} \times \frac{\not 4}{\not 5} \times \frac{\not 5}{\not 6} \times \frac{\not 6}{\not 7} \times \frac{\not 7}{\not 8} \times \frac{\not 8}{\not 9} \times \frac{\not 9}{10} = \frac{1}{10}$$

indicates 9 pourings.

OR

Make a table for the information:

Pouring	**Amount Poured**	**Amount Remaining**
1	$\frac{1}{2}$	$1 - \frac{1}{2} = \frac{1}{2}$
2	$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$	$\frac{1}{2} - \frac{1}{6} = \frac{1}{3}$
3	$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$	$\frac{1}{3} - \frac{1}{12} = \frac{1}{4}$
4	$\frac{1}{5} \times \frac{1}{4} = \frac{1}{20}$	$\frac{1}{4} - \frac{1}{20} = \frac{1}{5}$
$\vdots$	$\vdots$	$\vdots$
n	$\frac{1}{n+1} \times \frac{1}{n} = \frac{1}{n(n+1)}$	$\frac{1}{n} - \frac{1}{n(n+1)} = \frac{1}{n+1}$
$\vdots$	$\vdots$	$\vdots$
9	$\frac{1}{10} \times \frac{1}{9} = \frac{1}{90}$	$\frac{1}{9} - \frac{1}{90} = \frac{1}{10}$

Thus $1/10$ remains after the 9^{th} pouring.

AMERICAN MATHEMATICS COMPETITIONS

AJHSME SOLUTIONS PAMPHLET
FOR STUDENTS AND TEACHERS

9th ANNUAL

AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION (AJHSME)

THURSDAY, NOVEMBER 18, 1993

Sponsored by

Mathematical Association of America
Society of Actuaries   Mu Alpha Theta
National Council of Teachers of Mathematics
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This Solutions Pamphlet gives at least one solution for each problem on this year's Examination and shows that all the problems can be solved using material normally associated with the mathematics curriculum for the eighth grade or lower. These solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

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University of Nebraska
Lincoln, NE 68588-0658

1. **(C)** $\frac{1}{2} \times (-72) = -36.$

2. **(C)** $\frac{49}{84} = \frac{7 \times 7}{7 \times 12} = \frac{7}{12}.$
The sum of the numerator and the denominator is $7 + 12 = 19.$

3. **(B)** Factoring each number into prime factors yields

$$39 = 3 \times 13, \quad 51 = 3 \times 17, \quad 77 = 7 \times 11, \quad 91 = 7 \times 13 \quad \text{and} \quad 121 = 11 \times 11.$$

The largest of these prime factors is 17, which is a factor of 51.

4. **(E)**
$$\begin{aligned} 1000 \times 1993 \times 0.1993 \times 10 &= ((1000 \times 10) \times 0.1993) \times 1993 \\ &= (10,000 \times 0.1993) \times 1993 \\ &= 1993 \times 1993 = (1993)^2. \end{aligned}$$

5. **(C)** The unshaded area is half the total, and each of the shaded areas is one fourth of the total. This is represented in bar graph **(C)**.

6. **(B)** Three cans of soup are needed for 15 children, so the remaining 2 cans of soup will feed $2 \times 3 = 6$ adults.

7. **(A)** $3^3 + 3^3 + 3^3 = 3(3^3) = 3(3 \times 3 \times 3) = 3 \times 3 \times 3 \times 3 = 3^4.$

OR

$3^3 + 3^3 + 3^3 = 27 + 27 + 27 = 81 = 9 \times 9 = 3 \times 3 \times 3 \times 3 = 3^4.$

8. **(D)** Since she takes one half of a pill every other day, one pill will last 4 days. Hence 60 pills will last $60 \times 4 = 240$ days, or about 8 months.

9. **(D)** Substituting the values from the table yields

$$(2 * 4) * (1 * 3) = 3 * 3 = 4.$$

Query. Would evaluating the products

$$((2 * 4) * 1) * 3, \quad (2 * (4 * 1)) * 3, \quad 2 * ((4 * 1)) * 3) \quad \text{and} \quad 2 * (4 * (1 * 3)),$$

yield the same result?

10. **(B)** The graph shows the following changes in the price of the card:

Jan :	\$2.50 to \$2.00	**drop of \$0.50**
Feb :	\$2.00 to \$4.00	**rise of \$2.00**
Mar :	\$4.00 to \$1.50	**drop of \$2.50**
Apr :	\$1.50 to \$4.50	**rise of \$3.00**
May :	\$4.50 to \$3.00	**drop of \$1.50**
Jun :	\$3.00 to \$1.00	**drop of \$2.00**

The greatest drop occurred during March.

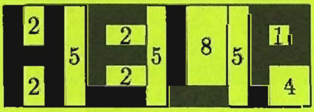
11. **(C)** Since 81 took the test, the median (middle) score is the 41^{st}. The test interval containing the 41^{st} score is labeled 70.

12. **(E)** The six permutations of $+, -$ and $\times$ yield these results:

$$5 \times 4 + 6 - 3 = 20 + 6 - 3 = 23$$
$$5 \times 4 - 6 + 3 = 20 - 6 + 3 = 17$$
$$5 + 4 \times 6 - 3 = 5 + 24 - 3 = 26$$
$$5 - 4 \times 6 + 3 = 5 - 24 + 3 = -16$$
$$5 + 4 - 6 \times 3 = 5 + 4 - 18 = -9$$
$$5 - 4 + 6 \times 3 = 5 - 4 + 18 = 19.$$

The only result listed is 19.

13. **(D)** The white portion can be partitioned into rectangles as shown.



The sum of the areas of the white rectangles is $4(2) + 3(5) + 8 + 1 + 4 = 36$.

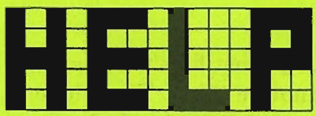
OR

Compute the area of the black letters and subtract it from $5 \times 15 = 75$, the total area of the sign:

H: $2(1 \times 5) + 1 \times 1 = 11$.
E: $1 \times 5 + 3(2 \times 1) = 11$.
L: $1 \times 5 + 1 \times 2 = 7$.
P: $1 \times 5 + 2(1 \times 1) + 1 \times 3 = 10$.
The area of the white portion is $75 - (11 + 11 + 7 + 10) = 36$.

OR

Superimpose a 1×1 grid on the sign and count the 36 white squares:



14. **(C)** Only 3's can complete the 2 by 2 square whose diagonal is given. If two entries in a row or column are known, the third is determined. Use this to complete the table:

1		
	2	

$\Rightarrow$

1	3	
3	2	

$\Rightarrow$

1	3	2
3	2	1
2	1	

$\Rightarrow$

1	3	2
3	2	A=1
2	1	B=3

Thus, $A + B = 4$.

15. **(A)** The sum of the four numbers is $4 \times 85 = 340$, so the sum of the remaining three numbers is $340 - 97 = 243$. Thus the mean of these three numbers is $243/3 = 81$.

16. **(C)** Using the common denominators and simplifying yield

$$\frac{1}{1+\frac{1}{2+\frac{1}{3}}} = \frac{1}{1+\frac{1}{\frac{7}{3}}} = \frac{1}{1+\frac{3}{7}} = \frac{1}{\frac{10}{7}} = \frac{7}{10}.$$

OR

Clearing fractions and simplifying yield

$$\frac{1}{1+\frac{1\times 3}{\left(2+\frac{1}{3}\right)\times 3}} = \frac{1}{1+\frac{3}{6+1}} = \frac{1\times 7}{\left(1+\frac{3}{7}\right)\times 7} = \frac{7}{7+3} = \frac{7}{10}.$$

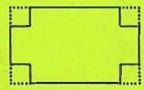
OR

Estimating, the result is between

$$\frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3} \quad \text{and} \quad \frac{1}{1+\frac{1}{3}} = \frac{1}{\frac{4}{3}} = \frac{3}{4}.$$

The only choice in this interval is $\frac{7}{10}$.

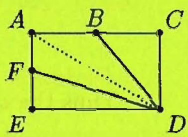
17. **(B)** The interior (or exterior) has the same surface area as one side of the sheet of cardboard after the corners have been removed. The area of the sheet is $30 \times 20 = 600$ and the area of each of the square corners removed is $5 \times 5 = 25$, so the answer is $600 - (4 \times 25) = 500$.



18. **(A)** Rectangle $ACDE$ has area $32 \times 20 = 640$. Triangle BCD has area $(16 \times 20)/2 = 160$, and triangle DEF has area $(10 \times 32)/2 = 160$. The remaining area, $ABDF$, is $640 - (160 + 160) = 320$.

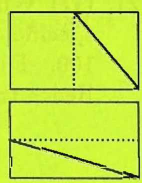
OR

Insert diagonal $\overline{AD}$. The areas of triangles ABD and BCD are $(AB)(CD)/2$ and $(BC)(CD)/2$ which are equal since $AB = BC$. Hence half the area in the rectangle above $\overline{AD}$ is in $ABDF$. Similarly, triangles ADF and DEF have equal areas, and half the area in the rectangle below $\overline{AD}$ is in $ABDF$. Thus, the area of $ABDF$ is $(32 \times 20)/2 = 320$.



OR

Draw a perpendicular from point B to $\overline{ED}$ to show that the area of $\triangle BCD$ is one fourth of the area of $ACDE$. Similarly, draw a perpendicular from point F to $\overline{CD}$ to show that the area of $\triangle DEF$ is one fourth of the area of $ACDE$. Thus the area of $ABDF$ is one half of the area of $ACDE$, or $(32 \times 20)/2 = 320$.



19. **(A)** Each number in the first set of numbers is 1800 more than the corresponding number in the second set:

$$\begin{array}{ccccc} 1901, & 1902, & 1903, & \ldots, & 1993 \\ \underline{-101}, & \underline{-102}, & \underline{-103}, & \ldots, & \underline{-193} \\ 1800, & 1800, & 1800, & \ldots, & 1800 \end{array}$$

Thus the sum of the first set of numbers is $93 \times 1800 = 167,400$ more than the sum of the second set.

20. **(D)** Since $10^{93} = 1\overbrace{00\cdots00}^{93\text{ zeros}}$, we have

$$\begin{array}{r} 100\cdots000 \\ -\quad 93 \\ \hline \underbrace{99\cdots9}_{91\text{ nines}}07 \end{array}$$ and the sum of the digits is $(91 \times 9) + 7 = 826$.

OR

Look for a pattern using simpler cases:

$$\begin{aligned} 10^2 - 93 &= 100 - 93 &&= 07 \\ 10^3 - 93 &= 1,000 - 93 &&= \underline{9}07 \\ 10^4 - 93 &= 10,000 - 93 &&= \underline{9}\underline{9}07 \\ 10^5 - 93 &= 100,000 - 93 &&= \underline{9}\underline{9}\underline{9}07 \\ &\vdots && \vdots \\ 10^{93} - 93 &= &&= \underbrace{999\ldots9}_{91\text{ nines}}07 \end{aligned}$$

Thus the sum of the digits is $(91 \times 9) + 7 = 826$.

21. **(D)** When a problem indicates a general result, then it must hold for any specific case. Therefore, suppose the original rectangle is 10 by 10 with area 100. The new length is $10 + 2 = 12$ and the new width is $10 + 5 = 15$. Hence the new area is $12 \times 15 = 180$ for an increase of 80%.

OR

The length is changed to 120%, or 1.2 times its original value, and the width is changed to 150%, or 1.5 times its original value. Since area is length times width, the new area is $1.2 \times 1.5 = 1.8$ times the original area. Thus the area is increased by 80%.

22. **(D)** Ten 2's are needed in the unit's place in counting to 100 and ten more 2's are used in the ten's place. With the remaining two 2's he can number 102 and 112 and continue all the way to 119 before needing another 2.

23. **(C)** Since P, T and Q must finish in front of S, S cannot be third. Since P is the winner, P cannot be third. Thus the only possible orders are $PRTQS$, $PTRQS$, $PTQRS$ and $PTQSR$, which show that anyone except P and S could finish third.

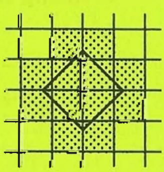
OR

Since $PTQS$ must finish in that order and R can finish anyplace except ahead of P, it follows that the only possible orders are $PRTQS$, $PTRQS$, $PTQRS$, $PTQSR$. Thus T, R and Q might have finished third, but P and S could not have finished third.

24. **(C)** After completing some more rows, the pattern of squares as the last entry in each row becomes apparent. Thus, the line containing 142 ends in 144, and the line above it ends in 121. Therefore 121 is directly above 143, and 120 is above 142.

$$\begin{array}{ccccccccc}
 & & & \boxed{1} & & & & & \\
 & & 2 & 3 & \boxed{4} & & & & \\
 & 5 & 6 & 7 & 8 & \boxed{9} & & & \\
10 & 11 & 12 & 13 & 14 & 15 & \boxed{16} & & \\
 & & & \vdots & & & & & \\
 & & & & \cdots & 119 & 120 & \boxed{121} & \\
 & & & & \cdots & 141 & 142 & 143 & \boxed{144}
\end{array}$$

25. **(E)** Using the Pythagorean Theorem, the length of the diagonal of the card is $\sqrt{(1.5)^2+(1.5)^2} = \sqrt{4.5} \approx 2.1$. This is longer than 2, the length of two adjacent squares. The figure shows 12 squares being touched.



Query. Is 12 the maximum number of squares that can be touched?

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FOR STUDENTS AND TEACHERS

10th ANNUAL
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(AJHSME)

THURSDAY, NOVEMBER 17, 1994

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1. **(D)** Express all the choices as fractions using their least common denominator:

$$\frac{1}{3}=\frac{8}{24},\quad \frac{1}{4}=\frac{6}{24},\quad \frac{3}{8}=\frac{9}{24},\quad \frac{5}{12}=\frac{10}{24},\quad \frac{7}{24}.$$

Thus, $5/12$ is the largest.

OR

Express each choice as a decimal:

$\frac{1}{3}=0.333\ldots$ $\frac{1}{4}=0.25$ $\frac{3}{8}=0.375$ $\frac{5}{12}=0.41666\ldots$ $\frac{7}{24}=0.291666\ldots$

Thus, $5/12$ is the largest.

OR

All choices are less than $1/2$, so the choice least distant from $1/2$ will be the largest:

fraction :	1/3	1/4	3/8	**5/12**	7/24
distance from 1/2 :	1/6	1/4	1/8	**1/12**	5/24

2. **(D)** The sum of all the numerators is 100. Consequently, the sum of all the fractions is $100/10=10$.

OR

Regroup the fractions before adding:

$$\left(\frac{1}{10}+\frac{9}{10}\right)+\left(\frac{2}{10}+\frac{8}{10}\right)+\left(\frac{3}{10}+\frac{7}{10}\right)+\left(\frac{4}{10}+\frac{6}{10}\right)+\left(\frac{5}{10}+\frac{55}{10}\right)=$$

$1+1+1+1+6=10.$

3. **(C)** Maria's working day ends 8 hours and 45 minutes later than 7:25 A.M. Eight hours later than 7:25 A.M. is 3:25 P.M. Forty-five minutes later than 3:25 P.M. is 4:10 P.M.

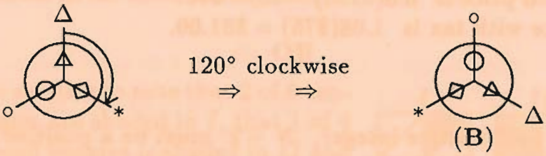
OR

7:25 A.M.	begin
+8:00	work time
+ :45	lunch time
15:70 = 16:10 = 4:10 P.M.	end

OR

Maria's total time at work including lunch is 15 minutes less than 9 hours. Nine hours later than 7:25 A.M. is 4:25 P.M. Fifteen minutes before 4:25 P.M. is 4:10 P.M.

4. **(B)**



5. **(B)** 1 mile = 8 furlongs = $8 \times (1 \text{ furlong}) = 8 \times (40 \text{ rods}) = 320$ rods.

OR

$$1\ \cancel{\text{mile}} \times \frac{8\ \cancel{\text{furlongs}}}{1\ \cancel{\text{mile}}} \times \frac{40 \text{ rods}}{1\ \cancel{\text{furlong}}} = (8 \times 40) \text{ rods} = 320 \text{ rods}.$$

6. **(A)** Any selection of six consecutive positive whole numbers has at least one multiple of 5 and at least one multiple of 2. Thus, the product has at least one factor of $5 \times 2 = 10$ and must end in 0.

OR

The statement of the problem implies that <u>any</u> six consecutive positive numbers can be used. Compute $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 120$ to find that the last digit is 0. Or, even easier, multiply any six consecutive integers containing 10 to see without multiplication that the last digit must be 0.

7. **(B)** Since the sum of the angles in any triangle is 180°,

$$\angle ABE = 180^\circ - (60^\circ + 40^\circ) = 80^\circ.$$

Since $\angle ABD$ and $\angle DBC$ together form a straight angle, their sum is 180°, so $\angle DBC = 180^\circ - 80^\circ = 100^\circ$. Thus $\angle BDC = 180^\circ - (100^\circ + 30^\circ) = 50^\circ$.

8. **(C)** Since the sum of the three digits must equal 25 and $8 + 8 + 8 = 24$, it follows that at least one of the digits must be a 9. If only one digit is 9, then the other two digits add up to 16 and are each 8. If two digits are 9, then the other digit is 7. There are six numbers that meet the requirement:

988, 898, 889, 997, 979, 799.

9. **(A)** The 20% discount lowers the price to \$80.
Then the \$5 coupon reduces the price to \$75.
The sales tax is 8% of \$75, or $0.08 \times \$75 = \$6$.
Thus, the final cost is $\$75 + \$6 = \$81.00$.

OR

The discounted price is $0.8(\$100) - \$5 = \$75$.
The final price with tax is $1.08(\$75) = \$81.00$.

10. **(A)** Since N is a positive integer, $N + 2$ must be a positive integer divisor of 36. There are 9 positive integer divisors of 36:

$$36, \quad 18, \quad 12, \quad 9, \quad 6, \quad 4, \quad 3, \quad 2, \quad 1.$$

Express these numbers in the form $N + 2$:

$$34{+}2, \quad 16{+}2, \quad 10{+}2, \quad 7{+}2, \quad 4{+}2, \quad 2{+}2, \quad 1{+}2, \quad 0{+}2, \quad -1{+}2.$$

Of these, seven have a positive value for N.

OR

Given $N > 0$, it follows that $N + 2 > 2$. Since $N + 2$ must be a divisor of 36, there are 7 such divisors greater than 2: $3, 4, 6, 9, 12, 18, 36$.

11. **(B)** Since the total number of girls was 48 and there were 20 girls from Jones, it follows that there were $48 - 20 = 28$ girls from Clay. The total number of students from Clay was 60. Thus, there were $60 - 28 = 32$ boys from Clay.

OR

Complete a table starting with the given information:

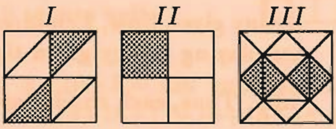
	B	G	Tot		B	G	Tot		B	G	Tot
JMS :		20	40	**JMS :**	$\boxed{20}$	20	40	**JMS :**	20	20	40
CMS :			60	**CMS :**			60	**CMS :**	$\boxed{32}$		60
Total :	52	48	100	**Total :**	52	48	100	**Total :**	52	48	100

There were 32 boys from Clay Middle School.

12. **(A)** The shaded area of square II is clearly 1/4 of the total area. In square I each shaded triangle is 1/2 of 1/4 or 1/8 of the total area, so the total shaded area is $\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$ of the total. In square III, each shaded diamond can be visualized as two triangles. Each of these triangles is 1/4 of 1/4 or 1/16 of the total area, so the four triangles make up 4/16 or 1/4 of the total. Thus the shaded areas in all three are equal.

OR

Partition the squares to note that 2 of 8 congruent triangles are shaded in I, that 1 of 4 congruent small squares is shaded in II and that 4 of 16 congruent triangles are shaded in III.



13. **(C)** Since $\frac{1}{6}=\frac{2}{12}=\frac{4}{24}$ and $\frac{1}{4}=\frac{3}{12}=\frac{6}{24}$, it follows that the number halfway between $\frac{1}{6}$ and $\frac{1}{4}$ is $\frac{5}{24}$.

OR

The number halfway between any two numbers is their arithmetic mean (average):

$$\frac{\frac{1}{6}+\frac{1}{4}}{2}=\frac{\frac{4+6}{24}}{2}=\frac{\frac{10}{24}}{2}=\frac{5}{24}.$$

OR

Estimating each fraction to three decimal places and calculating the average yields

$$\frac{\frac{1}{6}+\frac{1}{4}}{2}\approx\frac{0.167+0.250}{2}=\frac{0.417}{2}=0.2085$$

which is closest to $\frac{5}{24}\approx 0.2083$.

14. **(E)** Two children are playing at the same time, so there are $2\times 90=180$ minutes of playing time. This must be divided equally 5 ways, so each child gets $180/5=36$ minutes.

OR

If pairball were played by one person at a time, each child would get 1/5 of 90, or 18 minutes of playing time. Since two players are required, each child gets double this, or 36 minutes.

OR

There are 10 ways to pair the children, so each session lasts $90/10 = 9$ minutes. Each child is paired with four other children, so each child will play for $4 \times 9 = 36$ minutes.

Query. What are the 10 ways to pair the five children?

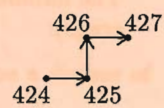
OR

At any given time, 2 children are playing and 3 are not, so $2/5$ of the children are playing at any given time. This implies that each child plays $2/5$ of the time. Thus, each child plays $\frac{2}{5} \times 90 = 36$ minutes.

15. **(A)** The path repeats every four numbers. Since 425 leaves a remainder of 1 when divided by 4 and 427 leaves a remainder of 3 when divided by 4, it follows that the pattern from point 425 to point 427 is the same as that from point 1 to point 3.

OR

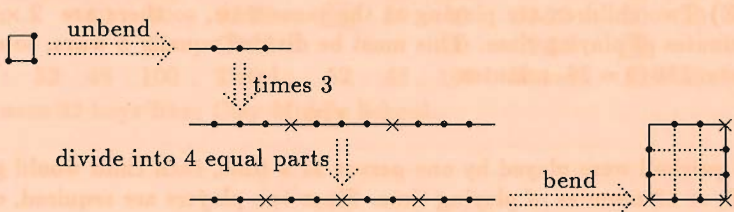
The pattern starts over every four numbers, that is, at 0, 4, 8, etc. All the multiples of 4 are in the same position in their section of the pattern, so draw the portion of the diagram using the multiple of 4 just less than 425.



16. **(E)** The perimeter being 3 times larger implies that a side of the larger square is 3 times a side of the smaller square. Thus, since the area of a square is the length of the side squared, it follows that the area of the larger square will be $3^2 = 9$ times the area of the smaller square.

OR

The sketch shows that when the perimeter is tripled, it encloses 9 squares equal to the original square:



OR

Since the ratio of areas of similar figures is the square of the ratio of any matching linear part, it follows that the ratio of the areas is $\left(\frac{3}{1}\right)^2 = \frac{9}{1}$.

OR

Use a sample case: Area of a 1 by 1 square is 1 square unit.
Area of a 3 by 3 square is 9 square units.

17. **(D)** The volume of the snow on the driveway is $4 \times 10 \times 3 = 120$ cubic yards. Adding the rates for 7 consecutive hours yields $20+19+18+17+16+15+14 = 119$, while 8 consecutive hours yields $20+19+18+17+16+15+14+13 = 132$. The 7-hour solution is closest to 120 cubic yards.

OR

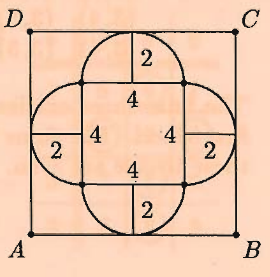
The volume of the snow on the driveway is $4 \times 10 \times 3 = 120$ cubic yards. Computing the amount left after each hour, we have:

$$120 - 20 = 100, \quad 100 - 19 = 81, \quad 81 - 18 = 63, \quad 63 - 17 = 46,$$
$$46 - 16 = 30, \quad 30 - 15 = 15, \quad 15 - 14 = 1, \quad 1 - 13 = -12.$$

Since 1 is closer to zero than is -12, the 7-hour solution is closest to 120 cubic yards.

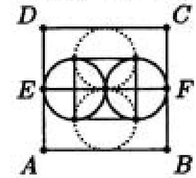
18. **(B)** Both graphs **(A)** and **(B)** show a flat segment indicating no change in distance, or a stop. Graph **(A)** shows a constant change in distance indicating a constant rate of driving before and after the stop. Graph **(B)** shows a slow change in distance (shallow graph) followed by a more rapid change in distance (steep graph) indicating a slower rate followed by a faster rate of driving before the stop. After the stop, it shows a faster rate of driving followed by a slower rate. Thus, graph **(B)** corresponds to Mike's trip.

19. **(E)** The radius of each semicircle is 2, since it is 1/2 the length of a side of the 4 by 4 square. Since the length of a side of $ABCD$ is the length of a side of the 4 by 4 square plus two radii of semicircles [see figure], each side of $ABCD$ measures $4 + 2(2) = 8$, so the area of $ABCD$ is $8^2 = 64$.



OR

Complete each semicircle to a circle, and note that since the radius of each circle is half the side-length of the smaller square, the four circles must intersect at the center of both squares. Thus, $AB = EF = 8$ since the length of $\overline{EF}$ is the length of two diameters. Hence, the area of $ABCD$ is $8^2 = 64$.



20. **(D)** Small numerators and large denominators yield small fractions. Use 1 and 2 for numerators and 8 and 9 for denominators to obtain the smallest fractions, then compare the sums

$$\frac{1}{8}+\frac{2}{9}=\frac{9+16}{72}=\frac{25}{72} \qquad \text{and} \qquad \frac{1}{9}+\frac{2}{8}=\frac{8+18}{72}=\frac{26}{72}$$

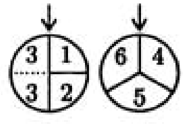
to see that $\frac{25}{72}$ is the answer.

Note. Analyzing the sums yields $\frac{1}{8}+\frac{1}{9}+\frac{1}{9}$ which is smaller than $\frac{1}{9}+\frac{1}{8}+\frac{1}{8}$.

21. **(C)** It is *possible* to get four gumballs of the same color by buying 4, 5, 6, 7, 8, or 9. However, the first nine gumballs *might* consist of three of each color, so nine gumballs will *not guarantee* four of the same color. In this case, the tenth gumball must match one of the previous colors, giving four of that color.

22. **(D)** Subdivide the 3-space on the first wheel so that wheel is divided into four equal regions. Each of the four regions has the same probability of occurring when the first wheel is spun. Hence, the sample space is

(1,4)	(1,5)	(1,6)
(2,4)	(2,5)	(2,6)
(3,4)	(3,5)	(3,6)
(3,4)	(3,5)	(3,6)



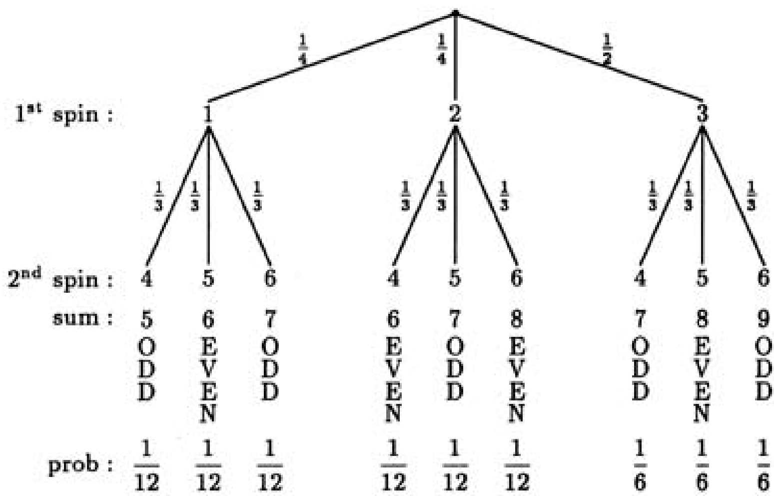
The final three entries were repeated to show the double space for 3 on the first wheel. The sums for these entries are 5, 6, 7, 6, 7, 8, 7, 8, 9, 7, 8, 9. Five of these twelve are even.

OR

There are two ways to get an even sum, (odd + odd) or (even + even). The first outcome happens if one spins 1 or 3 on the first wheel (3 chances out of 4) and 5 on the second wheel (1 chance out of 3) for a probability of $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$. The second outcome happens if one spins a 2 on the first wheel (1 chance out of 4) and 4 or 6 on the second wheel (2 chances out of 3) for a probability of $\frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$. Thus the total probability of an even sum is $\frac{1}{4} + \frac{1}{6} = \frac{5}{12}$.

OR

Use a probability tree diagram:



Thus, the probability that the sum is even is $\frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{6} = \frac{5}{12}$.

OR

List equally likely outcomes in a table. Even sums are marked. The probability is 5/12.

+	4	5	6
1	5	(6)	7
2	(6)	7	(8)
3	7	(8)	9
3	7	(8)	9

OR

$$P(\text{even}) = P(3+5) + P(1+5) + P(2+4) + P(2+6)$$
$$= \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{3}\right) = \frac{5}{12}.$$

23. **(D)** Since the sum must be only three digits, it follows that $X \neq 9$ because $X = 9$ would give a four-digit sum. Thus the largest value for X must be 8. Then, to obtain the largest sum, $Y = 9$. This yields

$$\begin{array}{r} 888 \\ 98 \\ +\ \ 8 \\ \hline 994 \end{array}$$

Thus, the largest sum has the form YYZ.

24. **(B)** If there is any square painted green, then all the squares above or to the right of it must also be green. Therefore, the possible patterns of colors are:

$$\begin{bmatrix} R & R \\ R & R \end{bmatrix} \quad \begin{bmatrix} R & G \\ R & R \end{bmatrix} \quad \begin{bmatrix} G & G \\ R & R \end{bmatrix}$$
$$\begin{bmatrix} R & G \\ R & G \end{bmatrix} \quad \begin{bmatrix} G & G \\ R & G \end{bmatrix} \quad \begin{bmatrix} G & G \\ G & G \end{bmatrix}$$

Thus, there are 6 different ways to paint the squares according to the requirements of the problem.

OR

All the green squares on any row must be to the right. The number of green squares in any row must be at least as large as the number of green squares in any lower row. Therefore, the number of ways to paint n squares green is the number of sums $a + b = n$ with $2 \geq a \geq b \geq 0$, and n can be any number from 0 through 4. There are 6 such sums:

$$\begin{array}{lll} 0+0=0 & 1+0=1 & 2+0=2 \\ 1+1=2 & 2+1=3 & 2+2=4 \end{array}$$

Therefore, there are 6 ways to paint the 2 by 2 square according to the requirements of the problem.

Query. What if the original square were 3 by 3? 4 by 4?

25. **(A)** Since $\underbrace{9999\cdots99}_{\text{94 nines}} = 1\underbrace{0000\cdots00}_{\text{94 zeros}} - 1$, we have

$$\underbrace{9999\cdots99}_{\text{94 nines}} \times \underbrace{4444\cdots44}_{\text{94 fours}} = (1\underbrace{0000\cdots00}_{\text{94 zeros}} - 1) \times \underbrace{4444\cdots44}_{\text{94 fours}}$$
$$= (\underbrace{4444\cdots44}_{\text{94 fours}}\underbrace{0000\cdots00}_{\text{94 zeros}} - \underbrace{4444\cdots44}_{\text{94 fours}}$$

which is

$$\begin{array}{r} 4444\cdots44\,0000\cdots00 \\ -\ 4444\cdots44 \\ \hline \underbrace{4444\cdots4}_{\text{93 fours}}3\underbrace{5555\cdots5}_{\text{93 fives}}6 \end{array}$$

The sum of the digits of this answer is

$$93(4) + 3 + 93(5) + 6 = 93(4+5) + (3+6) = 94(9) = 846.$$

OR

Try smaller cases to observe a pattern:

$$\begin{array}{ccccc} 9 \times 4 & = & 36 & \text{sum} = & 9 = 9 \times 1 \\ 99 \times 44 & = & 4356 & & = 18 = 9 \times 2 \\ 999 \times 444 & = & 443556 & & = 27 = 9 \times 3 \\ 9999 \times 4444 & = & 44435556 & & = 36 = 9 \times 4 \\ \vdots & = & \vdots & & \\ \underbrace{9999\cdots99}_{\text{94 nines}} \times \underbrace{4444\cdots44}_{\text{94 fours}} & = & \underbrace{4444\cdots4}_{\text{93 fours}}3\underbrace{5555\cdots5}_{\text{93 fives}}6 & & = 9 \times 94 \end{array}$$

The sum of the digits of this answer is

$$93(4) + 3 + 93(5) + 6 = 93(4+5) + (3+6) = 94(9) = 846.$$

Query. What is the sum of the digits when any 94-digit number is multiplied by $\underbrace{9999\cdots99}_{\text{94 nines}}$?

AMERICAN MATHEMATICS COMPETITIONS

AJHSME SOLUTIONS PAMPHLET
FOR STUDENTS AND TEACHERS

11th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 16, 1995

Sponsored by

Mathematical Association of America
Society of Actuaries   Mu Alpha Theta
National Council of Teachers of Mathematics
Casualty Actuarial Society   American Statistical Association
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Prof Walter E Mientka, AMC Executive Director
Department of Mathematics and Statistics
University of Nebraska
Lincoln, NE 68588-0658

1. **(D)**. The total value of the coins is

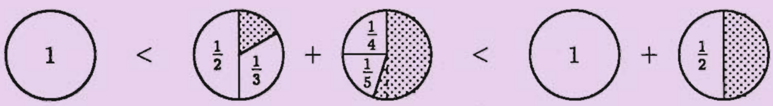
$$\$0.01 + \$0.05 + \$0.10 + \$0.25 = \$0.41,$$

which is $0.41/1.00$ or $41/100 = 41\%$ of a dollar.

2. **(C)** Zack must be $15+3=18$ years old, so Jose must be $18-4=14$ years old.

3. **(E)** Since $\frac{3}{4} \div \frac{3}{5} = \frac{3}{4} \times \frac{5}{3} = \frac{5}{4}$, multiplying by $\frac{5}{4}$ has the same effect.

4. **(C)** Ben adds 1 to 6 to get 7, and then doubles 7 to get 14. Sue subtracts 1 from 14 to get 13, and then doubles 13 to get 26.

5. **(C)** 

The sum of the fractions adds between 1 and $1\frac{1}{2}$ to the sum of the whole numbers, which is $2+3+4+5=14$. Thus the overall sum is between 15 and $15\frac{1}{2}$, so the answer is 16.

OR

Since

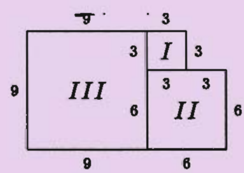
$$1 < \frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\frac{1}{5} < \mathbf{\frac{1}{2}}+\mathbf{\frac{1}{3}}+\mathbf{\frac{1}{4}}+\mathbf{\frac{1}{5}} < \frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2} = 2,$$

the sum of the fractions adds between 1 and 2 to the sum of the whole numbers, which is $2+3+4+5=14$. Thus the overall sum is between 15 and 16, so the answer is 16.

OR

Approximate the fractions as decimals and add $2.5+3.33+4.25+5.2$ which yields 15.28. Thus the answer is 16.

6. **(C)** Since the perimeter of I is 12, its side is 3. Similarly, the side of II is 6. Hence the side of III is $3+6=9$. Thus the perimeter of III is $4 \times 9 = 36$.



Query. Is it a coincidence that the perimeter of III equals the sum of the perimeters of I and II?

7. **(B)** $\frac{1}{2}+\frac{1}{4}+\frac{1}{10}=\frac{10}{20}+\frac{5}{20}+\frac{2}{20}=\frac{17}{20}$. The rest, $\frac{20}{20}-\frac{17}{20}=\frac{3}{20}$, walk home.

OR

Since $\frac{1}{2}$ or 50% of the students use the bus,

$\frac{1}{4}$ or 25% of the students use an auto, and

$\frac{1}{10}$ or 10% of the students use a bicycle,

it follows that $100\% - (50\% + 25\% + 10\%) = 15\%$ or $\frac{3}{20}$ of the students walk home.

OR

Assume there are 100 students. This yields

$\frac{1}{2}$		$\frac{1}{4}$		$\frac{1}{10}$				
50	+	25	+	10	+	?	=	100 total.
bus		automobile		bicycle		walk		

Thus 15 walk home, which gives $\frac{15}{100}=\frac{3}{20}$.

8. **(D)** $\$1.00 = \$1.00 \times \frac{3000 \text{ lire}}{\$1.60} = 1875$ lire.

OR

$\frac{\$1.00}{\$1.60}=\frac{5}{8}$, so \$1.00 is $\frac{5}{8}$ of \$1.60. Thus, the number of lire is $\frac{5}{8}$ of 3000 or 1875 lire.

OR

Use the proportion $\frac{3000 \text{ lire}}{160 \text{ cents}}=\frac{x \text{ lire}}{100 \text{ cents}}$. Solving for x gives 1875 lire.

9. **(C)** Since the length of $\overline{BC}$ is the same as the diameter of the circle with center Q, it follows that $BC = 4$. Since the circles with centers P and R are tangent to the parallel sides $\overline{AB}$ and $\overline{DC}$, the diameters of these circles are also 4. The sum of the diameters of the circles with centers P and R gives the length of $\overline{AB}$, so $AB = 4 + 4 = 8$. Hence the area of the rectangle is $8 \times 4 = 32$.

OR

The radius of the circle with center Q is $4/2 = 2$, so $PQ = RQ = 2$. But $\overline{PQ}$ and $\overline{RQ}$ are also radii of the circles with centers P and R, respectively, so all three circles have radius 2. Hence $AB = 8$ and $BC = 4$, so the area of the rectangle is $8 \times 4 = 32$.

10. **(A)** For the jacket, 40% of \$80 is a savings of \$32. For the shirt, 55% of \$40 is a savings of \$22. The total savings is \$32 + \$22 = \$54. The total of the original prices is \$120. Thus, \$54/\$120 = 0.45 = 45%.

11. **(D)** Jane covers twice as much distance as Hector in the same time. So Jane goes to A then B, as Hector goes to E. Jane continues to C then D as Hector goes to D.

OR

Jane covers twice as much distance as Hector in the same time. When they meet, they will have covered 18 blocks. Since $12 + 6 = 18$ and 12 is twice 6, they must meet 6 blocks counterclockwise from the start, and this is point D.

12. **(E)** The last two digits of 1994 can only be factored as $94 = 2 \times 47$. All the other choices have at least one date that makes them lucky:
    (A) 9/10/90 **(B)** 7/13/91 **(C)** 4/23/92 **(D)** 3/31/93

13. **(E)** In $\triangle BDE$, $\angle BED + \angle BDE + \angle B = 180°$. Since $\angle BED = \angle BDE$ and $\angle B = 90°$, it follows that $\angle BED = \angle BDE = 45°$. In $\triangle AEF$, $\angle A + \angle AEF + \angle AFE = 180°$. Since $\angle A = 90°$ and $\angle AEF = 40°$, it follows that $\angle AFE = 50°$. Consequently $\angle BFG = 50°$ in $\triangle BFG$ and, since $\angle B = 90°$, it follows that $\angle BGF = 40°$. Consequently $\angle CGD = 40°$ in $\triangle CDG$, and since $\angle C = 90°$, it follows that $\angle CDG = 50°$. Thus $\angle CDE = 50° + 45° = 95°$.

OR

As in the first solution, $\angle BED = \angle BDE = 45°$. Then $\angle AED = 40° + 45° = 85°$. Since the four angles of a quadrilateral sum to $360°$, we have $\angle A + \angle C + \angle AED + \angle CDE = 360°$. Thus $\angle CDE = 360° - 90° - 90° - 85° = 95°$.

14. **(B)** The total number of games for the season is $50+40=90$ games. Since 70% of 90 games is 63, it follows that $63-40=23$ more wins are needed.

15. **(B)** Since $4/37=0.108108\ldots$, it follows that the 3^{rd}, 6^{th}, 9^{th} $\ldots$, 99^{th} digits are 8. Thus the 100^{th} digit must be 1.

16. **(C)** Tabulate the data:

Allen School	: 7 students for 3 days	=	21 worker days
Balboa School	: 4 students for 5 days	=	20 worker days
Carver School	: 5 students for 9 days	=	45 worker days
	Total :		86 worker days

Hence, $\$774\div 86=\$9$ per worker day. Thus the students from Balboa School earned \$9 per worker day for 20 worker days, for a total of \$180.

17. **(D)** In Annville 11% of 100, or 11 students are in the 6^{th} grade. In Cleona 17% of 200, or 34 students are in the 6^{th} grade. Thus in the two schools combined, 45 out of 300 students are in the 6^{th} grade, so $45/300=0.15=15\%$ of all the students are in the 6^{th} grade.

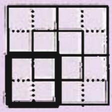
OR

Since 1/3 of the students are at Annville and 2/3 are at Cleona, using a weighted average yields $\frac{1}{3}(0.11)+\frac{2}{3}(0.17)=0.15$ or 15%.

18. **(C)** The four L-shaped regions account for $4\times\left(\frac{3}{16}\right)=\frac{12}{16}=\frac{3}{4}$ of the total area. That leaves $\frac{1}{4}$ of the total area for the center square, which yields $\left(\frac{1}{4}\right)(100\times 100)=2500$ square inches. Thus the length of the side of the center square is $\sqrt{2500}=50$ inches.

OR

As in the first solution, the center square is 1/4 of the total area. Dividing the large square into four equal parts as shown yields a square that is 50 by 50 inches.



19. **(D)** Putting the number of children in each family in order from least to greatest yields

$$1, 1, 2, 3, 3, 4, \boxed{4}, 5, 5, 5, 5, 5, 5.$$

The median is the middle, or 7^{th} value, which is 4.

OR

The graph shows 2 families with 1 child, 1 with 2, 2 with 3, 2 with 4 and 6 with 5. Since there are $2 + 1 + 2 + 2 + 6 = 13$ families under consideration, the middle entry is the 7^{th} entry, which is a family with 4 children.

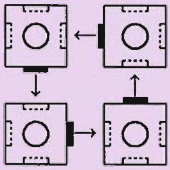
20. **(B)** There are $6 \times 6 = 36$ possible outcomes of rolling the dice. Since Diana and Apollo roll the same number in 6 of these, there are 30 in which the numbers on the two dice are different. By symmetry, Diana's number is larger than Apollo's number in exactly half of these. Thus the requested probability is $\frac{15}{36} = \frac{5}{12}$.

OR

Let (d, a) represent *"Diana rolled d and Apollo rolled a."* List the 36 outcomes and mark those where $d > a$.

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	**(3,2)**	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	**(4,2)**	**(4,3)**	(4,4)	(4,5)	(4,6)
(5,1)	**(5,2)**	**(5,3)**	**(5,4)**	(5,5)	(5,6)
(6,1)	**(6,2)**	**(6,3)**	**(6,4)**	**(6,5)**	(6,6)

Since there are 15 marked pairs, the probability that Diana rolls a larger number than Apollo is $15/36 = 5/12$.



21. **(B)** Using one, two or three cubes always leaves one protuding snap showing. The smallest number of cubes is four, arranged as shown (viewed from above).

22. **(A)** The prime factorization of 6545 is $5 \times 7 \times 11 \times 17$. Since the product of any three of these primes is a three-digit number and since 7×17 and 11×17 are both three-digit numbers, it follows that the only pair of two-digit numbers with product 6545 is $5 \times 17 = 85$ and $7 \times 11 = 77$. Thus the answer is $85 + 77 = 162$.

23. **(B)** There are 5 odd and 5 even digits that can be used for the two leftmost digits in the number. Once an odd and even digit have been selected and since all four digits are different, there are 8 choices remaining for the third digit, and then 7 choices for the fourth digit. Thus there are $5 \times 5 \times 8 \times 7 = 1400$ such whole numbers.

    Query. How many four-digit whole numbers have an even leftmost digit, an odd second digit, and four different digits? (It is NOT 1400.)

24. **(C)** Since opposite sides of a parallelogram are equal, $AB = 12$. Then $AE = 12 - 4 = 8$. Using the Pythagorean Theorem gives $AD = \sqrt{8^2 + 6^2} = 10$, and then $BC = 10$ also. The area of a parallelogram is base $\times$ altitude. Using base $AB = 12$ and altitude $DE = 6$ gives an area of $12 \times 6 = 72$. Using base $BC = 10$ and altitude $\overline{DF}$ must also give an area of 72. Thus $DF = 72/10 = 7.2$.

25. **(D)** A Houston-bound bus leaving Dallas at 6:00 p.m., for example, will arrive in Houston at 11:00 p.m., having passed buses that left Houston at 1:30 p.m., 2:30 p.m., 3:30 p.m., ..., 10:30 p.m. That is 10 buses.

AMERICAN MATHEMATICS COMPETITIONS

AJHSME SOLUTIONS PAMPHLET
FOR STUDENTS AND TEACHERS

12th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 21, 1996

Sponsored by

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American Mathematics Competitions
University of Nebraska-Lincoln
P.O. Box 81606
Lincoln, NE 68501-1606

1. **(B)** The positive factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36. Of these, 4, 12, and 36 are also multiples of 4.

OR

List the multiples of 4 through 36, then mark those which are factors of 36:

$$\boxed{4},\ 8,\ \boxed{12},\ 16,\ 20,\ 24,\ 28,\ 32,\ \boxed{36}.$$

OR

Since $36 = 2^2 3^2$, its positive factors are:

$$\begin{array}{lll} 2^0 3^0, & 2^1 3^0, & \boxed{2^2 3^0}, \\ 2^0 3^1, & 2^1 3^1, & \boxed{2^2 3^1}, \\ 2^0 3^2, & 2^1 3^2, & \boxed{2^2 3^2}. \end{array}$$

Of these, only three are also multiples of $4 = 2^2$.

2. **(C)** Starting with 10:
   - José computes, consecutively, 9, then 18, and finally 20.
   - Thuy computes, consecutively, 20, then 19, and finally 21.
   - Kareem computes, consecutively, 9, then 11, and finally 22.

   Thus Kareem gets the largest final answer.

   Note. Any number n could have been used instead of 10 to obtain the same result.

	José	**Thuy**	**Kareem**
Start :	n	n	n
First :	$n-1$	$2n$	$n-1$
Then :	$2(n-1) = 2n-2$	$2n-1$	$(n-1)+2 = n+1$
Finally :	$(2n-2)+2 = \underline{2n}$	$(2n-1)+2 = \underline{2n+1}$	$2(n+1) = \underline{2n+2}$

Since $2n+2 > 2n+1 > 2n$, Kareem gets the largest final answer.

3. **(A)** The first row is $1, 2, 3, \ldots, 7, 8$ and the last row is $57, 58, 59, \ldots, 63, 64$. Thus the four corner numbers are 1, 8, 57, and 64, and their sum is 130.

OR

Listing the array yields:

1	2	3	4	5	6	7	**8**
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	**64**

The sum of the numbers in the corners is 130.

4. **(B)** $\dfrac{2+4+6+\cdots+34}{3+6+9+\cdots+51} = \dfrac{2(1+2+3+\cdots+17)}{3(1+2+3+\cdots+17)} = \dfrac{2}{3}$.

5. **(A)** Examine the sign of each choice. Since P is to the left of Q, $P-Q$ is negative. All the other answers are positive:
   - Since P and Q are both negative, their product **(B)** is positive.
   - Since S and Q are of opposite signs, their quotient is negative. When this quotient is multiplied by a negative number, P, the result **(C)** is positive.
   - Since R and $P \cdot Q$ are both positive, their quotient **(D)** is positive.
   - Since $S+T$ and R are both positive, their quotient **(E)** is positive.

6. **(C)** To obtain the smallest result, use the three smallest numbers. This yields three choices:

$$3(5+7) = 36, \qquad 5(3+7) = 50, \quad \text{and} \quad 7(3+5) = 56.$$

Thus 36 is the smallest result.

OR

Since multiplication is repeated addition, it follows that the smallest result should use the smallest number as the multiplier and the other two of the three smallest numbers for the sum. Thus $3(5+7) = 36$ is the smallest result.

7. **(B)** Make a table.

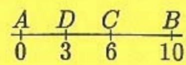
Month:	**0**	**1**	**2**	**3**	**4**	**5**
Brent:	4	16	64	256	1024	4096
Gretel:	128	256	512	1024	2048	4096

Thus it takes 5 months for the number of goldfish to be equal.

OR

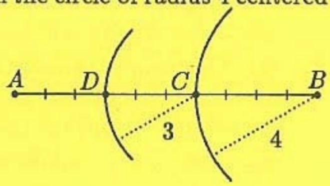
The ratio of the number of Gretel's goldfish to the number of Brent's goldfish decreases by a factor of 2 every month. Because they initially differ by a factor of $128/4 = 32 = 2^5$, this indicates that the process will take 5 months.

8. **(B)** The shortest distance occurs when the four points are collinear. Then $BC = 4$ and $CD = 3$ so that $AD = 3$ as shown.



OR

Once A and B are chosen, C can be anywhere on the circle of radius 4 centered at B. The closest point to A would be on the segment between B and A. Then D could be anywhere on the circle of radius 3 centered at C. Once again the closest point to A is on the line segment between C and A. Thus the distance from A to D is 3 units.

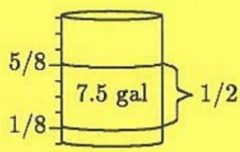


9. **(D)** To find the number, divide 2 by 5 to obtain 2/5. The reciprocal of 2/5 is 5/2, and 100 times 5/2 equals 250.

OR

Let n be the unknown number. Then $5 \cdot n = 2$ so $n = 2/5$. The reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$, and $100 \cdot \frac{5}{2} = 250$.

10. **(D)** The gasoline tank going from $1/8$ to $5/8$ full represents an increase of $5/8 - 1/8 = 1/2$ tank. Since half a tank is 7.5 gallons, it follows that a full tank is $2 \times 7.5 = 15$ gallons.



11. **(D)** Since x is near zero, $3 + x$ and $3 - x$ are near 3. Also $3 \cdot x$ and $x/3$ are near zero. However, $3/x$ is the number

$$\underbrace{30000\ldots0000}_{\text{1997 zeros}},$$

which is a 3 followed by 1997 zeros. This number is much larger than any of the other alternatives.

12. **(B)** The sum of the eleven numbers is 66. For the average of ten numbers to be 6.1, the sum of the ten numbers must be $10 \times 6.1 = 61$. Thus, remove the 5.

13. **(E)** Since 50% of 800 is 400,

the organizers expect $800 + 400 = 1200$ participants in 1997.
Since 50% of 1200 is 600, $1200 + 600 = 1800$ are expected in 1998.
Since 50% of 1800 is 900, $1800 + 900 = 2700$ are expected in 1999.

OR

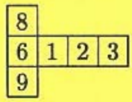
Make a table.

YEAR	**NUMBER OF PARTICIPANTS**
1996	800
1997	$800 + (0.5 \times 800) = 1200$
1998	$1200 + (0.5 \times 1200) = 1800$
1999	$1800 + (0.5 \times 1800) = 2700$

OR

An increase of 50% means there will be $100\% + 50\%$ or 1.5 times as many participants in each successive year. Thus in 1999, three years from 1996, there will be $800 \times 1.5^3 = 2700$ participants.

14. **(B)** Determine that one suitable arrangement of the digits is as indicated, and then compute



$$8 + 6 + 9 + 1 + 2 + 3 = 29.$$

OR

Since $7 + 8 + 9 = 24$, the digits in the column must be 6, 8, and 9 in some order. The sum of the digits in the row that are not also in the column must be at least $1 + 2 + 3 = 6$. Then the square common to both the column and the row contains at most $12 - (1 + 2 + 3) = 12 - 6 = 6$. Therefore, the 8 or 9 cannot be used in the common square, and hence that square must contain the digit 6. Thus the digits are 1, 2, 3, 6, 8, and 9, which have a sum of 29.

15. **(E)** The last digit of a product is determined by the product of the last digits of the factors. Since $2 \cdot 6 \cdot 2 \cdot 6 = 144$, the last digit of the product is 4. Since multiples of 5 end in 0 or 5, any number with last digit 4 leaves a remainder of 4 when divided by 5.

16. **(C)** Combining in groups of four yields

$$1-2-3+4=0, \quad 5-6-7+8=0, \quad 9-10-11+12=0,$$

and so on. Since there are 499 groups of four in 1996, it follows that the sum is zero.

OR

Combining in pairs yields

$$\underbrace{\underbrace{(1-2)+(-3+4)}_{(-1)+1}}_{0} + \underbrace{\underbrace{(5-6)+(-7+8)}_{(-1)+1}}_{0} + \cdots + \underbrace{\underbrace{(1993-1994)+(-1995+1996)}_{(-1)+1}}_{0}$$

so the sum is 0.

17. **(C)** Since $OPQR$ is a square and point Q has coordinates $(2,2)$, it follows that point P has coordinates $(2,0)$ and point R has coordinates $(0,2)$. Thus the side of square $OPQR$ is 2 and the area of the 2 by 2 square $OPQR$ is $2^2=4$. The area of triangle PQT is $(1/2)(PT)(PQ)=(1/2)(PT)(2)=PT$. Since the area of the triangle equals the area of the square, $PT=4$. Thus the point T has coordinates $(-2,0)$.

OR

If the area of square $OPQR$ and triangle PQT are equal, then the area of the small triangle with side $\overline{RQ}$ that is in the square but not in the triangle must equal the area of the small triangle with side $\overline{OT}$ that is in the triangle but not in the square. These two triangles will be the same size and shape when $\overline{QT}$ intersects $\overline{RO}$ at its midpoint and $OT=RQ=2$, so T must have coordinates $(-2,0)$.

18. **(A)** After the first change, Ana's salary was $\$2000+0.20(\$2000)=\$2400$. After the second change, Ana's salary was $\$2400-0.20(\$2400)=\$1920$.

OR

After the two changes, Ana's salary was 80% of 120% of \$2000, or $0.80(1.20)(\$2000)=\$1920$.

19. **(C)** The first pie chart shows that 22% of 2000, or 440 students at East prefer tennis. The second chart shows that 40% of 2500, or 1000 students at West prefer tennis. Thus 1440 of the total of 4500 students prefer tennis. This gives $1440/4500=0.32$, or 32% that prefer tennis.

OR

East has 2000 of the 4500 students, and West has 2500 of the 4500 students. Using a weighted average yields $\frac{2000}{4500} \cdot 0.22 + \frac{2500}{4500} \cdot 0.40 = 0.32$, or 32%.

20. **(A)** After the special key is pressed once, the calculator display reads -0.25 since $1/(1-5) = 1/(-4) = -0.25$. If the key is pressed again, the calculator display reads 0.8 since $1/(1-(-0.25)) = 1/(1.25) = 0.8$. If the key is pressed a third time, the calculator display reads 5, since $1/(1-0.8) = 1/(0.2) = 5$. Thus pressing the special key three times returns to the original calculator display. The calculator display will continue to cycle through the three answers -0.25, 0.8, and 5. Since 100 is 1 more than a multiple of 3, the calculator display will be -0.25.

21. **(D)** The sum of three numbers is even if all three numbers are even, or if two numbers are odd and one is even. Since there are only two even numbers in the set, it follows that the three numbers must include two odd numbers and one even. The possibilities are:

$$\{89,95,132\} \quad \{89,99,132\} \quad \{89,173,132\}$$
$$\{89,95,166\} \quad \{89,99,166\} \quad \{89,173,166\}$$

$$\{95,99,132\} \quad \{95,173,132\} \quad \{99,173,132\}$$
$$\{95,99,166\} \quad \{95,173,166\} \quad \{99,173,166\}.$$

Thus there are 12 possibilities.

OR

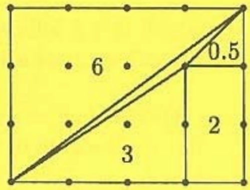
Let O stand for odd and E stand for even. The numbers given are O, O, O, E, E and O. There are only two ways that the sum of three numbers is even.

Case I: $E+E+E=E$, and

Case II: $O+O+E=E$.

Since there are only two even numbers, **Case I** cannot happen. For **Case II**, $O+O+E$, counting choices yields 4 choices for the first odd, 3 remaining choices for the second odd, and 2 choices for the even, for a total of $4 \times 3 \times 2 = 24$ choices. However, since $O_1 + O_2 + E = O_2 + O_1 + E$, the number of choices is reduced by a factor of 2. Hence there are $24/2 = 12$ choices.

22. **(B)** From the total area of 12, subtract the areas of the four surrounding polygons whose areas are indicated in the diagram. Thus the area of the remaining triangle ABC is



$$12 - 6 - 3 - 0.5 - 2 = 0.5 = 1/2.$$

Note. Points whose coordinates are integers are called *lattice* points. According to Pick's Theorem, if there are I lattice points in the interior of a triangle and B lattice points on the boundary, then the area of the triangle is $I + B/2 - 1$. In this problem, $I = 0$ and $B = 3$. Therefore the area of the triangle is $0 + 3/2 - 1 = 1/2$.

23. **(E)** Had there been \$5 more in the company fund, there would have been $\$95 + \$5 = \$100$ which would have been enough to give each employee another \$5. Thus there are $\$100/\$5 = 20$ employees. So the company fund contained $20 \cdot \$45 + \$95 = \$995$.

OR

The \$95 left over represents \$5 for each employee who would have received \$50. Thus there are $95/5 = 19$ of these employees. Hence the amount of money in the company fund before any bonuses were paid was $(19 \times \$50) + (1 \times \$45) =$ \$995.

24. **(C)** Since the sum of the measures of the angles of a triangle is 180°, in triangle ABC it follows that

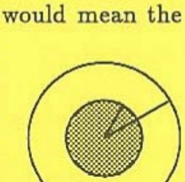
$$\angle BAC + \angle BCA = 180^\circ - 50^\circ = 130^\circ.$$

The measures of angles DAC and DCA are half that of angles BAC and BCA, respectively, so

$$\angle DAC + \angle DCA = \frac{130^\circ}{2} = 65^\circ.$$

In triangle ACD, we have $\angle ADC = 180^\circ - 65^\circ = 115^\circ$.

25. **(A)** Suppose that the circle has radius 1. Then, being closer to the center of the region than the boundary of the region (the circle) would mean the chosen point must be inside the circle of radius 1/2 with the same center as the larger circle of radius 1. The area of the smaller region is $\pi(1/2)^2 = \pi/4$, and the area of the total region is $\pi(1)^2 = \pi$. Since the area of the smaller region is 1/4 of the area of the total region, the required probability is 1/4.



Note. The odds that the point is closer to the center are $1:3$.

AMERICAN MATHEMATICS COMPETITIONS

AJHSME SOLUTIONS PAMPHLET
FOR STUDENTS AND TEACHERS

13th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 20, 1997

Sponsored by

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National Council of Teachers of Mathematics
Casualty Actuarial Society American Statistical Association
American Mathematical Association of Two-Year Colleges
American Mathematical Society
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This Solutions Pamphlet gives at least one solution for each problem on this year's Examination and shows that all the problems can be solved using material normally associated with the mathematics curriculum for students in eighth grade or below. These solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

PLEASE NOTE

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Orders for prior year Examination questions and Solutions Pamphlets should be addressed to:

Dr. Walter E. Mientka, AMC Executive Director
American Mathematics Competitions
University of Nebraska-Lincoln
P.O. Box 81606
Lincoln, NE 68501-1606

1. **(C)** In decimal form:

$$\begin{array}{r} 0.1 \\ 0.09 \\ 0.009 \\ +\ \underline{0.0007} \\ 0.1997 \end{array}$$

2. **(D)** The largest number is achieved by choosing the smallest number to subtract. The smallest two-digit number is 10, so the largest answer is (200-10) (2) = 380.

3. **(B)** Write each decimal to four places: and 0.9790 is seen to be the largest.

0.9700
0.9790
0.9709
0.9070
0.9089

4. **(E)** For one-half hour: 30 min. x 150 words/min. = 4500 words and for three-quarters of an hour: 45 min. x 150 words/min. = 6750 words. Only 5650 falls in this interval.

5. **(A)** The two-digit multiples of 7 are:

14, 21, (28), 35, 42, 49, 56, 63, 70, 77, 84, (91), and 98.

Only 28 and 91 have digit sums of 10. The sum of 28 and 91 is 119.

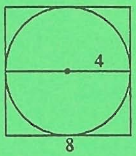
6. **(C)** Each shift of one place represents a multiple of 10:

7 4 9 8 2 1 0 3 5

100 10 1 0.1 0.01 0.001

Five shifts are needed and 10 x 10 x 10 x 10 x 10 = 10^5 = 100000.

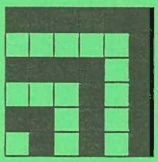
7. **(D)** The smallest square has side 8 and area $8^2 = 64$.



8. **(B)** Walter is gone from 7:30 a.m. until 4:00 p.m., a total of 8 hours and 30 minutes. He is in class for 6 (50 min.) = 300 min. or 5 hrs., at lunch for $\frac{1}{2}$ hr., and has 2 hours additional time. His total time at school is 7 hrs. and 30 min., so he was on the bus for 1 hour or 60 minutes.

9. **(C)** There are six ways to line up: ABC, ACB, BAC, BCA, CAB, CBA, and only one of these is in alphabetical order front-to-back. The probability is one in six or $\frac{1}{6}$.

10. **(C)** Of the 36 small squares, 21 are shaded, and $\frac{21}{36} = \frac{7}{12}$.



11. **(A)** Both 1 and 11 divide 11, so $\boxed{11} = 2$, and since 1, 2, 4, 5, 10, and 20 divide 20, then $\boxed{20} = 6$. The inner expression, $\boxed{11} \times \boxed{20} = 2 \times 6 = 12$. Finally, $\boxed{12} = 6$ because 1, 2, 3, 4, 6, and 12 divide 12.

12. **(D)** Since the sum of the angles of a triangle is 180°, $40° + 70° + \angle 1 = 180°$ and $\angle 1 = 70°$. This means that $\angle 2 = 110°$. Then $110° + \angle 3 + \angle 4 = 180°$, so $\angle 3 + \angle 4 = 70°$ and $\angle 3 = \angle 4 = 35°$.

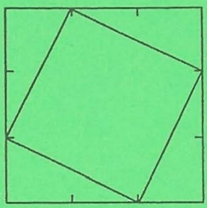
13. **(A)** There is a total of $26 + 28 + 30 = 84$ jelly beans:

50% of 26 =	13
25% of 28 =	7
20% of 30 =	6
	26

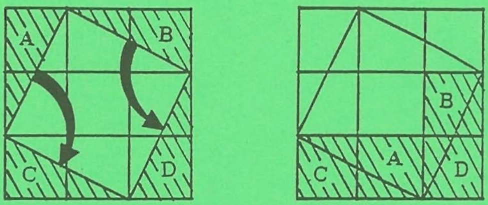
$\frac{26}{84} = 0.3095 \approx 31\%$

14. **(D)** Since 5 is the median, there must be two integers greater than 5 and two less than 5. The five integers may be arranged this way: ___ ___ 5 8 8. Since the mean is 5, the sum of all five integers is 25. The numbers 5, 8, and 8 total 21, leaving 4 for the sum of the first two. They can't both be 2 since 8 is the only mode, so they are 1 and 3. The difference between 8 and 1 is 7.

15. **(B)** Taking the side of the large square to be 3 inches gives an area of 9 square inches. Each of the four right triangles has an area of $\frac{1}{2}$ (2) (1) = 1 sq. inch. The area of the inscribed square is the area of the large square minus the area of the four right triangles, that is, 9 - 4 (1) = 5 sq. inches. The desired ratio is $\frac{5}{9}$.

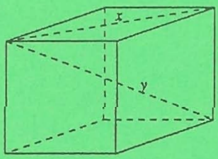


OR



16. **(E)** At the end of two years, stock values are:
Stock AA: \$100 (1.2) (0.8) = \$96
Stock BB: \$100 (0.75) (1.25) = \$93.75
Stock CC: \$100 (1) (1) = \$100
So, $B < A < C$.

17. **(E)** There are two diagonals, such as x, in each of the six faces for a total of twelve face diagonals. There are also four space diagonals, such as y, which are within the cube. This makes a total of 16.



18. **(B)** Last week one box cost \$1.25; this week one box costs \$0.80. The decrease, \$0.45, compared to the original price of \$1.25, is a decrease of $\frac{0.45}{1.25}$ = 0.36 or 36%, so choice (B) is closest.

OR

Last week there was a 4-box offer and this week a 5-box offer. Consider a purchase of 20 boxes (the smallest number divisible by both 4 and 5). Last week 20 boxes would have been \$25 (5 offers). This week it is \$16 (4 offers). The savings, \$9, compared to \$25 is 36%.

19. **(D)** Since $\frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} \cdot \frac{6}{5} \cdot \ldots \cdot \frac{a}{b} = 9$, we see that $\frac{a}{2} = 9$. Thus, $a = 18$, $b = 17$, and $a + b = 35$.

20. **(A)** There are 64 equally likely possibilities for the numbers on the two dice. Of these, only (5, 8), (6, 7), (6, 8), (7, 6), (7, 7), (7, 8), (8, 5), (8, 6), (8, 7), and (8, 8) give products exceeding 36, so the probability of this occurring is $\frac{10}{64} = \frac{5}{32}$.

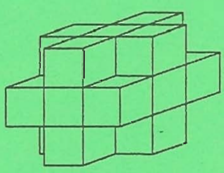
OR

Make a table for the sample space:

x	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32
5	5	10	15	20	25	30	35	(40)
6	6	12	18	24	30	36	(42)	(48)
7	7	14	21	28	35	(42)	(49)	(56)
8	8	16	24	32	(40)	(48)	(56)	(64)

The ten circled products exceed 36, so the probability of this occurring is $\frac{10}{64}$ or $\frac{5}{32}$.

21. **(D)** In terms of the original cube, three square centimeters are lost from each corner, but three new squares are added as the sides of the cavity in that corner. The total area remains at 54 square centimenters.



OR

Each face had an area of 9 square centimeters originally, so the total area was 6 (9) = 54 sq. cm. Four squares are lost from each face, leaving 54 - 4 (6) = 30 sq. cm. But the cavity in each of the eight corners has an area of 3 sq. cm, so 8 (3) = 24 sq. cm must be added. Finally, 30 + 24 = 54 sq. cm.

22. **(E)** The volume of a two-inch cube is $2^3 = 8$ cu. inches, while that of a three-inch cube is 27 cu. inches. Therefore, the weight and value of the larger cube is $\frac{27}{8}$ times that of the smaller. $\$200\left(\frac{27}{8}\right) = \$675$.

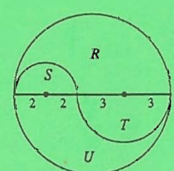
    Note: The actual weight of the cubes is not needed to solve the problem.

23. **(C)** To meet the first condition, numbers which sum to 50 must be chosen from the set of squares $\{1, 4, 9, 16, 25, 36, 49\}$. To meet the second condition, the squares selected must be different. Consequently, there are three possibilities: $1 + 49$, $1 + 4 + 9 + 36$, and $9 + 16 + 25$. These correspond to the integers 17, 1236, and 345, respectively. The largest is 1236, and the product of its digits is $1 \cdot 2 \cdot 3 \cdot 6 = 36$.

24. **(C)** Let the diameter of the large circle equal 10. Then the ratio is:

$$\frac{\begin{pmatrix}\text{Area of}\\ \text{semicircle}\\ \text{R+S}\end{pmatrix} - \begin{pmatrix}\text{Area of}\\ \text{semicircle}\\ \text{S}\end{pmatrix} + \begin{pmatrix}\text{Area of}\\ \text{semicircle}\\ \text{T}\end{pmatrix}}{\begin{pmatrix}\text{Area of}\\ \text{semicircle}\\ \text{T+U}\end{pmatrix} - \begin{pmatrix}\text{Area of}\\ \text{semicircle}\\ \text{T}\end{pmatrix} + \begin{pmatrix}\text{Area of}\\ \text{semicircle}\\ \text{S}\end{pmatrix}}$$

$$\frac{\frac{1}{2}\pi 5^2 - \frac{1}{2}\pi 2^2 + \frac{1}{2}\pi 3^2}{\frac{1}{2}\pi 5^2 - \frac{1}{2}\pi 3^2 + \frac{1}{2}\pi 2^2} = \frac{15\pi}{10\pi} = \frac{3}{2} \text{ or } 3:2.$$



25. **(D)** If the numbers 2, 4, 6, and 8 are multiplied, the product is 384, so 4 is the final digit of the product of a set of numbers ending in 2, 4, 6, and 8. Since there are ten such sets of numbers, the final digit of the overall product is the same as the final digit of 4^{10}. Now, $4^{10} = (4^2)^5 = 16^5$. Next, consider 6^5. Since any number of 6's multiply to give 6 as the final digit, the final digit of the required product is 6.

AMERICAN MATHEMATICS COMPETITIONS

AJHSME SOLUTIONS PAMPHLET
FOR STUDENTS AND TEACHERS

14th ANNUAL

AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION

(AJHSME)

Tuesday, NOVEMBER 17, 1998

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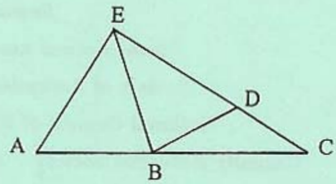
P.O. Box 81606
Lincoln, NE 68501-1606

1. (B): Only 6/7 (A) and 6/8 (B) are less than 1. For fractions: If the numerators are equal, the smaller fraction will have a larger denominator. Therefore, 6/8 (B) is smaller than 6/7 (A).

2. (E): $3 \cdot 2 - 4 \cdot 1 = 6 - 4 = 2$.

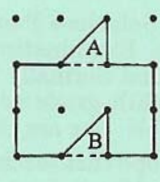
3. (B): $\frac{3}{8} + \frac{7}{8} = \frac{10}{8} = \frac{5}{4}$. Therefore, $\frac{5}{4} \div \frac{4}{5} = \frac{5}{4} \cdot \frac{5}{4} = \frac{25}{16}$.

4. (E): The triangles are ABE, ACE, BCD, BCE, BDE.



5. (B): (A) $9.12344 = 9.12344000\ldots$,
(B) $9.123\overline{4} = 9.12344444\ldots$,
(C) $9.12\overline{34} = 9.12343434\ldots$,
(D) $9.1\overline{234} = 9.12342342\ldots$,
(E) $9.\overline{1234} = 9.12341234\ldots$.

6. (B): Slide triangle A down to fill in triangle B. The resulting 2×3 rectangle has area 6.



7. (D): Use the associative property to group as follows:
$(100 \times 19.98) \times (1.998 \times 1000) = 1998 \times 1998 = (1998)^2$.

8. (C): 200 gallons $-$ 0.5(30) gallons $=$ 200 gallons $-$ 15 gallons $=$ 185 gallons.

9. (C): First reduction: $10 - 0.2(10) = 10 - 2 = 8$.
Second reduction: $8 - 0.5(8) = 8 - 4 = 4$.

OR

The sale price is 80% of the original price. After the next reduction, the final price is one-half the sale price, or one-half of 80% or 40%. Therefore, $0.4(10) = 4$.

10. (E): The only arrangement that produces a whole number is $\frac{3}{1} - \frac{4}{2} = 1$.
Therefore, $W + Y = 3 + 4 = 7$.

11. (C): Since Harry has 3 sisters and 5 brothers, there are 3 girls and 6 boys in the family. So Harriet has 2 sisters and 6 brothers. The product of 2 and 6 is 12.

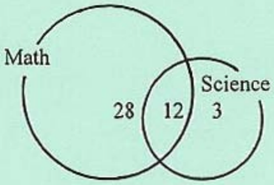
12. (A): $2(1-1/2) + 3(1-1/3) + 4(1-1/4) + \ldots + 10(1-1/10) =$
$2(1/2) + 3(2/3) + 4(3/4) + \ldots + 10(9/10) =$
$1 + 2 + 3 + \ldots + 9 = 45$.

13. (C): Divide the square into 16 smaller squares as shown. The shaded square is formed from 4 half-squares, so its area is 2. The ratio 2 to 16 is 1/8.

Note: There are several other ways to divide the region to show this.

14. (E): Since 80% of the Science Club members are also in the Math Club, there are $0.8(15) = 12$ students common to both clubs. Because 30% of the students in the Math Club are also in the Science Club, there are $12/0.3 = 40$ students in the Math Club.

OR



$28 + 12 = 40$

15. (D):

In the year 1998 the population is 200.
In 2023 the population will be $200(3) = 600$.
In 2048 the population will be $600(3) = 1800$.
In 2050 the population will be about 2000.

16. (B):

Year	Population
1998	200
2023	600
2048	1,800
2073	5,400
2098	16,200

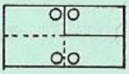
Of the choices available the year 2075 is the best estimate.

17. (C):

Year	Population	Area Needed
1998	200	300
2023	600	900
2048	1,800	2,700
2073	5,400	8,100
2098	16,200	24,300

The Isles can support $24{,}900 \div 1.5 = 16{,}600$ people. The chart shows that this will happen about the year 2098, or in about 100 years.

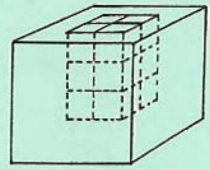
18. (B): The folded rectangle appears in the upper right corner of the sheet of paper, and the hole is punched in its upper left corner. Only Figure (B) has a hole in the upper left corner of the upper right rectangle of the unfolded sheet.



19. (A): Tamika can get the numbers $8 + 9 = 17$, $8 + 10 = 18$, or $9 + 10 = 19$. Carlos can get $3 \times 5 = 15$, $3 \times 6 = 18$, or $5 \times 6 = 30$. The possible ways to pair these are: (17, 15), (17, 18), (17, 30), (18, 15), (18, 18), (18, 30), (19, 15), (19, 18), (19, 30). Four of these nine pairs show Tamika with a higher result, so the probability is 4/9.

20. (D): After folding the square twice the resulting figure is an isosceles triangle with area 9 square inches. Since there are 4 such congruent triangles in the square, the area of the square is 36 square inches. Therefore, the sides of PQRS are 6 inches, and the perimeter is 24 inches.

21. (B): The $2 \times 2 \times 3$ core contains all of the small cubes that do not touch a side or the bottom. These 12 cubes are subtracted from 64 to leave 52.



OR

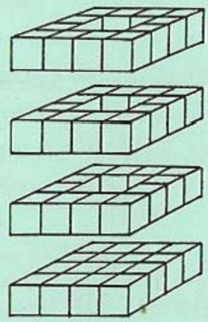
12

12

12

+ 16

52



22. (D): The sequence is 98, 49, 44, 22, 11, 6, 54, 27, 22,… . After 3 terms the cycle (22, 11, 6, 54, 27) is repeated. The 98^{th} term is the fifth term of the cycle, and this is 27.

.3. (C):

Step	Number of triangles	Number of shaded triangles
1	1	0
2	4	0+1=1
3	9	1+2=3
4	16	1+2+3=6
5	25	1+2+3+4=10
6	36	1+2+3+4+5=15
7	49	1+2+3+4+5+6=21
8	64	1+2+3+4+5+6+7=28

The ratio at step eight: $\frac{28}{64} = \frac{7}{16}$.

24. (E) The numbers in the first column all have remainders of 1 when divided by 8, those of the second column have remainders of 2 when divided by 8, and so on. We need to find numbered squares so that each remainder 0 through 7 appears at least once. The squares that are shaded are numbered 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105 , 120, and the remainders upon dividing by 8 are 1, 3, 6, 2, 7, 5, 4, 4, 5, 7, 2, 6, 3, 1, 0. Thus, we must shade square 120 to obtain the first shaded square in the last column.

25. (D): Since Toy begins with \$36 and her amount is doubled in the first two exchanges, her amounts are \$36, \$72, \$144, and \$36. This means that she gave away \$108, and this is exactly enough to double the amounts of Ami and Jan. So, the total must be 2(\$108) + \$36 = \$252.