

American Mathematics Competitions

30th Annual

AMC 8 American Mathematics Contest 8 Tuesday, November 18, 2014

INSTRUCTIONS

- 1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
- 2. This is a twenty-five question multiple choice test. For each question, only one answer choice is correct.
- 3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
- 4. There is no penalty for guessing. Your score is the number of correct answers.
- 5. Only scratch paper, graph paper, rulers, protractors, and erasers are allowed as aids. Calculators are NOT allowed. No problems on the test *require* the use of a calculator.
- 6. Figures are not necessarily drawn to scale.
- 7. Before beginning the test, your proctor will ask you to record your information on the answer form.
- 8. You will have 40 minutes to complete the test once your proctor tells you to begin.
- 9. When you finish the exam, *sign your name* in the space provided on the answer form.

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DO NOT OPEN UNTIL TUESDAY, NOVEMBER 18, 2014

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- 1. Harry and Terry are each told to calculate 8 (2 + 5). Harry gets the correct answer. Terry ignores the parentheses and calculates 8-2+5. If Harry's answer is H and Terry's answer is T, what is H T?
 - $(A) -10 \qquad (B) -6 \qquad (C) 0 \qquad (D) 6 \qquad (E) 10$
- 2. Paul owes Paula 35 cents and has a pocket full of 5-cent coins, 10-cent coins, and 25-cent coins that he can use to pay her. What is the difference between the largest and the smallest number of coins he can use to pay her?
 - (A) 1 (B) 2 (C) 3 (D) 4 (E) 5



- 3. Isabella had a week to read a book for a school assignment. She read an average of 36 pages per day for the first three days and an average of 44 pages per day for the next three days. She then finished the book by reading 10 pages on the last day. How many pages were in the book?
 - (A) 240 (B) 250 (C) 260 (D) 270 (E) 280



- 4. The sum of two prime numbers is 85. What is the product of these two prime numbers?
 - (A) 85 (B) 91 (C) 115 (D) 133 (E) 166
- 5. Margie's car can go 32 miles on a gallon of gas, and gas currently costs \$4 per gallon. How many miles can Margie drive on \$20 worth of gas?
 - (A) 64 (B) 128 (C) 160 (D) 320 (E) 640



6. Six rectangles each with a common base width of 2 have lengths of 1, 4, 9, 16, 25, and 36. What is the sum of the areas of the six rectangles?

(A) 91 (B) 93 (C) 162 (D) 182 (E) 202

7. There are four more girls than boys in Ms. Raub's class of 28 students. What is the ratio of the number of girls to the number of boys in her class?

(A) 3:4 (B) 4:3 (C) 3:2 (D) 7:4 (E) 2:1

8. Eleven members of the Middle School Math Club each paid the same amount for a guest speaker to talk about problem solving at their math club meeting. They paid their guest speaker $\underline{1A2}$. What is the missing digit A of this 3-digit number?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4



- 9. In $\triangle ABC$, *D* is a point on side \overline{AC} such that BD = DC and $\angle BCD$ measures 70°. What is the degree measure of $\angle ADB$?
 - (A) 100 (B) 120 (C) 135 (D) 140 (E) 150



10. The first AMC 8 was given in 1985 and it has been given annually since that time. Samantha turned 12 years old the year that she took the seventh AMC 8. In what year was Samantha born?

(A) 1979 (B) 1980 (C) 1981 (D) 1982 (E) 1983

11. Jack wants to bike from his house to Jill's house, which is located three blocks east and two blocks north of Jack's house. After biking each block, Jack can continue either east or north, but he needs to avoid a dangerous intersection one block east and one block north of his house. In how many ways can he reach Jill's house by biking a total of five blocks?

(A) 4 (B) 5 (C) 6 (D) 8 (E) 10



12. A magazine printed photos of three celebrities along with three photos of the celebrities as babies. The baby pictures did not identify the celebrities. Readers were asked to match each celebrity with the correct baby picture. What is the probability that a reader guessing at random will match all three correctly?

(A)
$$\frac{1}{9}$$
 (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

13. If n and m are integers and $n^2 + m^2$ is even, which of the following is impossible?

(A) n and m are even (B) n and m are odd (C) n + m is even

- (D) n + m is odd (E) none of these is impossible
- 14. Rectangle ABCD and right triangle DCE have the same area. They are joined to form a trapezoid, as shown. What is DE?
 - (A) 12 (B) 13 (C) 14 (D) 15 (E) 16



15. The circumference of the circle with center O is divided into 12 equal arcs, marked the letters A through L as seen below. What is the number of degrees in the sum of angles x and y?

(A) 75 **(B)** 80 (C) 90 (D) 120 **(E)** 150



- 16. The "Middle School Eight" basketball conference has 8 teams. Every season, each team plays every other conference team twice (home and away), and each team also plays 4 games against non-conference opponents. What is the total number of games in a season involving "Middle School Eight" teams?
 - **(B)** 88 (A) 60 (C) 96 (E) 160 (D) 144



- 17. George walks 1 mile to school. He leaves home at the same time each day, walks at a steady speed of 3 miles per hour, and arrives just as school begins. Today he was distracted by the pleasant weather and walked the first $\frac{1}{2}$ mile at a speed of only 2 miles per hour. At how many miles per hour must George run the last $\frac{1}{2}$ mile in order to arrive just as school begins today?
 - (C) 8 (D) 10 (A) 4 **(B)** 6 **(E)** 12



30th AMC 8 2014 Problems

- 18. Four children were born at City Hospital yesterday. Assume each child is equally likely to be a boy or a girl. Which of the following outcomes is most likely?
 - (A) all 4 are boys (B) all 4 are girls (C) 2 are girls and 2 are boys
 - (D) 3 are of one gender and 1 is of the other gender
 - (E) all of these outcomes are equally likely



19. A cube with 3-inch edges is to be constructed from 27 smaller cubes with 1-inch edges. Twenty-one of the cubes are colored red and 6 are colored white. If the 3-inch cube is constructed to have the smallest possible white surface area showing, what fraction of the surface area is white?

(A)
$$\frac{5}{54}$$
 (B) $\frac{1}{9}$ (C) $\frac{5}{27}$ (D) $\frac{2}{9}$ (E) $\frac{1}{3}$

20. Rectangle ABCD has sides CD = 3 and DA = 5. A circle of radius 1 is centered at A, a circle of radius 2 is centered at B, and a circle of radius 3 is centered at C. Which of the following is closest to the area of the region inside the rectangle but outside all three circles?

(A) 3.5 (B) 4.0 (C) 4.5 (D) 5.0 (E) 5.5



21. The 7-digit numbers $\underline{74}\underline{A52}\underline{B1}$ and $\underline{326}\underline{A}\underline{B4}\underline{C}$ are each multiples of 3. Which of the following could be the value of C?

(A) 1 (B) 2 (C) 3 (D) 5 (E) 8

22. A 2-digit number is such that the product of the digits plus the sum of the digits is equal to the number. What is the units digit of the number?

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

23. Three members of the Euclid Middle School girls' softball team had the following conversation.

Ashley: I just realized that our uniform numbers are all 2-digit primes.

Bethany: And the sum of your two uniform numbers is the date of my birthday earlier this month.

Caitlin: That's funny. The sum of your two uniform numbers is the date of my birthday later this month.

Ashley: And the sum of your two uniform numbers is today's date.

What number does Caitlin wear?

- (A) 11 (B) 13 (C) 17 (D) 19 (E) 23
- 24. One day the Beverage Barn sold 252 cans of soda to 100 customers, and every customer bought at least one can of soda. What is the maximum possible median number of cans of soda bought per customer on that day?
 - (A) 2.5 (B) 3.0 (C) 3.5 (D) 4.0 (E) 4.5



25. A straight one-mile stretch of highway, 40 feet wide, is closed. Robert rides his bike on a path composed of semicircles as shown. If he rides at 5 miles per hour, how many hours will it take to cover the one-mile stretch?

Note: 1 mile = 5280 feet

(A)
$$\frac{\pi}{11}$$
 (B) $\frac{\pi}{10}$ (C) $\frac{\pi}{5}$ (D) $\frac{2\pi}{5}$ (E) $\frac{2\pi}{3}$





SOLUTIONS

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WRITE TO US

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Prof. Norbert Kuenzi, AMC 8 Chair 934 Nicolet Ave Oshkosh, WI 54901-1634

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> American Mathematics Competitions PO Box 471 Annapolis Junction, MD 20701

AMC 10 & AMC 12

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PUBLICATIONS

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31st Annual

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2015 **AMC 8**

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Math for America

- 1. How many square yards of carpet are required to cover a rectangular floor that is 12 feet long and 9 feet wide? (There are 3 feet in a yard.)
 - (A) 12 (B) 36 (C) 108 (D) 324 (E) 972
- 2. Point O is the center of the regular octagon ABCDEFGH, and X is the midpoint of side \overline{AB} . What fraction of the area of the octagon is shaded?



3. Jack and Jill are going swimming at a pool that is one mile from their house. They leave home simultaneously. Jill rides her bicycle to the pool at a constant speed of 10 miles per hour. Jack walks to the pool at a constant speed of 4 miles per hour. How many minutes before Jack does Jill arrive?

(A) 5 (B) 6 (C) 8 (D) 9 (E) 10

4. The Centerville Middle School chess team consists of two boys and three girls. A photographer wants to take a picture of the team to appear in the local newspaper. She decides to have them sit in a row with a boy at each end and the three girls in the middle. How many such arrangements are possible?



5. Billy's basketball team scored the following points over the course of the first 11 games of the season:

42, 47, 53, 53, 58, 58, 58, 61, 64, 65, 73.

If his team scores 40 in the 12th game, which of the following statistics will show an increase?



- 6. In $\triangle ABC$, AB = BC = 29, and AC = 42. What is the area of $\triangle ABC$?
 - (A) 100 (B) 420 (C) 500 (D) 609 (E) 701
- 7. Each of two boxes contains three chips numbered 1, 2, 3. A chip is drawn randomly from each box and the numbers on the two chips are multiplied. What is the probability that their product is even?
 - (A) $\frac{1}{9}$ (B) $\frac{2}{9}$ (C) $\frac{4}{9}$ (D) $\frac{1}{2}$ (E) $\frac{5}{9}$
- 8. What is the smallest whole number larger than the perimeter of any triangle with a side of length 5 and a side of length 19?
 - (A) 24 (B) 29 (C) 43 (D) 48 (E) 57
- 9. On her first day of work, Janabel sold one widget. On day two, she sold three widgets. On day three, she sold five widgets, and on each succeeding day, she sold two more widgets than she had sold on the previous day. How many widgets in total had Janabel sold after working 20 days?

(A) 39 (B) 40 (C) 210 (D) 400 (E) 401

10. How many integers between 1000 and 9999 have four distinct digits?

(A) 3024 (B) 4536 (C) 5040 (D) 6480 (E) 6561

11. In the small country of Mathland, all automobile license plates have four symbols. The first must be a vowel (A, E, I, O, or U), the second and third must be two different letters among the 21 non-vowels, and the fourth must be a digit (0 through 9). If the symbols are chosen at random subject to these conditions, what is the probability that the plate will read "AMC8"?

(A)
$$\frac{1}{22,050}$$
 (B) $\frac{1}{21,000}$ (C) $\frac{1}{10,500}$ (D) $\frac{1}{2,100}$ (E) $\frac{1}{1,050}$

- 12. How many pairs of parallel edges, such as \overline{AB} and \overline{GH} or \overline{EH} and \overline{FG} , does a cube have?
 - (A) 6 (B) 12 (C) 18 (D) 24 (E) 36



13. How many subsets of two elements can be removed from the set

 $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

so that the mean (average) of the nine remaining numbers is 6?

(A) 1 (B) 2 (C) 3 (D) 5 (E) 6

14. Which of the following integers cannot be written as the sum of four consecutive odd integers?

(A) 16 (B) 40 (C) 72 (D) 100 (E) 200

15. At Euler Middle School, 198 students voted on two issues in a school referendum with the following results: 149 voted in favor of the first issue and 119 voted in favor of the second issue. If there were exactly 29 students who voted against both issues, how many students voted in favor of both issues?



16. In a middle-school mentoring program, a number of the sixth graders are paired with a ninth-grade student as a buddy. No ninth grader is assigned more than one sixth-grade buddy. If $\frac{1}{3}$ of all the ninth graders are paired with $\frac{2}{5}$ of all the sixth graders, what fraction of the total number of sixth and ninth graders have a buddy?

(A)
$$\frac{2}{15}$$
 (B) $\frac{4}{11}$ (C) $\frac{11}{30}$ (D) $\frac{3}{8}$ (E) $\frac{11}{15}$

17. Jeremy's father drives him to school in rush hour traffic in 20 minutes. One day there is no traffic, so his father can drive him 18 miles per hour faster and gets him to school in 12 minutes. How far in miles is it to school?

(E) 12

(D) 9



18. An arithmetic sequence is a sequence in which each term after the first is obtained by adding a constant to the previous term. For example, 2, 5, 8, 11, 14 is an arithmetic sequence with five terms, in which the first term is 2 and the constant added is 3. Each row and each column in this 5×5 array is an arithmetic sequence with five terms. What is the value of X?

1		25
	X	
17		81

19. A triangle with vertices at A = (1, 3), B = (5, 1), and C = (4, 4) is plotted on a 6×5 grid. What fraction of the grid is covered by the triangle?

(A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$



2015 AMC 8 Problems

(A) $6\sqrt{2}$

(B) 9

20. Ralph went to the store and bought 12 pairs of socks for a total of \$24. Some of the socks he bought cost \$1 a pair, some of the socks he bought cost \$3 a pair, and some of the socks he bought cost \$4 a pair. If he bought at least one pair of each type, how many pairs of \$1 socks did Ralph buy?



21. In the given figure hexagon ABCDEF is equiangular, ABJI and FEHG are squares with areas 18 and 32 respectively, $\triangle JBK$ is equilateral and FE = BC. What is the area of $\triangle KBC$?



22. On June 1, a group of students is standing in rows, with 15 students in each row. On June 2, the same group is standing with all of the students in one long row. On June 3, the same group is standing with just one student in each row. On June 4, the same group is standing with 6 students in each row. This process continues through June 12 with a different number of students per row each day. However, on June 13, they cannot find a new way of organizing the students. What is the smallest possible number of students in the group?

(A) 21 (B) 30 (C) 60 (D) 90 (E) 1080

- 23. Tom has twelve slips of paper which he wants to put into five cups labeled A, B, C, D, E. He wants the sum of the numbers on the slips in each cup to be an integer. Furthermore, he wants the five integers to be consecutive and increasing from A to E. The numbers on the papers are 2, 2, 2, 2.5, 2.5, 3, 3, 3, 3, 3, 3, 5, 4, and 4.5. If a slip with 2 goes into cup E and a slip with 3 goes into cup B, then the slip with 3.5 must go into what cup?
 - (A) A (B) B (C) C (D) D (E) E
- 24. A baseball league consists of two four-team divisions. Each team plays every other team in its division N games. Each team plays every team in the other division M games with N > 2M and M > 4. Each team plays a 76 game schedule. How many games does a team play within its own division?



25. One-inch squares are cut from the corners of this 5 inch square. What is the area in square inches of the largest square that can be fitted into the remaining space?





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1. The longest professional tennis match ever played lasted a total of 11 hours and 5 minutes. How many minutes was this?

(A) 605 (B) 655 (C) 665 (D) 1005 (E) 1105



- 2. In rectangle *ABCD*, *AB* = 6 and *AD* = 8. Point *M* is the midpoint of \overline{AD} . What is the area of $\triangle AMC$?
 - (A) 12 (B) 15 (C) 18 (D) 20 (E) 24
- 3. Four students take an exam. Three of their scores are 70, 80, and 90. If the average of their four scores is 70, then what is the remaining score?
 - (A) 40 (B) 50 (C) 55 (D) 60 (E) 70
- 4. When Cheenu was a boy he could run 15 miles in 3 hours and 30 minutes. As an old man he can now walk 10 miles in 4 hours. How many minutes longer does it take for him to walk a mile now compared to when he was a boy?
 - (A) 6 (B) 10 (C) 15 (D) 18 (E) 30
- 5. The number N is a two-digit number.
 - When *N* is divided by 9, the remainder is 1.
 - When *N* is divided by 10, the remainder is 3.

What is the remainder when *N* is divided by 11?

(A) 0 (B) 2 (C) 4 (D) 5 (E) 7

6. The following bar graph represents the length (in letters) of the names of 19 people. What is the median length of these names?



(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

- 7. Which of the following numbers is **not** a perfect square?
 - (A) 1^{2016} (B) 2^{2017} (C) 3^{2018} (D) 4^{2019} (E) 5^{2020}
- 8. Find the value of the expression

 $100 - 98 + 96 - 94 + 92 - 90 + \dots + 8 - 6 + 4 - 2$.

- (A) 20 (B) 40 (C) 50 (D) 80 (E) 100
- 9. What is the sum of the distinct prime integer divisors of 2016?
 - (A) 9 (B) 12 (C) 16 (D) 49 (E) 63
- 10. Suppose that a * b means 3a b. What is the value of x if

$$2*(5*x) = 1$$
?

- (A) $\frac{1}{10}$ (B) 2 (C) $\frac{10}{3}$ (D) 10 (E) 14
- 11. Determine how many two-digit numbers satisfy the following property: When the number is added to the number obtained by reversing its digits, the sum is 132.
 - (A) 5 (B) 7 (C) 9 (D) 11 (E) 12

12. Jefferson Middle School has the same number of boys and girls. Threefourths of the girls and two-thirds of the boys went on a field trip. What fraction of the students on the field trip were girls?

(A)
$$\frac{1}{2}$$
 (B) $\frac{9}{17}$ (C) $\frac{7}{13}$ (D) $\frac{2}{3}$ (E) $\frac{14}{15}$

13. Two different numbers are randomly selected from the set $\{-2, -1, 0, 3, 4, 5\}$ and multiplied together. What is the probability that the product is 0?

(A)
$$\frac{1}{6}$$
 (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

14. Karl's car uses a gallon of gas every 35 miles, and his gas tank holds 14 gallons when it is full. One day Karl started with a full tank of gas, drove 350 miles, bought 8 gallons of gas, and continued driving to his destination. When he arrived, his gas tank was half full. How many miles did Karl drive that day?

- 15. What is the largest power of 2 that is a divisor of $13^4 11^4$?
 - (A) 8 (B) 16 (C) 32 (D) 64 (E) 128
- 16. Annie and Bonnie are running laps around a 400-meter oval track. They started together, but Annie has pulled ahead, because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie?
 - (A) $1\frac{1}{4}$ (B) $3\frac{1}{3}$ (C) 4 (D) 5 (E) 25



17. An ATM password at Fred's Bank is composed of four digits from 0 to 9, with repeated digits allowable. If no password may begin with the sequence 9, 1, 1, then how many passwords are possible?

(A) 30 $(B) /290$ $(C) 9000$ $(D) 9990$	(E) 9999
---	----------

18. In an All-Area track meet, 216 sprinters enter a 100-meter dash competition. The track has 6 lanes, so only 6 sprinters can compete at a time. At the end of each race the five non-winners are eliminated, and the winner will compete again in a later race. How many races are needed to determine the champion sprinter?

19. The sum of 25 consecutive even integers is 10,000. What is the largest of these 25 consecutive even integers?

(A) 360 (B) 388 (C) 412 (D) 416 (E) 424

20. The least common multiple of *a* and *b* is 12, and the least common multiple of *b* and *c* is 15. What is the least possible value of the least common multiple of *a* and *c*?

21. A box contains 3 red chips and 2 green chips. Chips are drawn randomly, one at a time without replacement, until all 3 of the reds are drawn or until both green chips are drawn. What is the probability that the 3 reds are drawn?

(A)
$$\frac{3}{10}$$
 (B) $\frac{2}{5}$ (C) $\frac{1}{2}$ (D) $\frac{3}{5}$ (E) $\frac{2}{3}$

22. Rectangle *DEFA* below is a 3×4 rectangle with DC = CB = BA = 1. The area of the "bat wings" (the shaded area) is



- 23. Two congruent circles centered at points *A* and *B* each pass through the other's center. The line containing both *A* and *B* is extended to intersect the circles at points *C* and *D*. The two circles intersect at two points, one of which is *E*. What is the degree measure of $\angle CED$?
 - (A) 90 (B) 105 (C) 120 (D) 135 (E) 150
- 24. The digits 1, 2, 3, 4, and 5 are each used once to write a five-digit number *PQRST*. The three-digit number *PQR* is divisible by 4, the three-digit number *QRS* is divisible by 5, and the three-digit number *RST* is divisible by 3. What is *P*?
 - (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
- 25. A semicircle is inscribed in an isosceles triangle with base 16 and height 15 so that the diameter of the semicircle is contained in the base of the triangle as shown. What is the radius of the semicircle?





SOLUTIONS

Your School Manager will be sent at least one copy of the 2016 AMC 8 Solutions Pamphlet with the report. It is meant to be loaned to students (but not duplicated).

WRITE TO US

Comments about the problems and solutions for this AMC 8 should be addressed to:

Prof. Norbert Kuenzi, AMC 8 Chair 934 Nicolet Ave Oshkosh, WI 54901-1634

Comments about administrative arrangements should be addressed to: MAA American Mathematics Competitions / amcinfo@maa.org

> MAA American Mathematics Competitions PO Box 471 Annapolis Junction, MD 20701

AMC 10 & AMC 12

The AMC 10 and AMC 12 are 25-question, 75-minute, multiple choice contests. All schools participating in the AMC 8 receive a brochure and registration form for the 2015 AMC 10. Schools with high scoring students on the AMC 8 should consider administering the AMC 10. The best way to prepare for these contests is to study exams from previous years. Orders for all publications listed below should be addressed to:

MAA American Mathematics Competitions PO Box 471 Annapolis Junction, MD 20701

PUBLICATIONS

A complete listing of the current publications for sale can be found on our web site: maa.org/math-competitions



MAA American Mathematics Competitions



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Collaborator's Circle

American Statistical Association Casualty Actuarial Society Conference Board of the Mathematical Sciences Mu Alpha Theta Society for Industrial and Applied Mathematics 1. Which of the following values is largest?

(A) 2 + 0 + 1 + 7 (B) $2 \times 0 + 1 + 7$ (C) $2 + 0 \times 1 + 7$ (D) $2 + 0 + 1 \times 7$ (E) $2 \times 0 \times 1 \times 7$

2. Alicia, Brenda, and Colby were the candidates in a recent election for student president. The pie chart below shows how the votes were distributed among the three candidates. If Brenda received 36 votes, then how many votes were cast all together?



- 3. What is the value of the expression $\sqrt{16\sqrt{8\sqrt{4}}}$?
 - (A) 4 (B) $4\sqrt{2}$ (C) 8 (D) $8\sqrt{2}$ (E) 16
- 4. When 0.000315 is multiplied by 7,928,564 the product is closest to which of the following?

(A) 210 (B) 240 (C) 2,100 (D) 2,400 (E) 24,000

5. What is the value of the expression $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8}{1 + 2 + 3 + 4 + 5 + 6 + 7 + 8}$? (A) 1020 (B) 1120 (C) 1220 (D) 2240 (E) 3360 6. If the degree measures of the angles of a triangle are in the ratio 3 : 3 : 4, what is the degree measure of the largest angle of the triangle?

(A) 18 (B) 36 (C) 60 (D) 72 (E) 90

7. Let Z be a 6-digit positive integer, such as 247247, whose first three digits are the same as its last three digits taken in the same order. Which of the following numbers must be a factor of Z ?

(A) 11 (B) 19 (C) 101 (D) 111 (E) 1111

- 8. Malcolm wants to visit Isabella after school today and knows the street where she lives but doesn't know her house number. She tells him, "My house number has two digits, and exactly three of the following four statements about it are true."
 - (1) It is prime.
 - (2) It is even.
 - (3) It is divisible by 7.
 - (4) One of its digits is 9.

This information allows Malcolm to determine Isabella's house number. What is its units digit?

- (A) 4 (B) 6 (C) 7 (D) 8 (E) 9
- 9. All of Marcy's marbles are blue, red, green, or yellow. One third of her marbles are blue, one fourth of them are red, and six of them are green. What is the smallest number of yellow marbles that Marcy could have?
 - (A) 1 (B) 2 (C) 3 (D) 4 (E) 5



MAA AMC 8

10. A box contains five cards, numbered 1, 2, 3, 4, and 5. Three cards are selected randomly without replacement from the box. What is the probability that 4 is the largest value selected?

(A)
$$\frac{1}{10}$$
 (B) $\frac{1}{5}$ (C) $\frac{3}{10}$ (D) $\frac{2}{5}$ (E) $\frac{1}{2}$

11. A square-shaped floor is covered with congruent square tiles. If the total number of tiles that lie on the two diagonals is 37, how many tiles cover the floor?

(A) 148 (B) 324 (C) 361 (D) 1296 (E) 1369

12. The smallest positive integer greater than 1 that leaves a remainder of 1 when divided by 4, 5, and 6 lies between which of the following pairs of numbers?

(A) 2 and 19	(B) 20 and 39	(C) 40 and 59
(D) 60 and 79	(E) 80 and 124	

13. Peter, Emma, and Kyler played chess with each other. Peter won 4 games and lost 2 games. Emma won 3 games and lost 3 games. If Kyler lost 3 games, how many games did he win?



- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
- 14. Chloe and Zoe are both students in Ms. Demeanor's math class. Last night they each solved half of the problems in their homework assignment alone and then solved the other half together. Chloe had correct answers to only 80% of the problems she solved alone, but overall 88% of her answers were correct. Zoe had correct answers to 90% of the problems she solved alone. What was Zoe's overall percentage of correct answers?

(A) 89 (B) 92 (C) 93 (D) 96 (E) 98

MAA AMC 8

15. In the arrangement of letters and numerals below, by how many different paths can one spell AMC8? Beginning at the A in the middle, a path allows only moves from one letter to an adjacent (above, below, left, or right, but not diagonal) letter. One example of such a path is traced in the picture.

			8	C	8	
		8	С	M	C	8
		С	М	Α	M	С
		8	C	M	С	8
			8	С	8	
(A) 8	(B) 9	(C) 12	(D) 24		(E) 36

16. In the figure shown below, choose point *D* on side \overline{BC} so that $\triangle ACD$ and $\triangle ABD$ have equal perimeters. What is the area of $\triangle ABD$?



17. Starting with some gold coins and some empty treasure chests, I tried to put 9 gold coins in each treasure chest, but that left 2 treasure chests empty. So instead I put 6 gold coins in each treasure chest, but then I had 3 gold coins left over. How many gold coins did I have?



(A) 9 (B) 27 (C) 45 (D) 63 (E) 81

18. In the non-convex quadrilateral *ABCD* shown below, $\angle BCD$ is a right angle, AB = 12, BC = 4, CD = 3, and AD = 13.



What is the area of quadrilateral ABCD?

(A) 12 (B) 24 (C) 26 (D) 30 (E) 36

19. For any positive integer *M*, the notation *M*! denotes the product of the integers 1 through *M*. What is the largest integer *n* for which 5ⁿ is a factor of the sum 98! + 99! + 100! ?

(A) 23 (B) 24 (C) 25 (D) 26 (E) 27

- 20. An integer between 1000 and 9999, inclusive, is chosen at random. What is the probability that it is an odd integer whose digits are all distinct?
 - (A) $\frac{14}{75}$ (B) $\frac{56}{225}$ (C) $\frac{107}{400}$ (D) $\frac{7}{25}$ (E) $\frac{9}{25}$

21. Suppose *a*, *b*, and *c* are nonzero real numbers, and a + b + c = 0. What are the possible value(s) for $\frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|}$?

(A) 0 (B) 1 and -1 (C) 2 and -2 (D) 0, 2, and -2

(**E**) 0, 1, and −1

22. In the right triangle ABC, AC = 12, BC = 5, and angle C is a right angle. A semicircle is inscribed in the triangle as shown. What is the radius of the semicircle?



23. Each day for four days, Linda traveled for one hour at a speed that resulted in her traveling one mile in an integer number of minutes. Each day after the first, her speed decreased so that the number of minutes to travel one mile increased by 5 minutes over the preceding day. Each of the four days, her distance traveled was also an integer number of miles. What was the total number of miles for the four trips?

(A) 10 (B) 15 (C) 25 (D) 50 (E) 82

24. Mrs. Sanders has three grandchildren, who call her regularly. One calls her every three days, one calls her every four days, and one calls her every five days. All three called her on December 31, 2016. On how many days during the next year did she not receive a phone call from any of her grandchildren?



(A) 78 (B) 80 (C) 144 (D) 146 (E) 152

25. In the figure shown, \overline{US} and \overline{UT} are line segments each of length 2, and $m \angle TUS = 60^{\circ}$. Arcs \overrightarrow{TR} and \overrightarrow{SR} are each one-sixth of a circle with radius 2. What is the area of the region shown?







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American Statistical Association Casualty Actuarial Society Conference Board of the Mathematical Sciences Mu Alpha Theta Society for Industrial and Applied Mathematics 1. An amusement park has a collection of scale models, with ratio 1:20, of buildings and other sights from around the country. The height of the United States Capitol is 289 feet. What is the height in feet of its replica at this park, rounded to the nearest whole number?

(A) 14 (B) 15 (C) 16 (D) 18 (E) 20

2. What is the value of the product

$$\left(1 + \frac{1}{1}\right) \cdot \left(1 + \frac{1}{2}\right) \cdot \left(1 + \frac{1}{3}\right) \cdot \left(1 + \frac{1}{4}\right) \cdot \left(1 + \frac{1}{5}\right) \cdot \left(1 + \frac{1}{6}\right)?$$
(A) $\frac{7}{6}$ (B) $\frac{4}{3}$ (C) $\frac{7}{2}$ (D) 7 (E) 8

- 3. Students Arn, Bob, Cyd, Dan, Eve, and Fon are arranged in that order in a circle. They start counting: Arn first, then Bob, and so forth. When the number contains a 7 as a digit (such as 47) or is a multiple of 7 that person leaves the circle and the counting continues. Who is the last one present in the circle?
 - (A) Arn (B) Bob (C) Cyd (D) Dan (E) Eve
- 4. The twelve-sided figure shown has been drawn on $1 \text{ cm} \times 1 \text{ cm}$ graph paper. What is the area of the figure in cm^2 ?



(A) 12 (B) 12.5 (C) 13 (D) 13.5 (E) 14

MAA AMC 8

- 5. What is the value of $1+3+5+\dots+2017+2019-2-4-6-\dots-2016-2018$?
 - (A) -1010 (B) -1009 (C) 1008 (D) 1009 (E) 1010
- 6. On a trip to the beach, Anh traveled 50 miles on the highway and 10 miles on a coastal access road. He drove three times as fast on the highway as on the coastal road. If Anh spent 30 minutes driving on the coastal road, how many minutes did his entire trip take?
 - (A) 50 (B) 70 (C) 80 (D) 90 (E) 100
- 7. The 5-digit number 2 0 1 8 U is divisible by 9. What is the remainder when this number is divided by 8?
 - (A) 1 (B) 3 (C) 5 (D) 6 (E) 7
- 8. Mr. Garcia asked the members of his health class how many days last week they exercised for at least 30 minutes. The results are summarized in the following bar graph, where the heights of the bars represent the number of students.



What was the mean number of days of exercise last week, rounded to the nearest hundredth, reported by the students in Mr. Garcia's class?

(A) 3.50 (B) 3.57 (C) 4.36 (D) 4.50 (E) 5.00

9. Tyler is tiling the floor of his 12 foot by 16 foot living room. He plans to place one-foot by one-foot square tiles to form a border along the edges of the room and to fill in the rest of the floor with two-foot by two-foot square tiles. How many tiles will he use?

(A) 48 (B) 87 (C) 91 (D) 96 (E) 120

- 10. The *harmonic mean* of a set of non-zero numbers is the reciprocal of the average of the reciprocals of the numbers. What is the harmonic mean of 1, 2, and 4?
 - (A) $\frac{3}{7}$ (B) $\frac{7}{12}$ (C) $\frac{12}{7}$ (D) $\frac{7}{4}$ (E) $\frac{7}{3}$
- 11. Abby, Bridget, and four of their classmates will be seated in two rows of three for a group picture, as shown.

If the seating positions are assigned randomly, what is the probability that Abby and Bridget are adjacent to each other in the same row or the same column?

- (A) $\frac{1}{3}$ (B) $\frac{2}{5}$ (C) $\frac{7}{15}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$
- 12. The clock in Sri's car, which is not accurate, gains time at a constant rate. One day as he begins shopping he notes that his car clock and his watch (which is accurate) both say 12:00 noon. When he is done shopping, his watch says 12:30 and his car clock says 12:35. Later that day, Sri loses his watch. He looks at his car clock and it says 7:00. What is the actual time?

13. Laila took five math tests, each worth a maximum of 100 points. Laila's score on each test was an integer between 0 and 100, inclusive. Laila received the same score on the first four tests, and she received a higher score on the last test. Her average score on the five tests was 82. How many values are possible for Laila's score on the last test?

(A) 4 (B) 5 (C) 9 (D) 10 (E) 18

MAA AMC 8

14. Let N be the greatest five-digit number whose digits have a product of 120. What is the sum of the digits of N?

(A) 15 (B) 16 (C) 17 (D) 18 (E) 20

15. In the diagram below, a diameter of each of the two smaller circles is a radius of the larger circle. If the two smaller circles have a combined area of 1 square unit, then what is the area of the shaded region, in square units?



(A)
$$\frac{1}{4}$$
 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1 (E) $\frac{\pi}{2}$

16. Professor Chang has nine different language books lined up on a bookshelf: two Arabic, three German, and four Spanish. How many ways are there to arrange the nine books on the shelf keeping the Arabic books together and keeping the Spanish books together?

(A) 1440 (B) 2880 (C) 5760 (D) 182,440 (E) 362,880

17. Bella begins to walk from her house toward her friend Ella's house. At the same time, Ella begins to ride her bicycle toward Bella's house. They each maintain a constant speed, and Ella rides 5 times as fast as Bella walks. The distance between their houses is 2 miles, which is 10,560 feet, and Bella covers $2\frac{1}{2}$ feet with each step. How many steps will Bella take by the time she meets Ella?

(A) 704 (B) 845 (C) 1056 (D) 1760 (E) 3520

- 18. How many positive factors does 23,232 have?
 - (A) 9 (B) 12 (C) 28 (D) 36 (E) 42

19. In a sign pyramid a cell gets a "+" if the two cells below it have the same sign, and it gets a "-" if the two cells below it have different signs. The diagram below illustrates a sign pyramid with four levels. How many possible ways are there to fill the four cells in the bottom row to produce a "+" at the top of the pyramid?



(A) 2 (B) 4 (C) 8 (D) 12 (E) 16

20. In $\triangle ABC$, point *E* is on \overline{AB} with AE = 1 and $\overline{EB} = 2$. Point *D* is on \overline{AC} so that $\overline{DE} \parallel \overline{BC}$ and point *F* is on \overline{BC} so that $\overline{EF} \parallel \overline{AC}$. What is the ratio of the area of *CDEF* to the area of $\triangle ABC$?



- 21. How many positive three-digit integers have a remainder of 2 when divided by 6, a remainder of 5 when divided by 9, and a remainder of 7 when divided by 11?
 - (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
- 22. Point *E* is the midpoint of side \overline{CD} in square *ABCD*, and \overline{BE} meets diagonal \overline{AC} at *F*. The area of quadrilateral *AFED* is 45. What is the area of *ABCD*?



23. From a regular octagon, a triangle is formed by connecting three randomly chosen vertices of the octagon. What is the probability that at least one of the sides of the triangle is also a side of the octagon?



24. In the cube ABCDEFGH with opposite vertices C and E, J and I are the midpoints of edges \overline{FB} and \overline{HD} , respectively. Let R be the ratio of the area of the cross-section EJCI to the area of one of the faces of the cube. What is R^2 ?



25. How many perfect cubes lie between $2^8 + 1$ and $2^{18} + 1$, inclusive?

(A) 4 (B) 9 (C) 10 (D) 57 (E) 58



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- 1. Ike and Mike go into a sandwich shop with a total of \$30.00 to spend. Sandwiches cost \$4.50 each and soft drinks cost \$1.00 each. Ike and Mike plan to buy as many sandwiches as they can and use any remaining money to buy soft drinks. Counting both soft drinks and sandwiches, how many items will they buy?
 - (A) 6 (B) 7 (C) 8 (D) 9 (E) 10
- 2. Three identical rectangles are put together to form rectangle ABCD, as shown in the figure below. Given that the length of the shorter side of each of the smaller rectangles is 5 feet, what is the area in square feet of rectangle ABCD?



- 3. Which of the following is the correct order of the fractions $\frac{15}{11}$, $\frac{19}{15}$, and $\frac{17}{13}$, from least to greatest?
 - (A) $\frac{15}{11} < \frac{17}{13} < \frac{19}{15}$ (B) $\frac{15}{11} < \frac{19}{15} < \frac{17}{13}$ (C) $\frac{17}{13} < \frac{19}{15} < \frac{15}{11}$ (D) $\frac{19}{15} < \frac{15}{11} < \frac{17}{13}$ (E) $\frac{19}{15} < \frac{17}{13} < \frac{15}{11}$
- 4. Quadrilateral ABCD is a rhombus with perimeter 52 meters. The length of diagonal \overline{AC} is 24 meters. What is the area in square meters of rhombus ABCD?



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5. A tortoise challenges a hare to a race. The hare eagerly agrees and quickly runs ahead, leaving the slow-moving tortoise behind. Confident that he will win, the hare stops to take a nap. Meanwhile, the tortoise walks at a slow steady pace for the entire race. The hare awakes and runs to the finish line, only to find the tortoise already there. Which of the following graphs matches the description of the race, showing the distance d traveled by the two animals over time t from start to finish?



3

1

6. There are 81 grid points (uniformly spaced) in the square shown in the diagram below, including the points on the edges. Point P is the center of the square. Given that point Q is randomly chosen from among the other 80 points, what is the probability that the line PQ is a line of symmetry for the square?



7. Shauna takes five tests, each worth a maximum of 100 points. Her scores on the first three tests are 76, 94, and 87. In order to average 81 for all five tests, what is the lowest score she could earn on one of the other two tests?

(A) 48 (B) 52 (C) 66 (D) 70 (E) 74

8. Gilda has a bag of marbles. She gives 20% of them to her friend Pedro. Then Gilda gives 10% of what is left to another friend, Ebony. Finally, Gilda gives 25% of what is now left in the bag to her brother Jimmy. What percentage of her original bag of marbles does Gilda have left for herself?

(A) 20 (B) $33\frac{1}{3}$ (C) 38 (D) 45 (E) 54

9. Alex and Felicia each have cats as pets. Alex buys cat food in cylindrical cans that are 6 cm in diameter and 12 cm high. Felicia buys cat food in cylindrical cans that are 12 cm in diameter and 6 cm high. What is the ratio of the volume of one of Alex's cans to the volume of one of Felicia's cans?

(A) 1:4 (B) 1:2 (C) 1:1 (D) 2:1 (E) 4:1

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10. The diagram shows the number of students at soccer practice each weekday during last week. After computing the mean and median values, Coach discovers that there were actually 21 participants on Wednesday. Which of the following statements describes the change in the mean and median after the correction is made?



- (A) The mean increases by 1 and the median does not change.
- (B) The mean increases by 1 and the median increases by 1.
- (C) The mean increases by 1 and the median increases by 5.

(D) The mean increases by 5 and the median increases by 1.

(E) The mean increases by 5 and the median increases by 5.

11. The eighth grade class at Lincoln Middle School has 93 students. Each student takes a math class or a foreign language class or both. There are 70 eighth graders taking a math class, and there are 54 eighth graders taking a foreign language class. How many eighth graders take *only* a math class and *not* a foreign language class?

(A) 16 (B) 23 (C) 31 (D) 39 (E) 70

12. The faces of a cube are painted in six different colors: red (*R*), white (*W*), green (*G*), brown (*B*), aqua (*A*), and purple (*P*). Three views of the cube are shown below. What is the color of the face opposite the aqua face?



- 13. A *palindrome* is a number that has the same value when read from left to right or from right to left. (For example 12321 is a palindrome.) Let N be the least three-digit integer which is not a palindrome but which is the sum of three distinct two-digit palindromes. What is the sum of the digits of N?
 - (A) 2 (B) 3 (C) 4 (D) 5 (E) 6
- 14. Isabella has 6 coupons that can be redeemed for free ice cream cones at Pete's Sweet Treats. In order to make the coupons last, she decides that she will redeem one every 10 days until she has used them all. She knows that Pete's is closed on Sundays, but as she circles the 6 dates on her calendar, she realizes that no circled date falls on a Sunday. On what day of the week does Isabella redeem her first coupon?

(A) Monday	(B) Tuesday	(C) Wednesday	(D) Thursday
(E) Friday			

15. On a beach 50 people are wearing sunglasses and 35 people are wearing caps. Some people are wearing both sunglasses and caps. If one of the people wearing a cap is selected at random, the probability that this person is also wearing sunglasses is $\frac{2}{5}$. If instead, someone wearing sunglasses is selected at random, what is the probability that this person is also wearing a cap?

(A)
$$\frac{14}{85}$$
 (B) $\frac{7}{25}$ (C) $\frac{2}{5}$ (D) $\frac{4}{7}$ (E) $\frac{7}{10}$

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16. Qiang drives 15 miles at an average speed of 30 miles per hour. How many additional miles will he have to drive at 55 miles per hour to average 50 miles per hour for the entire trip?

(A) 45 (B) 62 (C) 90 (D) 110 (E) 135

17. What is the value of the product

$$\left(\frac{1\cdot 3}{2\cdot 2}\right)\left(\frac{2\cdot 4}{3\cdot 3}\right)\left(\frac{3\cdot 5}{4\cdot 4}\right)\cdots\left(\frac{97\cdot 99}{98\cdot 98}\right)\left(\frac{98\cdot 100}{99\cdot 99}\right)$$

- (A) $\frac{1}{2}$ (B) $\frac{50}{99}$ (C) $\frac{9800}{9801}$ (D) $\frac{100}{99}$ (E) 50
- 18. The faces on each of two fair dice are numbered 1, 2, 3, 5, 7, and 8. When the two dice are tossed, what is the probability that their sum will be an even number?
 - (A) $\frac{4}{9}$ (B) $\frac{1}{2}$ (C) $\frac{5}{9}$ (D) $\frac{3}{5}$ (E) $\frac{2}{3}$
- 19. In a tournament there are six teams that play each other twice. A team earns 3 points for a win, 1 point for a draw, and 0 points for a loss. After all the games have been played it turns out that the top three teams earned the same number of total points. What is the greatest possible number of total points for each of the top three teams?
 - (A) 22 (B) 23 (C) 24 (D) 26 (E) 30
- 20. How many different real numbers x satisfy the equation

$$(x^2 - 5)^2 = 16?$$

(A) 0 (B) 1 (C) 2 (D) 4 (E) 8

- 21. What is the area of the triangle formed by the lines y = 5, y = 1 + x, and y = 1 x?
 - (A) 4 (B) 8 (C) 10 (D) 12 (E) 16

- **

22. A store increased the original price of a shirt by a certain percent and then decreased the new price by the same percent. Given that the resulting price was 84% of the original price, by what percent was the price increased and decreased?

(A) 16 (B) 20 (C) 28 (D) 36 (E) 40

- 23. After Euclid High School's last basketball game, it was determined that $\frac{1}{4}$ of the team's points were scored by Alexa and $\frac{2}{7}$ were scored by Brittany. Chelsea scored 15 points. None of the other 7 team members scored more than 2 points. What was the total number of points scored by the other 7 team members?
 - (A) 10 (B) 11 (C) 12 (D) 13 (E) 14
- 24. In triangle ABC, point D divides side \overline{AC} so that AD : DC = 1 : 2. Let E be the midpoint of \overline{BD} and let F be the point of intersection of line BC and line AE. Given that the area of $\triangle ABC$ is 360, what is the area of $\triangle EBF$?



(A) 24 (B) 30 (C) 32 (D) 36 (E) 40

25. Alice has 24 apples. In how many ways can she share them with Becky and Chris so that each of the three people has at least two apples?

(A) 105 (B) 114 (C) 190 (D) 210 (E) 380

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Scores and official competition solutions will be sent to your competition manager who can share that information with you.

For more information about the MAA American Mathematics Competitions program and our other competitions, please visit maa.org/amc.

Questions and comments about this competition should be sent to:

amcinfo@maa.org

or

MAA American Mathematics Competitions P.O. Box 471 Annapolis Junction, MD 20701

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DO NOT OPEN UNTIL COMPETITION DAY.

The AMC 8 may be administered from Tuesday, November 12, 2019 through Monday, November 18, 2019.

Administration on an earlier date will disqualify your school's results.

- All the information needed to administer this competition is contained in the AMC 8 Teacher's Manual. PLEASE READ THE MANUAL BEFORE TUESDAY, NOVEMBER 12, 2019.
- Answer sheets must be returned to the MAA AMC office within 24 hours of the competition administration. Use an overnight or 2-day shipping service, with a tracking number, to guarantee timely arrival of these answer sheets. FedEx, UPS, or USPS overnight are strongly recommended.
- The publication, reproduction, or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.