

AMERICAN MATHEMATICS COMPETITIONS

1st ANNUAL
AMERICAN JUNIOR HIGH SCHOOL
MATHEMATICS EXAMINATION
(AJHSME)

TUESDAY, DECEMBER 10, 1985

Sponsors:

MATHEMATICAL ASSOCIATION OF AMERICA

SOCIETY OF ACTUARIES MU ALPHA THETA

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CASUALTY ACTUARIAL SOCIETY

AMERICAN STATISTICAL ASSOCIATION

AMERICAN MATHEMATICAL ASSOCIATION OF TWO-YEAR COLLEGES



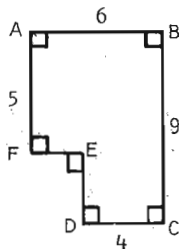
INSTRUCTIONS AND INFORMATION

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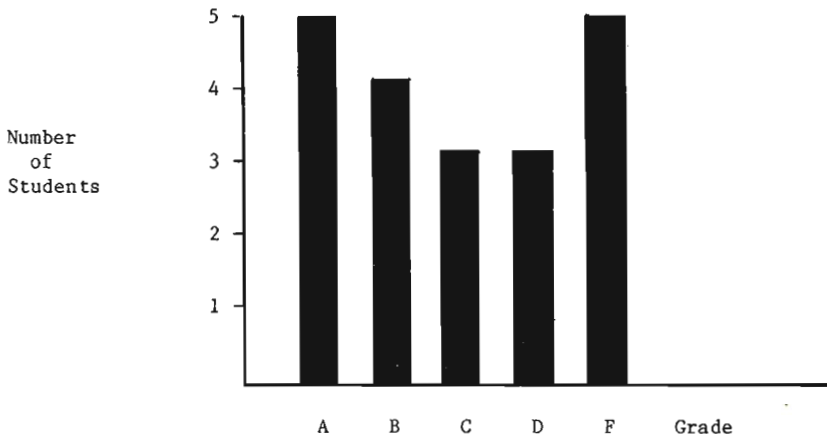
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1. $\frac{3 \times 5}{9 \times 11} \times \frac{7 \times 9 \times 11}{3 \times 5 \times 7} =$
- A) 1 B) 0 C) 49 D) $\frac{1}{49}$ E) 50
2. $90 + 91 + 92 + 93 + 94 + 95 + 96 + 97 + 98 + 99 =$
- A) 845 B) 945 C) 1005 D) 1025 E) 1045
3. $\frac{10^7}{5 \times 10^4} =$
- A) .002 B) .2 C) 20 D) 200 E) 2000

4. The area of polygon ABCDEF, in square units, is
- A) .24 B) 30 C) 46
- D) 66 E) 74



5.



The bar graph shows the grades in a mathematics class for the last grading period. If A, B, C and D are satisfactory grades, what fraction of the grades shown in the graph are satisfactory?

- A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) $\frac{4}{5}$ E) $\frac{9}{10}$

6. A ream of paper containing 500 sheets is 5 cm thick. Approximately how many sheets of this type of paper would there be in a stack 7.5 cm high?

A) 250 B) 550 C) 667 D) 750 E) 1250

7. A "stair-step" figure is made up of alternating black and white squares in each row. Rows 1 through 4 are shown. All rows begin and end with a white square. The number of black squares in the 37th row is



A) 34 B) 35 C) 36 D) 37 E) 38

8. If $a = -2$, the largest number in the set $\{-3a, 4a, \frac{24}{a}, a^2, 1\}$ is

A) $-3a$ B) $4a$ C) $\frac{24}{a}$ D) a^2 E) 1

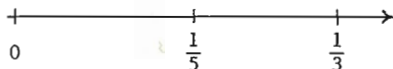
9. The product of the 9 factors $(1 - \frac{1}{2})(1 - \frac{1}{3})(1 - \frac{1}{4}) \dots (1 - \frac{1}{10}) =$

A) $\frac{1}{10}$ B) $\frac{1}{9}$ C) $\frac{1}{2}$ D) $\frac{10}{11}$ E) $\frac{11}{2}$

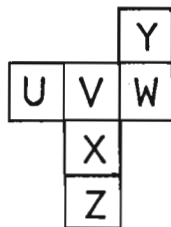
10. The fraction halfway between $\frac{1}{5}$ and $\frac{1}{3}$ (on the number line) is

A) $\frac{1}{4}$ B) $\frac{2}{15}$ C) $\frac{4}{15}$

D) $\frac{53}{200}$ E) $\frac{8}{15}$



11. A piece of paper containing six joined squares labeled as shown in the diagram is folded along the edges of the squares to form a cube. The label of the face opposite the face labeled X is



A) Z B) U C) V D) W E) Y

12. A square and a triangle have equal perimeters. The lengths of the three sides of the triangle are 6.2 cm, 8.3 cm and 9.5 cm. The area of the square is

A) 24 cm^2 B) 36 cm^2 C) 48 cm^2 D) 64 cm^2 E) 144 cm^2

13. If you walk for 45 minutes at a rate of 4 mph and then run for 30 minutes at a rate of 10 mph, how many miles have you gone at the end of one hour and 15 minutes?
- A) 3.5 miles B) 8 miles C) 9 miles D) $25\frac{1}{3}$ miles E) 480 miles
14. The difference between a 6.5% sales tax and a 6% sales tax on an item priced at \$20 before tax is
- A) \$.01 B) \$.10 C) \$.50 D) \$1 E) \$10
15. How many whole numbers between 100 and 400 contain the digit 2 ?
- A) 100 B) 120 C) 138 D) 140 E) 148
16. The ratio of boys to girls in Mr. Brown's math class is 2:3. If there are 30 students in the class, how many more girls than boys are in the class?
- A) 1 B) 3 C) 5 D) 6 E) 10
17. If your average score on your first six mathematics tests was 84 and your average score on your first seven mathematics tests was 85, then your score on the seventh test was
- A) 86 B) 88 C) 90 D) 91 E) 92
18. Nine copies of a certain pamphlet cost less than \$10.00 while ten copies of the same pamphlet (at the same price) cost more than \$11.00. How much does one copy of this pamphlet cost?
- A) \$1.07 B) \$1.08 C) \$1.09 D) \$1.10 E) \$1.11
19. If the length and width of a rectangle are each increased by 10%, then the perimeter of the rectangle is increased by
- A) 1% B) 10% C) 20% D) 21% E) 40%
20. In a certain year, January had exactly four Tuesdays and four Saturdays. On what day did January 1 fall that year?
- A) Monday B) Tuesday C) Wednesday D) Friday E) Saturday

21. Mr. Green receives a 10% raise every year. His salary after four such raises has gone up by what percent?

- A) less than 40% B) 40% C) 44% D) 45% E) More than 45%

22. Assume every 7-digit whole number is a possible telephone number except those which begin with 0 or 1. What fraction of telephone numbers begin with 9 and end with 0?

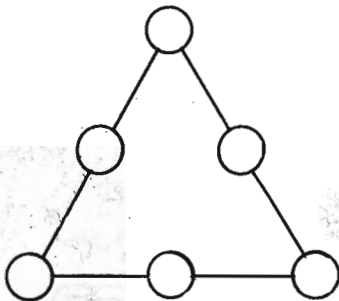
- A) $\frac{1}{63}$ B) $\frac{1}{80}$ C) $\frac{1}{81}$ D) $\frac{1}{90}$ E) $\frac{1}{100}$

23. King Middle School has 1200 students. Each student takes 5 classes a day. Each teacher teaches 4 classes. Each class has 30 students and 1 teacher. How many teachers are there at King Middle School?

- A) 30 B) 32 C) 40 D) 45 E) 50

24. In a magic triangle, each of the six whole numbers 10 - 15 is placed in one of the circles so that the sum, S , of the three numbers on each side of the triangle is the same. The largest possible value for S is

- A) 36 B) 37 C) 38
D) 39 E) 40



25. Five cards are lying on a table as shown. Each card has a letter on one side and a whole number on the other side. Jane said, "If a vowel is on one side of any card, then an even number is on the other side." Mary showed Jane was wrong by turning over one card. Which card did Mary turn over?



- A) 3 B) 4 C) 6
D) P E) Q

SOLUTIONS

A Solutions Pamphlet will be mailed to your school AJHSME Examination Manager along with your score.

WRITE TO US

Questions and comments about the problems and solutions for this AJHSME should be addressed to:

Professor Thomas Butts, AJHSME Subcommittee Chairman
Science Education Department
University of Texas at Dallas
PO Box 688 FN32
Richardson, Texas 75080

Remarks about administrative arrangements, and orders for any of the publications listed below should be addressed to:

Professor Walter E. Mientka, CAMC Executive Director
Department of Mathematics & Statistics
University of Nebraska-Lincoln
Lincoln, Nebraska, 68588-0322

1985 AHSME

The 37th American High School Mathematics Examination (AHSME) will be held on February 25, 1986. The Committee on the American Mathematics Competitions (CAMC) would like to encourage high scoring students on the AJHSME to participate in the AHSME, which is administered each year to over 380,000 students in the USA, Canada and abroad. Your teacher may obtain information about the AHSME by writing to the Executive Director of the CAMC whose address is indicated above.

PUBLICATIONS

In order to review prior year American High School Mathematics Examinations, the following publications are available in either pamphlet or book format.

Examinations. Each price is for one copy of an exam and its solutions for one year. Specify the years you want and how many copies of each. All prices effective to October 1, 1986

AHSME 1972-85, 40c per copy per year.

Books.

Contest Problem Book I (\$6.50), AHSME exams & solution, 1950-60.
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Contest Problem Book IV, (\$8.50), AHSMEs 1973-82.

Minimum Order: \$4.00 U.S. FUNDS ONLY. Make checks payable to MAA/AMC. Orders from U.S.A. and Canada must be prepaid. We usually mail Fourth Class, unless you wish First Class, in which case add 20% to the cost of your order. Overseas orders should not be prepaid, a Pro Forma Invoice will be provided.

AMERICAN MATHEMATICS COMPETITIONS

2nd ANNUAL
AMERICAN JUNIOR HIGH SCHOOL
MATHEMATICS EXAMINATION
(AJHSME)

TUESDAY, DECEMBER 9, 1986

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1. In July 1861, 366 inches of rain fell in Cherrapunji, India. What was the average rainfall in inches per hour during that month?

A) $\frac{366}{31 \times 24}$ B) $\frac{366 \times 31}{24}$ C) $\frac{366 \times 24}{31}$
 D) $\frac{31 \times 24}{366}$ E) $366 \times 31 \times 24$

2. Which of the following numbers has the largest reciprocal?

A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) 1 D) 5 E) 1986

3. The smallest sum one could get by adding three different numbers from the set { 7, 25, -1, 12, -3 } is

A) -3 B) -1 C) 3 D) 5 E) 21

4. The product $(1.8)(40.3 + .07)$ is closest to

A) 7 B) 42 C) 74 D) 84 E) 737

5. A contest began at noon one day and ended 1000 minutes later.

At what time did the contest end?

A) 10:00 p.m. B) midnight C) 2:30 a.m.
 D) 4:40 a.m. E) 6:40 a.m.

6. $\frac{2}{1 - \frac{2}{3}} =$

A) -3 B) $-\frac{4}{3}$ C) $\frac{2}{3}$ D) 2 E) 6

7. How many whole numbers are between $\sqrt{8}$ and $\sqrt{80}$?

A) 5 B) 6 C) 7 D) 8 E) 9

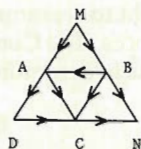
8. In the product shown, B is a digit.

The value of B is

$$\begin{array}{r} B2 \\ \times 7B \\ \hline 6396 \end{array}$$

A) 3 B) 5 C) 6 D) 7 E) 8

9. Using only the paths and the directions shown, how many different routes are there from M to N?



A) 2 B) 3 C) 4 D) 5 E) 6

10. A picture 3 feet across is hung in the center of a wall that is 19 feet wide. How many feet from the end of the wall is the nearest edge of the picture?

A) $1\frac{1}{2}$ B) 8 C) $9\frac{1}{2}$ D) 16 E) 22

11. If $A*B$ means $\frac{A+B}{2}$, then $(3*5)*8$ is

A) 6 B) 8 C) 12 D) 16 E) 30

12. The table to the right displays the grade distribution of the 30 students in a mathematics class on the last two tests. For example, exactly one student received a 'D' on Test 1 and a 'C' on Test 2 (see circled entry). What percent of the students received the same grade on both tests?

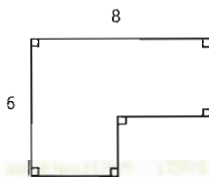
| | | TEST 2 | | | | |
|--------|---|--------|---|---|---|---|
| | | A | B | C | D | F |
| TEST 1 | A | 2 | 2 | 1 | 0 | 0 |
| | B | 1 | 4 | 3 | 0 | 0 |
| | C | 1 | 3 | 5 | 2 | 0 |
| | D | 0 | 0 | 1 | 1 | 1 |
| | E | 0 | 0 | 2 | 1 | 0 |
| | F | 0 | 0 | 2 | 1 | 0 |

A) 12% B) 25% C) $33\frac{1}{3}\%$ D) 40% E) 50%

13. The perimeter of the polygon shown is

A) 14 B) 20 C) 28 D) 48

E) cannot be determined from the information given



14. If $200 \leq a \leq 400$ and $600 \leq b \leq 1200$, then the largest value of the quotient $\frac{b}{a}$ is

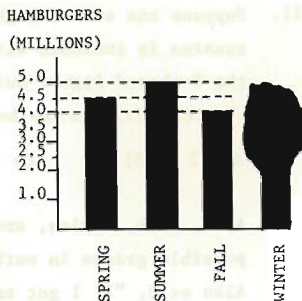
A) $\frac{3}{2}$ B) 3 C) 6 D) 300 E) 600

15. Sale prices at the Ajax Outlet Store are 50% below original prices. On Saturdays an additional discount of 20% off the sale price is given. What is the Saturday price of a coat whose original price is \$180?

A) \$54 B) \$72 C) \$90 D) \$108 E) \$110

16. A bar graph shows the number of hamburgers sold by a fast food chain each season.

However, the bar indicating the number sold during the winter is covered by a smudge. If exactly 25% of the chain's hamburgers are sold in the fall, how many million hamburgers are sold in the winter?



- A) 2.5 B) 3 C) 3.5 D) 4 E) 4.5

17. Let o be an odd whole number and let n be any whole number.

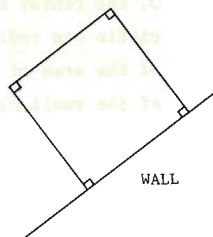
Which of the following statements about the whole number $(o^2 + no)$ is always true?

- A) it is always odd B) it is always even
 C) it is even only if n is even D) it is odd only if n is odd
 E) it is odd only if n is even

18. A rectangular grazing area is to be fenced off on three sides using part of a 100 meter rock wall as the fourth side. Fence posts are to be placed every 12 meters along the fence including the two posts where the fence meets the rock wall.

What is the fewest number of posts required to fence an area 36 m by 60 m?

- A) 11 B) 12 C) 13 D) 14 E) 16



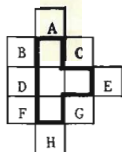
19. At the beginning of a trip, the mileage odometer read 56,200 miles. The driver filled the gas tank with 6 gallons of gasoline. During the trip, the driver filled his tank again with 12 gallons of gasoline when the odometer read 56,560. At the end of the trip, the driver filled the tank again with 20 gallons of gasoline. The odometer read 57,060. To the nearest tenth, what was the car's average miles-per-gallon for the entire trip?

- A) 22.5 B) 22.6 C) 24.0 D) 26.9 E) 27.5

20. The value of the expression $\frac{(304)^5}{(29.7)(399)^4}$ is closest to

- A) .003 B) .03 C) .3 D) 3 E) 30

21. Suppose one of the eight lettered identical squares is included with the four squares in the T-shaped figure outlined. How many of the resulting figures can be folded into a topless cubical box?

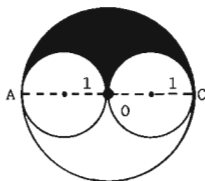


- A) 2 B) 3 C) 4 D) 5 E) 6

22. Alan, Beth, Carlos, and Diana were discussing their possible grades in mathematics class this grading period. Alan said, "If I get an A, then Beth will get an A." Beth said, "If I get an A, then Carlos will get an A." Carlos said, "If I get an A, then Diana will get an A." All of these statements were true, but only two of the students received an A. Which two received A's?

- A) Alan, Beth B) Beth, Carlos C) Carlos, Diana
D) Alan, Diana E) Beth, Diana

23. The large circle has diameter AC. The two small circles have their centers on AC and just touch at O, the center of the large circle. If each small circle has radius 1, what is the value of the ratio of the area of the shaded region to the area of one of the small circles?



- A) between $\frac{1}{2}$ and 1 B) 1 C) between 1 and $\frac{3}{2}$
D) between $\frac{3}{2}$ and 2 E) cannot be determined from the information given
24. The 600 students at King Middle School are divided into three groups of equal size for lunch. Each group has lunch at a different time. A computer randomly assigns each student to one of the three lunch groups. The probability that three friends, Al, Bob, and Carol, will be assigned to the same lunch group is approximately

- A) $\frac{1}{27}$ B) $\frac{1}{9}$ C) $\frac{1}{8}$ D) $\frac{1}{6}$ E) $\frac{1}{3}$

25. Which of the following sets of whole numbers has the largest average?

- A) multiples of 2 between 1 and 101 B) multiples of 3 between 1 and 101
C) multiples of 4 between 1 and 101 D) multiples of 5 between 1 and 101
E) multiples of 6 between 1 and 101

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PUBLICATIONS

MINIMUM ORDER: \$5.00 (before handling fee for first class postage) U.S. FUNDS ONLY. Individual orders from U.S. and Canada must be prepaid. Orders are mailed fourth class unless you wish first class. In that case, add a 20% handling fee. Make checks payable to MAA/CAMC.

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- AJHSME, 1985 American Junior High School Mathematics Examination, 50¢ per copy.
- AHSME, 1972-86, Spanish editions, 1978-86, 50¢ per copy per year.
- AJHSME National Summary of Results and Awards, 1985, \$3.00 per copy per year.
- AHSME National Summary of Results and Awards, 1976-86, \$3.00 per copy per year.

BOOKS

- Contest Problem Book I (\$7.50), AHSME exams and solutions, 1950-60.
- Contest Problem Book II (\$7.50), AHSME, 1961-65.
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3rd ANNUAL
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1. $.4 + .02 + .006 =$

- A) .012 B) .066 C) .12 D) .24 E) .426

2. $\frac{2}{25} =$

- A) .008 B) .08 C) .8 D) 1.25 E) 12.5

3. $2(81 + 83 + 85 + 87 + 89 + 91 + 93 + 95 + 97 + 99) =$

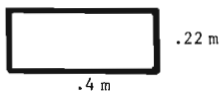
- A) 1600 B) 1650 C) 1700 D) 1750 E) 1800

4. Martians measure angles in clerts. There are 500 clerts in a full circle. How many clerts are there in a right angle?

- A) 90 B) 100 C) 125 D) 180 E) 250

5. The area of the rectangular region is

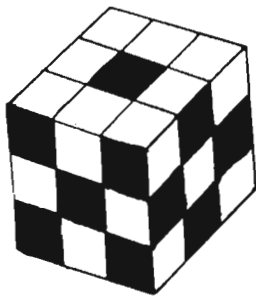
- A)
- $.088 \text{ m}^2$
- B)
- $.62 \text{ m}^2$
- C)
- $.88 \text{ m}^2$
-
- D)
- 1.24 m^2
- E)
- 4.22 m^2

6. The smallest product one could obtain by multiplying two numbers in the set $\{-7, -5, -1, 1, 3\}$ is

- A) -35 B) -21 C) -15 D) -1 E) 3

7. The large cube shown is made up of 27 identical sized smaller cubes. For each face of the large cube, the opposite face is shaded the same way. The total number of smaller cubes that must have at least one face shaded is

- A) 10 B) 16 C) 20 D) 22 E) 24

8.
$$\begin{array}{r} 9876 \\ A32 \\ \hline B1 \end{array}$$
 If A and B are nonzero digits, then the number of digits (not necessarily different) in the sum of the three whole numbers is

- A) 4 B) 5 C) 6 D) 9 E) depends on the values of A and B

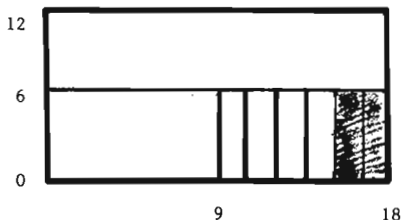
9. When finding the sum $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$, the least common denominator used is
 A) 120 B) 210 C) 420 D) 840 E) 5040

10. $4(299) + 3(299) + 2(299) + 298 =$
 A) 2889 B) 2989 C) 2991 D) 2999 E) 3009

11. The sum $2\frac{1}{7} + 3\frac{1}{2} + 5\frac{1}{19}$ is between
 A) 10 and $10\frac{1}{2}$ B) $10\frac{1}{2}$ and 11 C) 11 and $11\frac{1}{2}$
 D) $11\frac{1}{2}$ and 12 E) 12 and $12\frac{1}{2}$

12. What fraction of the large 12 by 18 rectangular region is shaded?

- A) $\frac{1}{108}$ B) $\frac{1}{18}$ C) $\frac{1}{12}$
 D) $\frac{2}{9}$ E) $\frac{1}{3}$



13. Which of the following fractions has the largest value?
 A) $\frac{3}{7}$ B) $\frac{4}{9}$ C) $\frac{17}{35}$ D) $\frac{100}{201}$ E) $\frac{151}{301}$

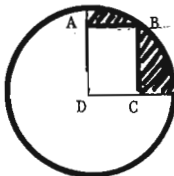
14. A computer can do 10,000 additions per second. How many additions can it do in one hour?
 A) 6 million B) 36 million C) 60 million
 D) 216 million E) 360 million

15. The sale ad read: "Buy three tires at the regular price and get the fourth tire for \$3." Sam paid \$240 for a set of four tires at the sale. What was the regular price of one tire?
 A) \$59.25 B) \$60 C) \$70 D) \$79 E) \$80

16. Joyce made 12 of her first 30 shots in the first three games of this basketball season, so her seasonal shooting average was 40%. In her next game, she took 10 shots and raised her seasonal shooting average to 50%. How many of these 10 shots did she make?
 A) 2 B) 3 C) 5 D) 6 E) 8

17. Abby, Bret, Carl, and Dana are seated in a row on four seats numbered #1 to #4. Joe looks at them and says:
- "Bret is next to Carl."
- "Abby is between Bret and Carl."
- However each one of Joe's statements is false. Bret is actually sitting in seat #3. Who is sitting in seat #2?
- A) Abby B) Bret C) Carl D) Dana
E) There is not enough information to be sure.
18. Half the people in a room left. One third of those remaining started to dance. There were then 12 people who were not dancing. The original number of people in the room was
- A) 24 B) 30 C) 36 D) 42 E) 72
19. A calculator has a squaring key $\boxed{x^2}$ which replaces the current number displayed with its square. For example, if the display is $\boxed{3}$ and the $\boxed{x^2}$ key is depressed, then the display becomes $\boxed{9}$. If the display reads $\boxed{2}$, how many times must you depress the $\boxed{x^2}$ key to produce a displayed number greater than 500?
- A) 4 B) 5 C) 8 D) 9 E) 250
20. "If a whole number n is not prime, then the whole number $n - 2$ is not prime." A value of n which shows this statement to be false is
- A) 9 B) 12 C) 13 D) 16 E) 23
21. Suppose n^* means $\frac{1}{n}$, the reciprocal of n . For example, $5^* = \frac{1}{5}$.
- How many of the following statements are true?
- i) $3^* + 6^* = 9^*$ iii) $2^* \cdot 6^* = 12^*$
ii) $6^* - 4^* = 2^*$ iv) $10^* \div 2^* = 5^*$
- A) 0 B) 1 C) 2 D) 3 E) 4

22. ABCD is a rectangle, D is the center of the circle, and B is on the circle. If $AD = 4$ and $CD = 3$, then the area of the shaded region is between



- A) 4 and 5 B) 5 and 6 C) 6 and 7
D) 7 and 8 E) 8 and 9

23. Assume the adjoining chart shows the 1980 U.S. population, in millions, for each region by ethnic group. To the nearest percent, what percent of the U.S. Black population lived in the South?

| | NE | MW | South | West |
|-------|----|----|-------|------|
| White | 42 | 52 | 57 | 35 |
| Black | 5 | 5 | 15 | 2 |
| Asian | 1 | 1 | 1 | 3 |
| Other | 1 | 1 | 2 | 4 |

- A) 20% B) 25% C) 40% D) 56% E) 80%
24. A multiple choice examination consists of 20 questions. The scoring is +5 for each correct answer, -2 for each incorrect answer, and 0 for each unanswered question. John's score on the examination is 48. What is the maximum number of questions he could have answered correctly?
- A) 9 B) 10 C) 11 D) 12 E) 16
25. Ten balls numbered 1 to 10 are in a jar. Jack reaches into the jar and randomly removes one of the balls. Then Jill reaches into the jar and randomly removes a different ball. The probability that the sum of the two numbers on the balls removed is even is
- A) $\frac{4}{9}$ B) $\frac{9}{19}$ C) $\frac{1}{2}$ D) $\frac{10}{19}$ E) $\frac{5}{9}$

SOLUTIONS

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AJHSME, 1985-86 American Junior High School Mathematics Examination, \$.50 per copy.
AHSME, 1972-87, Spanish editions, 1978-87, \$.50 per copy per year.
AJHSME National Summary of Results and Awards, 1985-86 \$3.00 per copy per year.
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BOOKS

Contest Problem Book I (\$8.50), AHSME exams and solutions, 1950-60
Contest Problem Book II (\$8.50), AHSME, 1961-65
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AMERICAN MATHEMATICS COMPETITIONS

4th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL
MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, DECEMBER 1, 1988

Sponsored by

Mathematical Association of America

Society of Actuaries Mu Alpha Theta

National Council of Teachers of Mathematics

Casualty Actuarial Society American Statistical Association

American Mathematical Association of Two-Year Colleges

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1. The diagram shows part of a scale of a measuring device. The arrow indicates an approximate reading of



- A) 10.05 B) 10.15 C) 10.25
D) 10.3 E) 10.6

2. The product $8 \times .25 \times 2 \times .125 =$

- A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 1 E) 2

3. $\frac{1}{10} + \frac{2}{20} + \frac{3}{30} =$

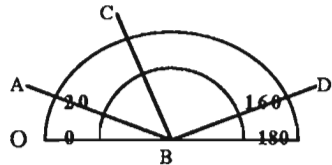
- A) .1 B) .123 C) .2 D) .3 E) .6

4. The figure consists of alternating light and dark squares. The number of dark squares exceeds the number of light squares by

- A) 7 B) 8 C) 9
D) 10 E) 11



5. If $\angle CBD$ is a right angle, then this protractor indicates that the measure of $\angle ABC$ is approximately



- A) 20° B) 40° C) 50°
D) 70° E) 120°

6. $\frac{(.2)^3}{(.02)^2} =$

- A) .2 B) 2 C) 10 D) 15 E) 20

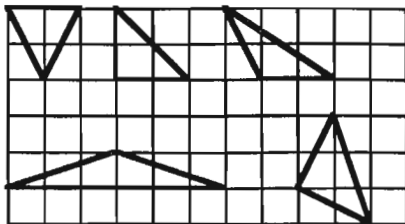
7. $2.46 \times 8.163 \times (5.17 + 4.829)$ is closest to

- A) 100 B) 200 C) 300 D) 400 E) 500

8. Betty used a calculator to find the product 0.075×2.56 . She forgot to enter the decimal points. The calculator showed 19200. If Betty had entered the decimal points correctly, the answer would have been

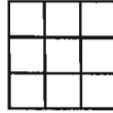
- A) .0192 B) .192 C) 1.92 D) 19.2 E) 192

9. An isosceles triangle is a triangle with two sides of equal length. How many of the five triangles on the square grid below are isosceles?



- A) 1 B) 2 C) 3 D) 4 E) 5
10. Chris' birthday is on a Thursday this year. What day of the week will it be 60 days after her birthday?
- A) Monday B) Wednesday C) Thursday D) Friday E) Saturday
11. $\sqrt{164}$ is .
- A) 42 B) less than 10 C) between 10 and 11
D) between 11 and 12 E) between 12 and 13
12. Suppose the estimated 20 billion dollar cost to send a person to the planet Mars is shared equally by the 250 million people in the U.S. Then each person's share is
- A) \$40 B) \$50 C) \$80 D) \$100 E) \$125
13. If rose bushes are spaced about 1 foot apart, approximately how many bushes are needed to surround a circular patio whose radius is 12 feet?
- A) 12 B) 38 C) 48 D) 75 E) 450
14. \diamond and Δ are whole numbers and $\diamond \times \Delta = 36$. The largest possible value of $\diamond + \Delta$ is
- A) 12 B) 13 C) 15 D) 20 E) 37
15. The reciprocal of $(\frac{1}{2} + \frac{1}{3})$ is
- A) $\frac{1}{6}$ B) $\frac{2}{5}$ C) $\frac{6}{5}$ D) $\frac{5}{2}$ E) 5

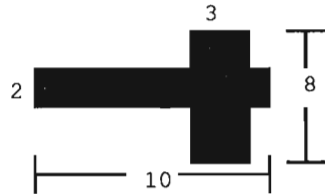
16. Placing no more than one X in each small square, what is the greatest number of X's that can be put on the grid shown without getting three X's in a row vertically, horizontally, or diagonally?



- A) 2 B) 3 C) 4
D) 5 E) 6

17. The shaded area formed by the two intersecting perpendicular rectangles, in square units, is

- A) 23 B) 38 C) 44 D) 46
E) unable to be determined from the information given



18. The average weight of 6 boys is 150 pounds and the average weight of 4 girls is 120 pounds. The average weight of the 10 children is

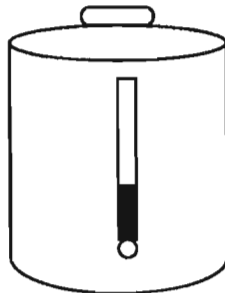
- A) 135 pounds B) 137 pounds C) 138 pounds
D) 140 pounds E) 141 pounds

19. What is the 100th number in the arithmetic sequence: 1, 5, 9, 13, 17, 21, 25, ... ?

- A) 397 B) 399 C) 401 D) 403 E) 405

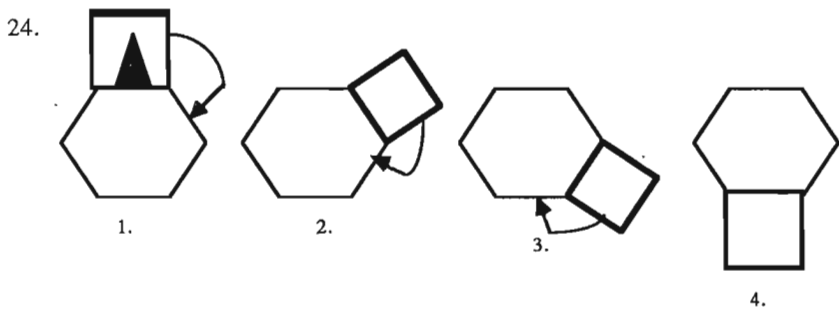
20. The glass gauge on a cylindrical coffee maker shows there are 45 cups left when the coffee maker is 36% full. How many cups of coffee does it hold when it is full?

- A) 80 B) 100 C) 125
D) 130 E) 262



4th AJHSME 1988

21. A fifth number, n , is added to the set of numbers $\{3,6,9,10\}$ to make the mean of the set of five numbers equal to its median. The number of possible values for n is
- A) 1 B) 2 C) 3 D) 4 E) more than 4
22. Tom's Hat Shoppe increased all original prices by 25%. Now the shoppe is having a sale where all prices are 20% off these increased prices. Which statement best describes the sale price of an item?
- A) The sale price is 5% higher than the original price.
 B) The sale price is higher than the original price, but by less than 5%.
 C) The sale price is higher than the original price, but by more than 5%.
 D) The sale price is lower than the original price.
 E) The sale price is the same as the original price.
23. Maria buys computer disks at a price of 4 for \$5 and sells them at a price of 3 for \$5. How many computer disks must she sell in order to make a profit of \$100?
- A) 100 B) 120 C) 200 D) 240 E) 1200



The square in the first diagram "rolls" clockwise around the fixed regular hexagon until it reaches the bottom. In which position will the solid triangle be in diagram 4?

- A) B) C) D) E)

25. A **palindrome** is a whole number that reads the same forwards as backwards. If one neglects the colon, certain times displayed on a digital watch are palindromes. Three examples are: $\boxed{1:01}$, $\boxed{4:44}$ and $\boxed{12:21}$. How many times during a 12-hour period will be palindromes?
- A) 57 B) 60 C) 63 D) 90 E) 93

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- **AJHSME** (Junior High Exam), 1985-1988, 50 cents per copy per year.
- **AJHSME Combination Set** containing 10 copies each of 1985, 1986, and 1987 AJHSME at \$12 per set.
- **AHSME 1972-88**, 50 cents per copy per year.
- **AIME 1983-88**, \$1 per copy per year.
- **AJHSME Summary of Results and Awards, 1985-87**, \$3 per copy per year.
- **AHSME Summary of Results and Awards, 1976-88**, \$4 per copy per year.

Books (Exams and solutions):

- Contest Problem Book I, AHSMEs 1950-60, \$8.50.
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5th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL
MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 30, 1989

Sponsored by

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Society of Actuaries Mu Alpha Theta
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1. $(1 + 11 + 21 + 31 + 41) + (9 + 19 + 29 + 39 + 49) =$

- A) 150 B) 199 C) 200 D) 249 E) 250

2. $\frac{2}{10} + \frac{4}{100} + \frac{6}{1000} =$

- A) .012 B) .0246 C) .12 D) .246 E) 246

3. Which of the following numbers is the largest?

- A) .99 B) .9099 C) .9 D) .909 E) .9009

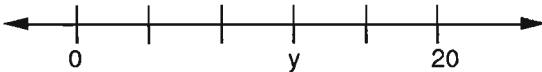
4. Estimate to determine which of the following numbers is closest to $\frac{401}{.205}$.

- A) .2 B) 2 C) 20 D) 200 E) 2000

5. $-15 + 9 \times (6 \div 3) =$

- A) -48 B) -12 C) -3 D) 3 E) 12

6. If the markings on the number line are equally spaced, what is the number y?



- A) 3 B) 10 C) 12 D) 15 E) 16

7. If the value of 20 quarters and 10 dimes equals the value of 10 quarters and n dimes, then n =

- A) 10 B) 20 C) 30 D) 35 E) 45

8. $(2 \times 3 \times 4) \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} \right) =$

- A) 1 B) 3 C) 9 D) 24 E) 26

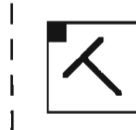
9. There are 2 boys for every 3 girls in Ms. Johnson's math class. If there are 30 students in her class, what percent of them are boys?

- A) 12% B) 20% C) 40% D) 60% E) $66\frac{2}{3}\%$

10. What is the number of degrees in the smaller angle between the hour hand and the minute hand on a clock that reads seven o'clock?

- A) 50° B) 120° C) 135° D) 150° E) 165°

11. Which of the five "T-like shapes" would be symmetric to the one shown with respect to the dashed line?



- A) B) C) D) E)

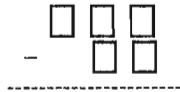
12. $\frac{1 - \frac{1}{3}}{1 - \frac{1}{2}} =$

- A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) $\frac{3}{2}$ E) $\frac{4}{3}$

13. $\frac{9}{7 \times 53} =$

- A) $\frac{.9}{.7 \times 53}$ B) $\frac{.9}{.7 \times .53}$ C) $\frac{.9}{.7 \times 5.3}$ D) $\frac{.9}{7 \times .53}$ E) $\frac{.09}{.07 \times .53}$

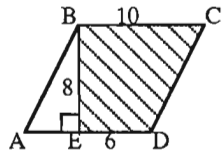
14. When placing each of the digits 2,4,5,6,9 in exactly one of the boxes of this subtraction problem, what is the smallest difference that is possible?



- A) 58 B) 123 C) 149 D) 171 E) 176

15. The area of the shaded region BEDC in parallelogram ABCD is

- A) 24 B) 48 C) 60 D) 64 E) 80



16. In how many ways can 47 be written as the sum of two primes?

- A) 0 B) 1 C) 2 D) 3 E) more than 3

17. The number N is between 9 and 17. The average of 6, 10, and N could be

- A) 8 B) 10 C) 12 D) 14 E) 16

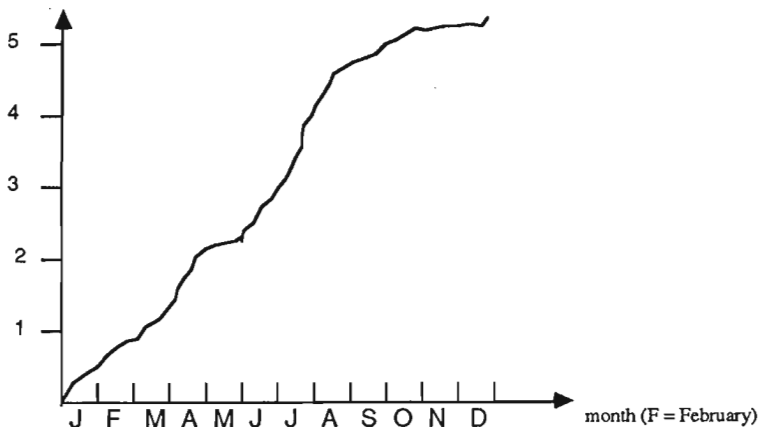
18. Many calculators have a reciprocal key $\frac{1}{x}$ that replaces the current number displayed with its reciprocal. For example, if the display is $\boxed{4}$ and the $\frac{1}{x}$ key is depressed, then the display becomes $\boxed{.25}$. If $\boxed{32}$ is currently displayed, what is the fewest number of times you must depress the $\frac{1}{x}$ key so the display again reads $\boxed{32}$?

A) 1 B) 2 C) 3 D) 4 E) 5

19. The graph below shows the total accumulated dollars (in millions) spent by the Surf City government during 1988. For example, about .5 million had been spent by the beginning of February and approximately 2 million by the end of April. Approximately how many millions of dollars were spent during the summer months of June, July, and August?

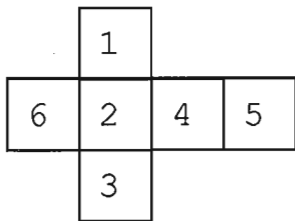
A) 1.5 B) 2.5 C) 3.5 D) 4.5 E) 5.5

dollars in millions



20. The figure may be folded along the lines shown to form a number cube. Three number faces come together at each corner of the cube. What is the largest sum of three numbers whose faces come together at a corner?

A) 11 B) 12 C) 13 D) 14 E) 15



21. Jack had a bag of 128 apples. He sold 25% of them to Jill. Next he sold 25% of those remaining to June. Of those apples still in his bag, he gave the shiniest one to his teacher. How many apples did Jack have then?

A) 7 B) 63 C) 65 D) 71 E) 111

22. The letters A,J,H,S,M,E and the digits 1,9,8,9 are "cycled" separately as follows and put in a numbered list:

AJHSME 1989

1. JHSMEA 9891
2. HSMEAJ 8919
3. SMEAJH 9198

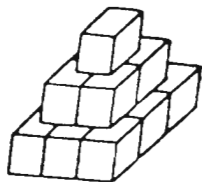
.....

What is the number of the line on which AJHSME 1989 will appear for the first time?

- A) 6 B) 10 C) 12 D) 18 E) 24

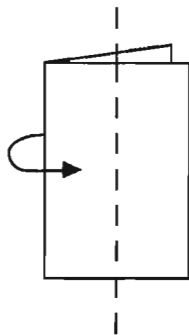
23. An artist has 14 cubes, each with an edge of 1 meter. She stands them on the ground to form a sculpture as shown. She then paints the exposed surface of the sculpture. How many square meters does she paint?

- A) 21 B) 24 C) 33 D) 37 E) 42



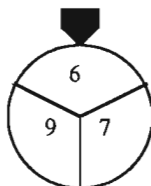
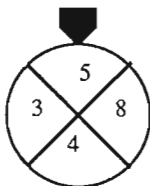
24. Suppose a square piece of paper is folded in half vertically. The folded paper is then cut in half along the dashed line. Three rectangles are formed—a large one and two small ones. What is the ratio of the perimeter of one of the small rectangles to the perimeter of the large rectangle?

- A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) $\frac{4}{5}$ E) $\frac{5}{6}$



25. Every time these two wheels are spun, two numbers are selected by the pointers. What is the probability that the sum of the two selected numbers is even?

- A) $\frac{1}{6}$ B) $\frac{3}{7}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) $\frac{5}{7}$



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- AJHSME (Junior High Exam), 1985-1989, 50 cents per copy per year.
- AJHSME Combination Set containing 10 copies each of 1985, 1986, 1987, and 1988 AJHSME at \$16 per set.
- AHSME 1972-89, 50 cents per copy per year.
- AIME 1983-89, \$2 per copy per year.
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AMERICAN MATHEMATICS COMPETITIONS

6th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL
MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 29, 1990

Sponsored by

Mathematical Association of America
Society of Actuaries Mu Alpha Theta
National Council of Teachers of Mathematics
Casualty Actuarial Society American Statistical Association
American Mathematical Association of Two-Year Colleges
American Mathematical Society

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
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1. What is the smallest sum of two 3-digit numbers that can be obtained by placing each of the six digits 4,5,6,7,8,9 in one of the six boxes in this addition problem?

A) 947 B) 1037 C) 1047 D) 1056 E) 1245

$$\begin{array}{r} \square \quad \square \quad \square \\ + \quad \square \quad \square \quad \square \\ \hline \end{array}$$

2. Which digit of .12345, when changed to 9, gives the largest number?

A) 1 B) 2 C) 3 D) 4 E) 5

3. What fraction of the square is shaded?

A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) $\frac{5}{12}$ D) $\frac{3}{7}$ E) $\frac{1}{2}$



4. Which of the following could **not** be the unit's digit [one's digit] of the square of a whole number?

A) 1 B) 4 C) 5 D) 6 E) 8

5. Which of the following is closest to the product $(.48017)(.48017)(.48017)$?

A) 0.011 B) 0.110 C) 1.10 D) 11.0 E) 110

6. Which of these five numbers is the largest?

A) $13579 + \frac{1}{2468}$ B) $13579 - \frac{1}{2468}$ C) $13579 \times \frac{1}{2468}$
 D) $13579 \div \frac{1}{2468}$ E) 13579.2468

7. When three different numbers from the set $\{-3, -2, -1, 4, 5\}$ are multiplied, the largest possible product is

A) 10 B) 20 C) 30 D) 40 E) 60

8. A dress originally priced at \$80 was put on sale at 25% off. If 10% tax was added to the sale price, then the total selling price of the dress was

A) \$45 B) \$52 C) \$54 D) \$66 E) \$68

9. The grading scale shown is used at Jones Junior High. The fifteen scores in Mr. Freeman's class were: 89, 72, 54, 97, 77, 92, 85, 74, 75, 63, 84, 78, 71, 80, 90.

| |
|-------------|
| A: 93 - 100 |
| B: 85 - 92 |
| C: 75 - 84 |
| D: 70 - 74 |
| F: 0 - 69 |

In Mr. Freeman's class, what percent of the students received a grade of C?

- A) 20% B) 25% C) 30% D) $33\frac{1}{3}\%$ E) 40%

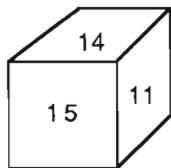
10. On this monthly calendar, the date behind one of the letters is added to the date behind C. If this sum equals the sum of the dates behind A and B, then the letter is

- A) P B) Q C) R D) S E) T

| | Tues. | Wed. | Thurs. | Fri. | Sat. |
|--|-------|------|--------|------|------|
| | | | C | A | |
| | | | Q | | |
| | S | B | P | T | R |
| | | | | | |

11. The numbers on the faces of this cube are consecutive whole numbers. The sums of the two numbers on each of the three pairs of opposite faces are equal. The sum of the six numbers on this cube is

- A) 75 B) 76 C) 78 D) 80 E) 81



12. There are twenty-four 4-digit whole numbers that use each of the four digits 2, 4, 5, and 7 exactly once. Listed in numerical order from smallest to largest, the number in the 17th position in the list is

- A) 4527 B) 5724 C) 5742 D) 7245 E) 7524

13. One proposal for new postage rates for a letter was 30¢ for the first ounce and 22¢ for each additional ounce (or fraction of an ounce). The postage for a letter weighing 4.5 ounces was

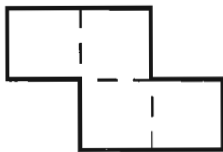
- A) 96¢ B) \$1.07 C) \$1.18 D) \$1.20 E) \$1.40

14. A bag contains only blue balls and green balls. There are 6 blue balls. If the probability of drawing a blue ball at random from this bag is $\frac{1}{4}$, then the number of green balls in the bag is

- A) 12 B) 18 C) 24 D) 30 E) 36

15. The area of this figure is 100 cm^2 .
Its perimeter is

A) 20 cm B) 25 cm C) 30 cm
D) 40 cm E) 50 cm



[figure consists
of four identical
squares]

16. $1990 - 1980 + 1970 - 1960 + \dots - 20 + 10 =$

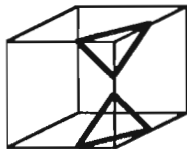
A) -990 B) -10 C) 990 D) 1000 E) 1990

17. A straight concrete sidewalk is to be 3 feet wide, 60 feet long and 3 inches thick. How many cubic yards of concrete must a contractor order for the sidewalk if concrete must be ordered in a whole number of cubic yards?

A) 2 B) 5 C) 12 D) 20 E) more than 20

18. Each corner of a rectangular prism is cut off. Two (of the eight) cuts are shown. How many edges does the new figure have?

A) 24 B) 30 C) 36 D) 42 E) 48



19. There are 120 seats in a row. What is the fewest number of seats that must be occupied so the next person to be seated must sit next to someone?

A) 30 B) 40 C) 41 D) 60 E) 119

20. The annual incomes of 1,000 families range from \$8200 to \$98,000. In error, the largest income was entered on the computer as \$980,000. The difference between the mean of the incorrect data and the mean of the actual data is

A) \$882 B) \$980 C) \$1078 D) \$482,000 E) \$882,000

21. A list of 8 numbers is formed by beginning with two given numbers. Each new number in the list is the product of the two previous numbers. Find the first number if the last three are shown:

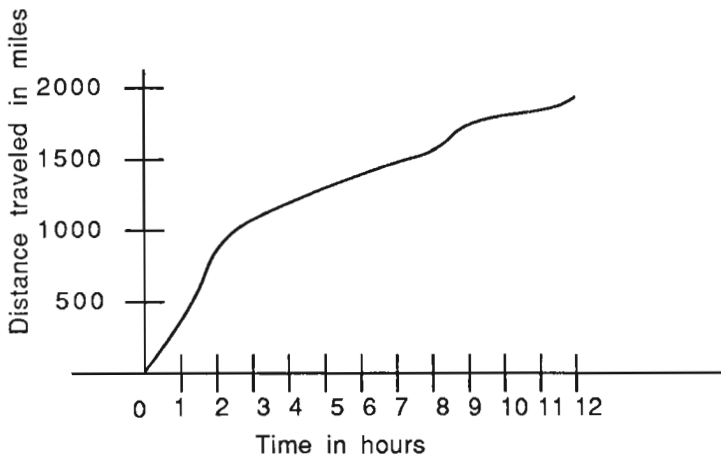
 , , , , , 16, 64, 1024

A) $\frac{1}{64}$ B) $\frac{1}{4}$ C) 1 D) 2 E) 4

22. Several students are seated at a large circular table. They pass around a bag containing 100 pieces of candy. Each person receives the bag, takes one piece of candy and then passes the bag to the next person. If Chris takes the first and the last piece of candy, then the number of students at the table could be

A) 10 B) 11 C) 19 D) 20 E) 25

23. The graph relates the distance traveled [in miles] to the time elapsed [in hours] on a trip taken by an experimental airplane. During which hour was the average speed of this airplane the largest?



- A) first (0-1) B) second (1-2) C) third (2-3) D) ninth (8-9) E) last (11-12)
24. Three Δ 's and a \diamond will balance nine \bullet 's. One Δ will balance a \diamond and a \bullet .



How many \bullet 's will balance the two \diamond 's in this balance?

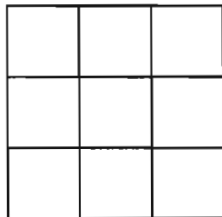
- A) 1 B) 2 C) 3 D) 4 E) 5



25. How many different patterns can be made by shading exactly two of the nine squares? Patterns that can be matched by flips and/or turns are not considered different. For example, the patterns shown below are **not** considered different.



- A) 3 B) 6 C) 8 D) 12 E) 18



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WRITE TO US!

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Prof Thomas Butts, AJHSME Chairman
Science Education Department
The University of Texas at Dallas
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Prof Walter E Mientka, AMC Executive Director
Department of Mathematics and Statistics
University of Nebraska, Lincoln, NE 68588-0322

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7th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL
MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 21, 1991

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1. $1,000,000,000,000 - 777,777,777,777 =$
 (A) 222,222,222,222 (B) 222,222,222,223 (C) 233,333,333,333
 (D) 322,222,222,223 (E) 333,333,333,333

2. $\frac{16 + 8}{4 - 2} =$
 (A) 4 (B) 8 (C) 12 (D) 16 (E) 20

3. Two hundred thousand times two hundred thousand equals
 (A) four hundred thousand
 (B) four million
 (C) forty thousand
 (D) four hundred million
 (E) forty billion

4. If $991 + 993 + 995 + 997 + 999 = 5000 - N$, then $N =$
 (A) 5 (B) 10 (C) 15 (D) 20 (E) 25

5. A "domino" is made up of two small squares: .
 Which of the "checkerboards" illustrated below CANNOT be covered exactly and completely by a whole number of non-overlapping dominoes?

(A) 3×4 (B) 3×5 (C) 4×4 (D) 4×5 (E) 6×3 

6. Which number in the array below is both the largest in its column and the smallest in its row? (Columns go up and down, rows go right and left.)

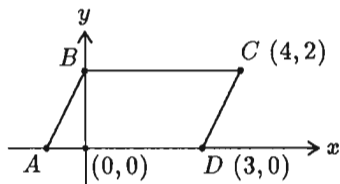
| | | | | |
|----|---|----|----|---|
| 10 | 6 | 4 | 3 | 2 |
| 11 | 7 | 14 | 10 | 8 |
| 8 | 3 | 4 | 5 | 9 |
| 13 | 4 | 15 | 12 | 1 |
| 8 | 2 | 5 | 9 | 3 |

- (A) 1 (B) 6 (C) 7 (D) 12 (E) 15

7. The value of $\frac{(487,000)(12,027,300) + (9,621,001)(487,000)}{(19,367)(.05)}$ is closest to
(A) 10,000,000 (B) 100,000,000 (C) 1,000,000,000
(D) 10,000,000,000 (E) 100,000,000,000
8. What is the largest quotient that can be formed using two numbers chosen from the set $\{-24, -3, -2, 1, 2, 8\}$?
(A) -24 (B) -3 (C) 8 (D) 12 (E) 24
9. How many whole numbers from 1 through 46 are divisible by either 3 or 5 or both?
(A) 18 (B) 21 (C) 24 (D) 25 (E) 27

10. The area in square units of the region enclosed by parallelogram $ABCD$ is

(A) 6 (B) 8 (C) 12
(D) 15 (E) 18



11. There are several sets of three different numbers whose sum is 15 which can be chosen from $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. How many of these sets contain a 5?
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

12. If $\frac{2+3+4}{3} = \frac{1990+1991+1992}{N}$, then $N =$
(A) 3 (B) 6 (C) 1990 (D) 1991 (E) 1992

13. How many zeros are at the end of the product

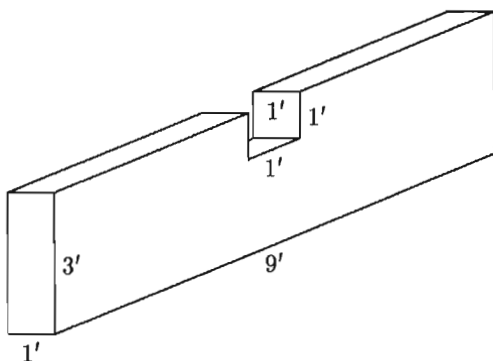
$$25 \times 25 \times 25 \times 25 \times 25 \times 25 \times 25 \times 8 \times 8 \times 8?$$

(A) 3 (B) 6 (C) 9 (D) 10 (E) 12

14. Several students are competing in a series of three races. A student earns 5 points for winning a race, 3 points for finishing second and 1 point for finishing third. There are no ties. What is the smallest number of points that a student must earn in the three races to be guaranteed of earning more points than any other student?

(A) 9 (B) 10 (C) 11 (D) 13 (E) 15

15. All six sides of a rectangular solid were rectangles. A one-foot cube was cut out of the rectangular solid as shown. The total number of square feet in the surface of the new solid is how many more or less than that of the original solid?



(A) 2 less (B) 1 less
(C) the same
(D) 1 more (E) 2 more

16. The 16 squares on a piece of paper are numbered as shown in the diagram. While lying on a table, the paper is folded in half four times in the following sequence:

- (1) fold the top half over the bottom half
- (2) fold the bottom half over the top half
- (3) fold the right half over the left half
- (4) fold the left half over the right half.

| | | | |
|----|----|----|----|
| 1 | 2 | 3 | 4 |
| 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 |

Which numbered square is on top after step 4?

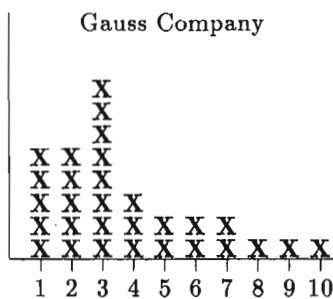
(A) 1 (B) 9 (C) 10 (D) 14 (E) 16

17. An auditorium with 20 rows of seats has 10 seats in the first row. Each successive row has one more seat than the previous row. If students taking an exam are permitted to sit in any row, but not next to another student in that row, then the maximum number of students that can be seated for an exam is

(A) 150 (B) 180 (C) 200 (D) 400 (E) 460

18. The vertical axis indicates the number of employees, but the scale was accidentally omitted from this graph. What percent of the employees at the Gauss Company have worked there for 5 years or more?

- (A) 9% (B) $23\frac{1}{3}\%$
 (C) 30% (D) $42\frac{6}{7}\%$
 (E) 50%



Number of years with company

19. The average (arithmetic mean) of 10 different positive whole numbers is 10. The largest possible value of any of these numbers is

- (A) 10 (B) 50 (C) 55 (D) 90 (E) 91

20. In the addition problem, each digit has been replaced by a letter. If different letters represent different digits then $C =$

- (A) 1 (B) 3 (C) 5 (D) 7 (E) 9

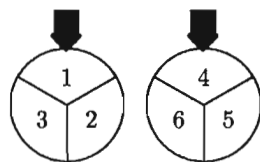
$$\begin{array}{r} A B C \\ B \\ + A \\ \hline 3 0 0 \end{array}$$

21. For every 3° rise in temperature, the volume of a certain gas expands by 4 cubic centimeters. If the volume of the gas is 24 cubic centimeters when the temperature is 32° , what was the volume of the gas in cubic centimeters when the temperature was 20° ?

- (A) 8 (B) 12 (C) 15 (D) 16 (E) 40

22. Each spinner is divided into 3 equal parts. The results obtained from spinning the two spinners are multiplied. What is the probability that this product is an even number?

- (A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{7}{9}$ (E) 1



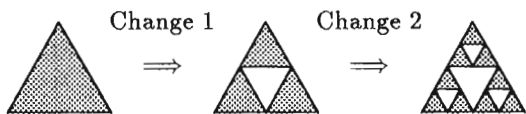
23. The Pythagoras High School band has 100 female and 80 male members. The Pythagoras High School orchestra has 80 female and 100 male members. There are 60 females who are members in both band and orchestra. Altogether, there are 230 students who are in either band or orchestra or both. The number of males in the band who are NOT in the orchestra is

(A) 10 (B) 20 (C) 30 (D) 50 (E) 70

24. A cube of edge 3 cm is cut into N smaller cubes, not all the same size. If the edge of each of the smaller cubes is a whole number of centimeters, then $N =$

(A) 4 (B) 8 (C) 12 (D) 16 (E) 20

25. An equilateral triangle is originally painted black. Each time the triangle is changed, the middle fourth of each black triangle turns white. After five changes, what fractional part of the original area of the black triangle remains black?



(A) $\frac{1}{1024}$ (B) $\frac{15}{64}$ (C) $\frac{243}{1024}$ (D) $\frac{1}{4}$ (E) $\frac{81}{256}$

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- Problem Book IV, AHSMEs 1973-82, \$11.
- USA Mathematical Olympiad Book 1972-86, \$13.
- International Mathematical Olympiad Book I, 1959-77, \$10.
- International Mathematical Olympiad Book II, 1978-85, \$11.

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8th ANNUAL

AMERICAN JUNIOR HIGH SCHOOL
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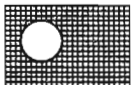
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INSTRUCTIONS

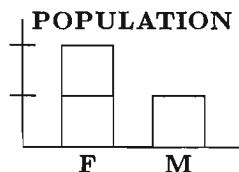
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1. $\frac{10 - 9 + 8 - 7 + 6 - 5 + 4 - 3 + 2 - 1}{1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + 9} =$
 (A) -1 (B) 1 (C) 5 (D) 9 (E) 10
2. Which of the following is not equal to $\frac{5}{4}$?
 (A) $\frac{10}{8}$ (B) $1\frac{1}{4}$ (C) $1\frac{3}{12}$ (D) $1\frac{1}{5}$ (E) $1\frac{10}{40}$
3. What is the largest difference that can be formed by subtracting two numbers chosen from the set $\{-16, -4, 0, 2, 4, 12\}$?
 (A) 10 (B) 12 (C) 16 (D) 28 (E) 48
4. During the softball season, Judy had 35 hits. Among her hits were 1 home run, 1 triple and 5 doubles. The rest of her hits were singles. What percent of her hits were singles?
 (A) 28% (B) 35% (C) 70% (D) 75% (E) 80%
5. A circle of diameter 1 is removed from a 2×3 rectangle, as shown. Which whole number is closest to the area of the shaded region?
 (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
- 
6. Suppose that $\begin{array}{c} \triangle \\ a \\ b \quad c \end{array}$ means $a + b - c$. For example, $\begin{array}{c} \triangle \\ 5 \\ 4 \quad 6 \end{array}$ is $5 + 4 - 6 = 3$.
 Then the sum $\begin{array}{c} \triangle \\ 1 \\ 3 \quad 4 \end{array} + \begin{array}{c} \triangle \\ 2 \\ 5 \quad 6 \end{array}$ is
 (A) -2 (B) -1 (C) 0 (D) 1 (E) 2
7. The digit-sum of 998 is $9 + 9 + 8 = 26$. How many 3-digit whole numbers, whose digit-sum is 26, are even?
 (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
8. A store owner bought 1500 pencils at \$0.10 each. If he sells them for \$0.25 each, how many of them must he sell to make a profit of exactly \$100.00?
 (A) 400 (B) 667 (C) 1000 (D) 1500 (E) 1900

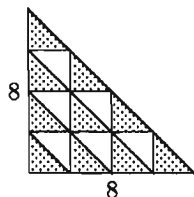
9. The population of a small town is 480. The graph indicates the number of females and males in the town, but the vertical scale-values are omitted. How many males live in the town?

(A) 120 (B) 160 (C) 200
(D) 240 (E) 360



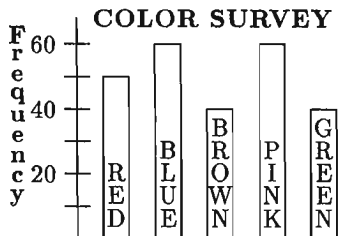
10. An isosceles right triangle with legs of length 8 is partitioned into 16 congruent triangles as shown. The shaded area is

(A) 10 (B) 20 (C) 32 (D) 40 (E) 64



11. The bar graph shows the results of a survey on color preferences. What percent preferred blue?

(A) 20% (B) 24% (C) 30%
(D) 36% (E) 42%



12. The five tires of a car (four road tires and a full-sized spare) were rotated so that each tire was used the same number of miles during the first 30,000 miles the car traveled. For how many miles was each tire used?

(A) 6000 (B) 7500 (C) 24,000 (D) 30,000 (E) 37,500

13. Five test scores have a mean (average score) of 90, a median (middle score) of 91 and a mode (most frequent score) of 94. The sum of the two lowest test scores is

(A) 170 (B) 171 (C) 176 (D) 177
(E) not determined by the information given

14. When four gallons are added to a tank that is one-third full, the tank is then one-half full. The capacity of the tank in gallons is

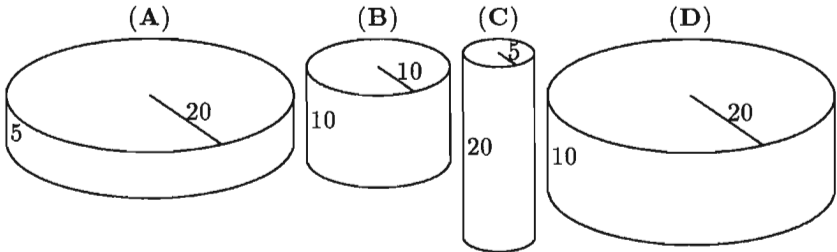
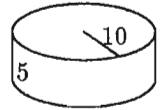
(A) 8 (B) 12 (C) 20 (D) 24 (E) 48

15. What is the 1992nd letter in this sequence?

ABCDED CBAABCDED CBAABCDED CBAABCDED C...

- (A) A (B) B (C) C (D) D (E) E

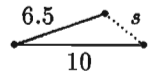
16. Which cylinder has twice the volume of the cylinder shown to the right?



- (E) None of the above

17. The sides of a triangle have lengths 6.5, 10, and s , where s is a whole number. What is the smallest possible value of s ?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7



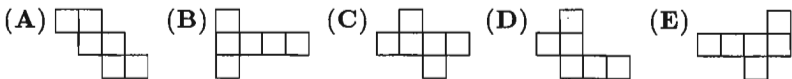
18. On a trip, a car traveled 80 miles in an hour and a half, then was stopped in traffic for 30 minutes, then traveled 100 miles during the next 2 hours. What was the car's average speed in miles per hour for the 4-hour trip?

- (A) 45 (B) 50 (C) 60 (D) 75 (E) 90

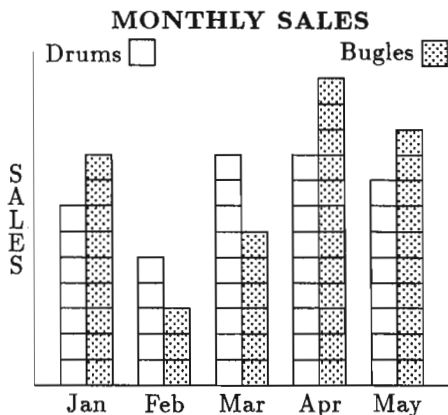
19. The distance between the 5th and 26th exits on an interstate highway is 118 miles. If any two exits are at least 5 miles apart, then what is the largest number of miles there can be between two consecutive exits that are between the 5th and 26th exits?

- (A) 8 (B) 13 (C) 18 (D) 47 (E) 98

20. Which pattern of identical squares could NOT be folded along the lines shown to form a cube?

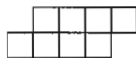


21. Northside's Drum and Bugle Corps raised money for a trip. The drummers and bugle players kept separate sales records. According to the double bar graph, in what month did one group's sales exceed the other's by the greatest percent?



- (A) Jan (B) Feb
(C) Mar (D) Apr
(E) May

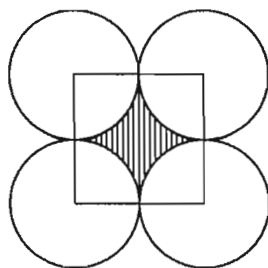
22. Eight 1×1 square tiles are arranged as shown so their outside edges form a polygon with a perimeter of 14 units. Two additional tiles of the same size are added to the figure so that at least one side of each tile is shared with a side of one of the squares in the original figure. Which of the following could be the perimeter of the new figure?



- (A) 15 (B) 17 (C) 18 (D) 19 (E) 20
23. If two dice are tossed, the probability that the product of the numbers showing on the tops of the dice is greater than 10 is

- (A) $\frac{3}{7}$ (B) $\frac{17}{36}$ (C) $\frac{1}{2}$ (D) $\frac{5}{8}$ (E) $\frac{11}{12}$

24. Four circles of radius 3 are arranged as shown. Their centers are the vertices of a square. The area of the shaded region is closest to



- (A) 7.7 (B) 12.1 (C) 17.2
(D) 18 (E) 27

25. One half of the water is poured out of a full container. Then one third of the remainder is poured out. Continue the process: one fourth of the remainder for the third pouring, one fifth of the remainder for the fourth pouring, etc. After how many pourings does exactly one tenth of the original water remain?

- (A) 6 (B) 7 (C) 8 (D) 9 (E) 10

SOLUTIONS

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WRITE TO US!

Questions and comments about the problems and solutions for this AJHSME (but not requests for the Solutions Pamphlet) should be addressed to:

Mr Bruce Brombacher, AJHSME Chairman
Jones Middle School
Upper Arlington, OH 43221

Comments about administrative arrangements and orders for any publications listed below should be addressed to:

Prof Walter E Mientka, AMC Executive Director
Department of Mathematics and Statistics
University of Nebraska, Lincoln, NE 68588-0658

1993 AHSME

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PUBLICATIONS

MINIMUM ORDER: \$5 (before handling fee), US FUNDS ONLY. Canada and US orders must be prepaid. Orders are mailed 4th class, unless you specify 1st class, in which case add \$3 or 20% of total order, whichever is larger, with a maximum of \$15. Make checks payable to the American Mathematics Competitions.

FOREIGN ORDERS: do NOT prepay; an invoice will be sent.

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Examinations: Each price is for an examination and its solutions for one year. Specify the years you want and how many copies of each. All prices effective to July 1, 1993.

- **AJHSME** (Junior High Exam), 1985-1992, \$1 per copy per year.
- **AHSME** 1972-92, \$1 per copy per year.
- **AIME** 1983-92, \$2 per copy per year.
- **AJHSME Summary of Results and Awards**, 1985-91, \$4 per copy per year.
- **AHSME Summary of Results and Awards**, 1980-92, \$4 per copy per year.

Books (Exams and solutions):

- Problem Book I, AHSMEs 1950-60, \$8.50.
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- Problem Book IV, AHSMEs 1973-82, \$11.
- USA Mathematical Olympiad Book 1972-86, \$13.
- International Mathematical Olympiad Book I, 1959-77, \$10.
- International Mathematical Olympiad Book II, 1978-85, \$11.

1992
American Junior High School Mathematics Examination

**DO NOT OPEN UNTIL
THURSDAY, NOVEMBER 19, 1992**

(AJHSME)

1. All information (Rules and Instructions) needed to administer the AJHSME is contained in the AJHSME TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE NOVEMBER 19.** Nothing is needed from inside this package until November 19.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AJHSME CERTIFICATION Form that all rules associated with the conduct of the examination were followed.
3. Results must be mailed by First Class Mail to Dr. Mientka no later than 48 hours following the Examination.
4. THE AJHSME IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAMINATION MAY BE GIVEN DURING THE REGULAR MATHEMATICS CLASS PERIOD OF THE STUDENTS IF IT IS NOT POSSIBLE TO ADMINISTER THE EXAMINATION TO ALL STUDENTS DURING ONE 40 MINUTE PERIOD.

AMERICAN MATHEMATICS COMPETITIONS

9th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL
MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 18, 1993

Sponsored by

Mathematical Association of America
Society of Actuaries Mu Alpha Theta
National Council of Teachers of Mathematics
Casualty Actuarial Society American Statistical Association
American Mathematical Association of Two-Year Colleges
American Mathematical Society

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AJHSME ANSWER FORM with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer sheet will be graded.
4. There is no penalty for guessing. Your score on this test is the number of correct answers.
5. No aids other than calculators, scratch paper, graph paper, rulers and erasers are permitted. No problems on the test will *require* the use of a calculator.
6. Unless specified otherwise, figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working the problems. You will have **40 MINUTES** working time for the test.

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1. Which pair of numbers does NOT have a product equal to 36?

- (A) $\{-4, -9\}$ (B) $\{-3, -12\}$ (C) $\left\{\frac{1}{2}, -72\right\}$
 (D) $\{1, 36\}$ (E) $\left\{\frac{3}{2}, 24\right\}$

2. When the fraction $\frac{49}{84}$ is expressed in simplest form, then the sum of the numerator and the denominator will be

- (A) 11 (B) 17 (C) 19 (D) 33 (E) 133

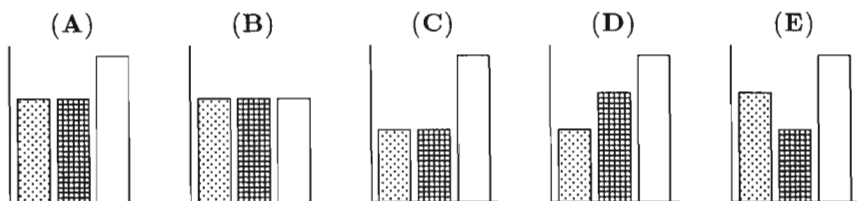
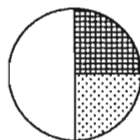
3. Which of the following numbers has the largest prime factor?

- (A) 39 (B) 51 (C) 77 (D) 91 (E) 121

4. $1000 \times 1993 \times 0.1993 \times 10 =$

- (A) 1.993×10^3 (B) 1993.1993 (C) $(199.3)^2$
 (D) 1,993,001.993 (E) $(1993)^2$

5. Which one of the following bar graphs could represent the data from the circle graph?



6. A can of soup can feed 3 adults or 5 children. If there are 5 cans of soup and 15 children are fed, then how many adults would the remaining soup feed?

- (A) 5 (B) 6 (C) 7 (D) 8 (E) 10

7. $3^3 + 3^3 + 3^3 =$

- (A) 3^4 (B) 9^3 (C) 3^9 (D) 27^3 (E) 3^{27}

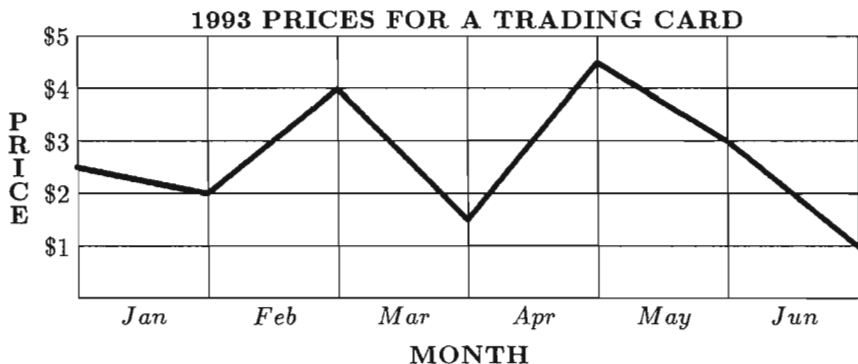
8. To control her blood pressure, Jill's grandmother takes one half of a pill every other day. If one supply of medicine contains 60 pills, then the supply of medicine will last approximately
- (A) 1 month (B) 4 months (C) 6 months
(D) 8 months (E) 1 year

9. Consider the operation $*$ defined by the following table:

| $*$ | 1 | 2 | 3 | 4 |
|-----|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 |
| 2 | 2 | 4 | 1 | 3 |
| 3 | 3 | 1 | 4 | 2 |
| 4 | 4 | 3 | 2 | 1 |

For example, $3 * 2 = 1$. Then $(2 * 4) * (1 * 3) =$

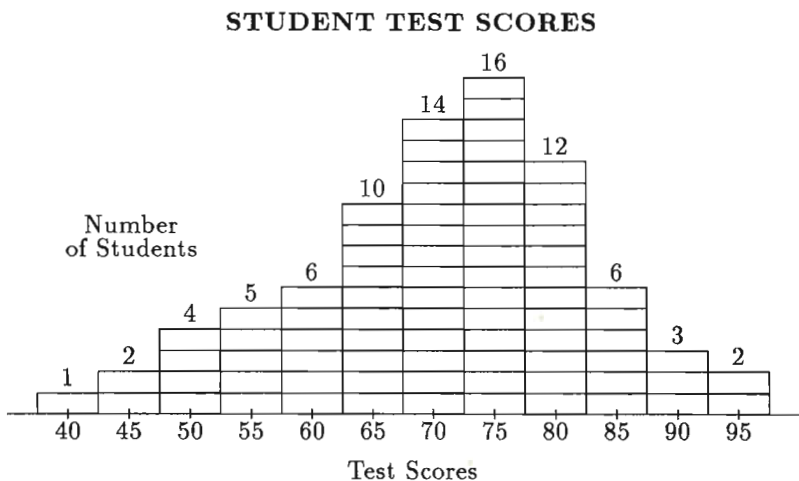
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
10. This line graph represents the price of a trading card during the first 6 months of 1993.



The greatest monthly drop in price occurred during

- (A) January (B) March (C) April (D) May (E) June

11. Consider this histogram of the scores for 81 students taking a test:



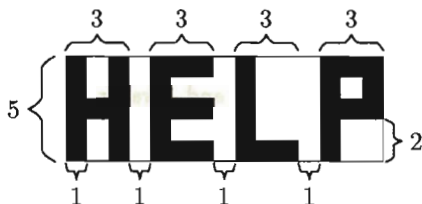
The median is in the interval labeled

- (A) 60 (B) 65 (C) 70 (D) 75 (E) 80
12. If each of the three operation signs, $+$, $-$, \times , is used exactly ONCE in one of the blanks in the expression

$$5 _ 4 _ 6 _ 3$$

then the value of the result could equal

- (A) 9 (B) 10 (C) 15 (D) 16 (E) 19
13. The word "HELP" in block letters is painted in black with strokes 1 unit wide on a 5 by 15 rectangular white sign with dimensions as shown. The area of the white portion of the sign, in square units, is



- (A) 30 (B) 32 (C) 34 (D) 36 (E) 38

14. The nine squares in the table shown are to be filled so that every row and every column contains each of the numbers 1, 2, 3. Then $A + B =$

| | | |
|---|---|---|
| 1 | | |
| | 2 | A |
| | | B |

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

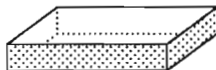
15. The arithmetic mean (average) of four numbers is 85. If the largest of these numbers is 97, then the mean of the remaining three numbers is

- (A) 81.0 (B) 82.7 (C) 83.0 (D) 84.0 (E) 84.3

16.
$$\frac{1}{1 + \frac{1}{2 + \frac{1}{3}}} =$$

- (A) $\frac{1}{6}$ (B) $\frac{3}{10}$ (C) $\frac{7}{10}$ (D) $\frac{5}{6}$ (E) $\frac{10}{3}$

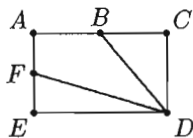
17. Square corners, 5 units on a side, are removed from a 20 unit by 30 unit rectangular sheet of cardboard. The sides are then folded to form an open box. The surface area, in square units, of the interior of the box is



- (A) 300 (B) 500 (C) 550 (D) 600 (E) 1000

18. The rectangle shown has length $AC = 32$, width $AE = 20$, and B and F are midpoints of \overline{AC} and \overline{AE} , respectively.

The area of the quadrilateral $ABDF$ is



- (A) 320 (B) 325 (C) 330
(D) 335 (E) 340

19. $(1901 + 1902 + 1903 + \cdots + 1993) - (101 + 102 + 103 + \cdots + 193) =$

- (A) 167,400 (B) 172,050 (C) 181,071 (D) 199,300 (E) 362,142

20. When $10^{93} - 93$ is expressed as a single whole number, the sum of the digits is

- (A) 10 (B) 93 (C) 819 (D) 826 (E) 833

21. If the length of a rectangle is increased by 20% and its width is increased by 50%, then the area is increased by
(A) 10% (B) 30% (C) 70% (D) 80% (E) 100%
22. Pat Peano has plenty of 0's, 1's, 3's, 4's, 5's, 6's, 7's, 8's and 9's, but he has only twenty-two 2's. How far can he number the pages of his scrapbook with these digits?
(A) 22 (B) 99 (C) 112 (D) 119 (E) 199
23. Five runners, P, Q, R, S, T , have a race, and P beats Q , P beats R , Q beats S , and T finishes after P and before Q . Who could NOT have finished third in the race?
(A) P and Q (B) P and R (C) P and S
(D) P and T (E) P, S and T
24. What number is directly above 142 in this array of numbers?

| | | | | | |
|----|----|----|-----|---|---|
| | | | 1 | | |
| | | 2 | 3 | 4 | |
| | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | ... | | |

- (A) 99 (B) 119 (C) 120 (D) 121 (E) 122
25. A checkerboard consists of one-inch squares. A square card, 1.5 inches on a side, is placed on the board so that it covers part or all of the area of each of n squares. The maximum possible value of n is
(A) 4 or 5 (B) 6 or 7 (C) 8 or 9 (D) 10 or 11 (E) 12 or more

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Prof Walter E Mientka, AMC Executive Director
Department of Mathematics and Statistics, University of Nebraska
Lincoln, NE 68588-0658; Phone: 402-472-2257; Fax: 402-472-6087

1994 AHSME

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1993
American Junior High School Mathematics Examination

(AJHSME)

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AMERICAN MATHEMATICS COMPETITIONS

10th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL
MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 17, 1994

Sponsored by

Mathematical Association of America
Society of Actuaries Mu Alpha Theta
National Council of Teachers of Mathematics
Casualty Actuarial Society American Statistical Association
American Mathematical Association of Two-Year Colleges
American Mathematical Society
American Society of Pension Actuaries

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1. Which of the following is the largest?

- (A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{3}{8}$ (D) $\frac{5}{12}$ (E) $\frac{7}{24}$

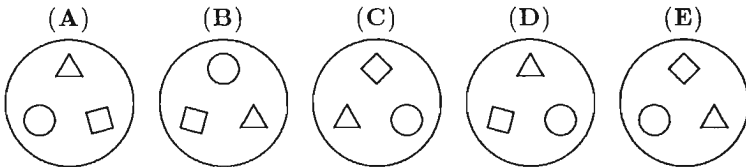
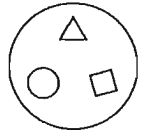
2. $\frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{4}{10} + \frac{5}{10} + \frac{6}{10} + \frac{7}{10} + \frac{8}{10} + \frac{9}{10} + \frac{55}{10} =$

- (A) $4\frac{1}{2}$ (B) 6.4 (C) 9 (D) 10 (E) 11

3. Each day Maria must work 8 hours. This does not include the 45 minutes she takes for lunch. If she begins working at 7:25 A.M. and takes her lunch break at noon, then her working day will end at

- (A) 3:40 P.M. (B) 3:55 P.M. (C) 4:10 P.M.
(D) 4:25 P.M. (E) 4:40 P.M.

4. Which of the following represents the result when the figure shown at the right is rotated clockwise 120° about its center?



5. Given that 1 mile = 8 furlongs and 1 furlong = 40 rods, the number of rods in one mile is

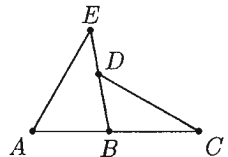
- (A) 5 (B) 320 (C) 660 (D) 1760 (E) 5280

6. The unit's digit (one's digit) of the product of any six consecutive positive whole numbers is

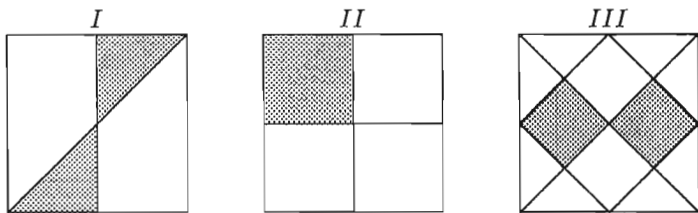
- (A) 0 (B) 2 (C) 4 (D) 6 (E) 8

7. If $\angle A = 60^\circ$, $\angle E = 40^\circ$ and $\angle C = 30^\circ$, then $\angle BDC =$

- (A) 40° (B) 50° (C) 60°
(D) 70° (E) 80°



8. For how many three-digit whole numbers does the sum of the digits equal 25?
 (A) 2 (B) 4 (C) 6 (D) 8 (E) 10
9. A shopper buys a \$100 coat on sale for 20% off. An additional \$5 is taken off the sale price by using a discount coupon. A sales tax of 8% is paid on the final selling price. The total amount the shopper pays for the coat is
 (A) \$81.00 (B) \$81.40 (C) \$82.00 (D) \$82.08 (E) \$82.40
10. For how many positive integer values of N ($N > 0$) is the expression $\frac{36}{N+2}$ an integer?
 (A) 7 (B) 8 (C) 9 (D) 10 (E) 12
11. Last summer 100 students attended basketball camp. Of those attending, 52 were boys and 48 were girls. Also, 40 students were from Jones Middle School and 60 were from Clay Middle School. Twenty of the girls were from Jones Middle School. How many of the boys were from Clay Middle School?
 (A) 20 (B) 32 (C) 40 (D) 48 (E) 52
12. Each of the three large squares shown below is the same size. Segments that intersect the sides of the squares intersect at the midpoints of the sides. How do the shaded areas of these squares compare?



- (A) The shaded areas in all three are equal.
 (B) Only the shaded areas of I and III are equal.
 (C) Only the shaded areas of I and III are equal.
 (D) Only the shaded areas of II and III are equal.
 (E) The shaded areas of I , II and III are all different.

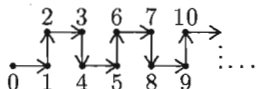
13. The number halfway between $\frac{1}{6}$ and $\frac{1}{4}$ is

(A) $\frac{1}{10}$ (B) $\frac{1}{5}$ (C) $\frac{5}{24}$ (D) $\frac{7}{24}$ (E) $\frac{5}{12}$

14. Two children at a time can play pairball. For 90 minutes, with only two children playing at one time, five children take turns so that each one plays the same amount of time. The number of minutes each child plays is

(A) 9 (B) 10 (C) 18 (D) 20 (E) 36

15. If this path is to continue in the same pattern:



then which sequence of arrows goes from point 425 to point 427?

(A) (B) (C) (D) (E)

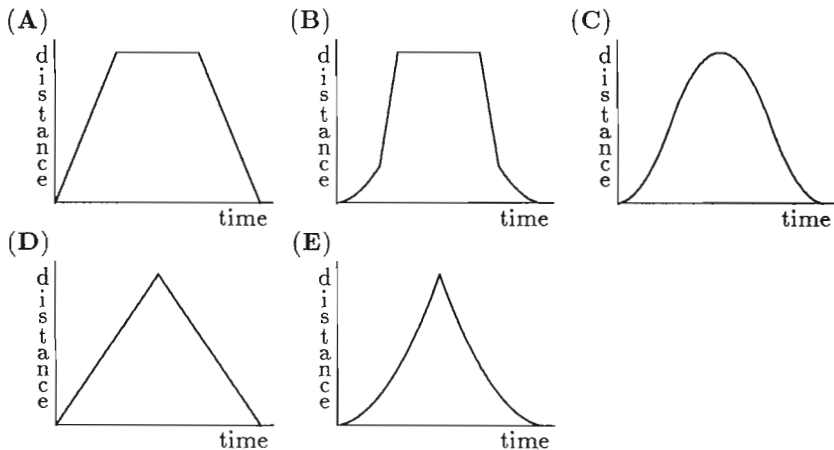
16. The perimeter of one square is 3 times the perimeter of another square. The area of the larger square is how many times the area of the smaller square?

(A) 2 (B) 3 (C) 4 (D) 6 (E) 9

17. Pauline Bunyan can shovel snow at the rate of 20 cubic yards for the first hour, 19 cubic yards for the second, 18 for the third, etc., always shoveling one cubic yard less per hour than the previous hour. If her driveway is 4 yards wide, 10 yards long, and covered with snow 3 yards deep, then the number of hours it will take her to shovel it clean is closest to

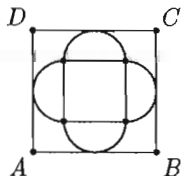
(A) 4 (B) 5 (C) 6 (D) 7 (E) 12

18. Mike leaves home and drives slowly east through city traffic. When he reaches the highway he drives east more rapidly until he reaches the shopping mall where he stops. He shops at the mall for an hour. Mike returns home by the same route as he came, driving west rapidly along the highway and then slowly through city traffic. Each graph shows the distance from home on the vertical axis versus the time elapsed since leaving home on the horizontal axis. Which graph is the best representation of Mike's trip?



19. Around the outside of a 4 by 4 square, construct four semicircles (as shown in the figure) with the four sides of the square as their diameters. Another square, $ABCD$, has its sides parallel to the corresponding sides of the original square, and each side of $ABCD$ is tangent to one of the semicircles. The area of the square $ABCD$ is

(A) 16 (B) 32 (C) 36 (D) 48 (E) 64



20. Let W , X , Y and Z be four different digits selected from the set

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

If the sum $\frac{W}{X} + \frac{Y}{Z}$ is to be as small as possible, then $\frac{W}{X} + \frac{Y}{Z}$ must equal

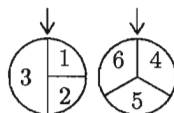
(A) $\frac{2}{17}$ (B) $\frac{3}{17}$ (C) $\frac{17}{72}$ (D) $\frac{25}{72}$ (E) $\frac{13}{36}$

21. A gumball machine contains 9 red, 7 white, and 8 blue gumballs. The least number of gumballs a person must buy to be sure of getting four gumballs of the same color is

(A) 8 (B) 9 (C) 10 (D) 12 (E) 18

22. The two wheels shown at the right are spun and the two resulting numbers are added. The probability that the sum of the two numbers is even is

(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{5}{12}$ (E) $\frac{4}{9}$



23. If X , Y and Z are different digits, then the largest possible 3-digit sum for

$$\begin{array}{r} XXX \\ YX \\ + \underline{X} \end{array}$$

has the form

(A) XXY (B) XYZ (C) YYX (D) YYZ (E) ZZY

24. A 2 by 2 square is divided into four 1 by 1 squares. Each of the small squares is to be painted either green or red. In how many different ways can the painting be accomplished so that no green square shares its top or right side with any red square? There may be as few as zero or as many as four small green squares.

(A) 4 (B) 6 (C) 7 (D) 8 (E) 16

25. Find the sum of the digits in the answer to

$$\underbrace{9999 \dots 99}_{94 \text{ nines}} \times \underbrace{4444 \dots 44}_{94 \text{ fours}}$$

where a string of 94 nines is multiplied by a string of 94 fours.

(A) 846 (B) 855 (C) 945 (D) 954 (E) 1072

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Upper Arlington, OH 43221

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Prof Walter E Mientka, AMC Executive Director
Department of Mathematics and Statistics, University of Nebraska
Lincoln, NE 68588-0658; Phone: 402-472-2257; Fax: 402-472-6087

1995 AHSME

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- **AHSME** (High School Exam) 1980-94, \$1 per copy per year.
- **AJHSME Summary of Results and Awards**, 1985-93, \$4 per copy per year.
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- Problem Book II, AHSMEs 1961-65, \$9.50
- Problem Book III, AHSMEs 1966-72, \$11.50
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- International Mathematical Olympiad Book I, 1959-77, \$11.50
- International Mathematical Olympiad Book II, 1978-85, \$11.50

1994

American Junior High School Mathematics Examination
(AJHSME)

DO NOT OPEN UNTIL
THURSDAY, NOVEMBER 17, 1994

*****Administration On An Earlier Date Will Disqualify
Your School's Results*****

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AMERICAN MATHEMATICS COMPETITIONS

11th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL
MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 16, 1995

Sponsored by

Mathematical Association of America
Society of Actuaries Mu Alpha Theta
National Council of Teachers of Mathematics
Casualty Actuarial Society American Statistical Association
American Mathematical Association of Two-Year Colleges
American Mathematical Society
American Society of Pension Actuaries

INSTRUCTIONS

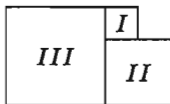
1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
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3. The answers to the problems are to be marked on the AJHSME ANSWER FORM with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer sheet will be graded.
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1. Walter has exactly one penny, one nickel, one dime and one quarter in his pocket. What percent of one dollar is in his pocket?
 (A) 4% (B) 25% (C) 40% (D) 41% (E) 59%
2. Jose is 4 years younger than Zack. Zack is 3 years older than Inez. Inez is 15 years old. How old is Jose?
 (A) 8 (B) 11 (C) 14 (D) 16 (E) 22
3. Which of the following operations has the same effect on a number as multiplying by $\frac{3}{4}$ and then dividing by $\frac{3}{5}$?
 (A) dividing by $\frac{4}{3}$ (B) dividing by $\frac{9}{20}$ (C) multiplying by $\frac{9}{20}$
 (D) dividing by $\frac{5}{4}$ (E) multiplying by $\frac{5}{4}$
4. A teacher tells the class,
"Think of a number, add 1 to it, and double the result. Give the answer to your partner. Partner, subtract 1 from the number you are given and double the result to get your answer."
 Ben thinks of 6, and gives his answer to Sue. What should Sue's answer be?
 (A) 18 (B) 24 (C) 26 (D) 27 (E) 30
5. Find the smallest whole number that is larger than the sum

$$2\frac{1}{2} + 3\frac{1}{3} + 4\frac{1}{4} + 5\frac{1}{5}$$

- (A) 14 (B) 15 (C) 16 (D) 17 (E) 18
6. Figures *I*, *II* and *III* are squares. The perimeter of *I* is 12 and the perimeter of *II* is 24. The perimeter of *III* is
 (A) 9 (B) 18 (C) 36 (D) 72 (E) 81



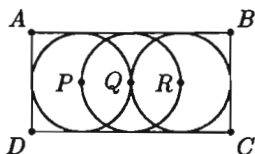
7. At Clover View Junior High, one half of the students go home on the school bus. One fourth go home by automobile. One tenth go home on their bicycles. The rest walk home. What fractional part of the students walk home?
 (A) $\frac{1}{16}$ (B) $\frac{3}{20}$ (C) $\frac{1}{3}$ (D) $\frac{17}{20}$ (E) $\frac{9}{10}$

8. An American traveling in Italy wishes to exchange American money (dollars) for Italian money (lire). If 3000 lire = \$1.60, how many lire will the traveler receive in exchange for \$1.00?

(A) 180 (B) 480 (C) 1800 (D) 1875 (E) 4875

9. Three congruent circles with centers P , Q and R are tangent to the sides of rectangle $ABCD$ as shown. The circle centered at Q has diameter 4 and passes through points P and R . The area of the rectangle is

(A) 16 (B) 24 (C) 32
(D) 64 (E) 128

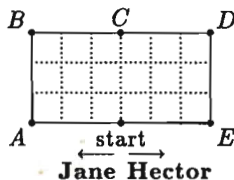


10. A jacket and a shirt originally sold for \$80 and \$40, respectively. During a sale Chris bought the \$80 jacket at a 40% discount and the \$40 shirt at a 55% discount. The total amount saved was what percent of the total of the original prices?

(A) 45% (B) $47\frac{1}{2}\%$ (C) 50% (D) $79\frac{1}{6}\%$ (E) 95%

11. Jane can walk any distance in half the time it takes Hector to walk the same distance. They set off in opposite directions around the outside of the 18-block area as shown. When they meet for the first time, they will be closest to

(A) A (B) B (C) C
(D) D (E) E

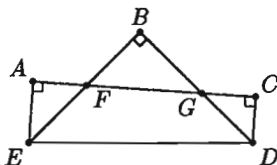


12. A *lucky* year is one in which at least one date, when written in the form month/day/year, has the following property: *The product of the month times the day equals the last two digits of the year.* For example, 1956 is a lucky year because it has the date 7/8/56 and $7 \times 8 = 56$. Which of the following is NOT a lucky year?

(A) 1990 (B) 1991 (C) 1992 (D) 1993 (E) 1994

13. In the figure, $\angle A$, $\angle B$ and $\angle C$ are right angles. If $\angle AEB = 40^\circ$ and $\angle BED = \angle BDE$, then $\angle CDE =$

(A) 75° (B) 80° (C) 85°
(D) 90° (E) 95°



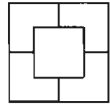
14. A team won 40 of its first 50 games. How many of the remaining 40 games must this team win so it will have won exactly 70% of its games for the season?
 (A) 20 (B) 23 (C) 28 (D) 30 (E) 35
15. What is the 100th digit to the right of the decimal point in the decimal form of $4/37$?
 (A) 0 (B) 1 (C) 2 (D) 7 (E) 8
16. Students from three middle schools worked on a summer project.
 Seven students from Allen School worked for 3 days.
 Four students from Balboa School worked for 5 days.
 Five students from Carver School worked for 9 days.
 The total amount paid for the students' work was \$774. Assuming each student received the same amount for a day's work, how much did the students from Balboa School earn altogether?
 (A) \$9.00 (B) \$48.38 (C) \$180.00 (D) \$193.50 (E) \$258.00
17. The table below gives the percent of students in each grade at Annville and Cleona elementary schools:

| | <u>K</u> | <u>1</u> | <u>2</u> | <u>3</u> | <u>4</u> | <u>5</u> | <u>6</u> |
|------------|----------|----------|----------|----------|----------|----------|----------|
| Annville : | 16% | 15% | 15% | 14% | 13% | 16% | 11% |
| Cleona : | 12% | 15% | 14% | 13% | 15% | 14% | 17% |

Annville has 100 students and Cleona has 200 students. In the two schools combined, what percent of the students are in grade 6?

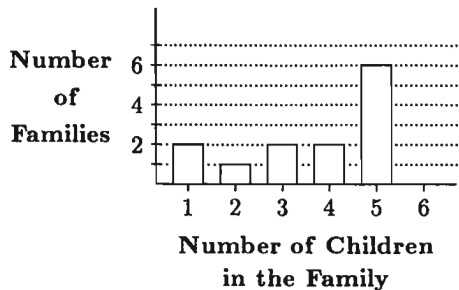
- (A) 12% (B) 13% (C) 14% (D) 15% (E) 28%
18. The area of each of the four congruent L-shaped regions of this 100-inch by 100-inch square is $3/16$ of the total area. How many inches long is the side of the center square?

- (A) 25 (B) 44 (C) 50 (D) 62 (E) 75



19. The graph shows the distribution of the number of children in the families of the students in Ms. Jordan's English class. The median number of children in the family for this distribution is

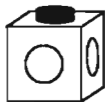
- (A) 1 (B) 2 (C) 3
 (D) 4 (E) 5



20. Diana and Apollo each roll a standard die obtaining a number at random from 1 to 6. What is the probability that Diana's number is larger than Apollo's number?

(A) $\frac{1}{3}$ (B) $\frac{5}{12}$ (C) $\frac{4}{9}$ (D) $\frac{17}{36}$ (E) $\frac{1}{2}$

21. A plastic snap-together cube has a protruding snap on one side and receptacle holes on the other five sides as shown. What is the smallest number of these cubes that can be snapped together so that only receptacle holes are showing?



(A) 3 (B) 4 (C) 5 (D) 6 (E) 8

22. The number 6545 can be written as a product of a pair of positive two-digit numbers. What is the sum of this pair of numbers?

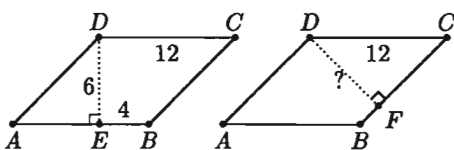
(A) 162 (B) 172 (C) 173 (D) 174 (E) 222

23. How many four-digit whole numbers are there such that the leftmost digit is odd, the second digit is even, and all four digits are different?

(A) 1120 (B) 1400 (C) 1800 (D) 2025 (E) 2500

24. In parallelogram $ABCD$, \overline{DE} is the altitude to the base \overline{AB} and \overline{DF} is the altitude to the base \overline{BC} . [Note: Both pictures represent the same parallelogram.] If $DC = 12$, $EB = 4$ and $DE = 6$, then $DF =$

(A) 6.4 (B) 7 (C) 7.2
(D) 8 (E) 10



25. Buses from Dallas to Houston leave every hour on the hour. Buses from Houston to Dallas leave every hour on the half hour. The trip from one city to the other takes 5 hours. Assuming the buses travel on the same highway, how many Dallas-bound buses does a Houston-bound bus pass on the highway (not in the station)?

(A) 5 (B) 6 (C) 9 (D) 10 (E) 11

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1995
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(AJHSME)

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AMERICAN MATHEMATICS COMPETITIONS

12th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL
MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 21, 1996

Sponsored by

Mathematical Association of America
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9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

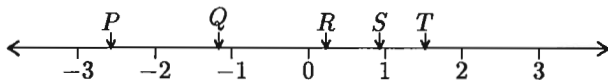
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1. How many positive factors of 36 are also multiples of 4?
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6
2. José, Thuy, and Kareem each start with the number 10. José subtracts 1 from the number 10, doubles his answer, and then adds 2. Thuy doubles the number 10, subtracts 1 from her answer, and then adds 2. Kareem subtracts 1 from the number 10, adds 2 to his answer, and then doubles the result. Who gets the largest final answer?
(A) José (B) Thuy (C) Kareem
(D) José and Thuy (E) Thuy and Kareem
3. The 64 whole numbers from 1 through 64 are written, one per square, on a checkerboard (an 8 by 8 array of 64 squares). The first 8 numbers are written in order across the first row, the next 8 across the second row, and so on. After all 64 numbers are written, the sum of the numbers in the four corners will be
(A) 130 (B) 131 (C) 132 (D) 133 (E) 134

4. $\frac{2+4+6+\cdots+34}{3+6+9+\cdots+51} =$

- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{17}{3}$ (E) $\frac{34}{3}$

5. The letters P , Q , R , S , and T represent numbers located on the number line as shown.



Which of the following expressions represents a negative number?

- (A) $P - Q$ (B) $P \cdot Q$ (C) $\frac{S}{Q} \cdot P$ (D) $\frac{R}{P \cdot Q}$ (E) $\frac{S+T}{R}$
6. What is the smallest result that can be obtained by the following process?
- Choose three different numbers from the set $\{3, 5, 7, 11, 13, 17\}$.
 - Add two of these numbers.
 - Multiply their sum by the third number.
- (A) 15 (B) 30 (C) 36 (D) 50 (E) 56

7. Brent has goldfish that quadruple (become four times as many) every month, and Gretel has goldfish that double every month. If Brent has 4 goldfish at the same time that Gretel has 128 goldfish, then in how many months from that time will they have the same number of goldfish?
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8
8. Points A and B are 10 units apart. Points B and C are 4 units apart. Points C and D are 3 units apart. If A and D are as close as possible, then the number of units between them is
(A) 0 (B) 3 (C) 9 (D) 11 (E) 17
9. If 5 times a number is 2, then 100 times the reciprocal of the number is
(A) 2.5 (B) 40 (C) 50 (D) 250 (E) 500
10. When Walter drove up to the gasoline pump, he noticed that his gasoline tank was $\frac{1}{8}$ full. He purchased 7.5 gallons of gasoline for \$10. With this additional gasoline, his gasoline tank was then $\frac{5}{8}$ full. The number of gallons of gasoline his tank holds when it is full is
(A) 8.75 (B) 10 (C) 11.5 (D) 15 (E) 22.5

11. Let x be the number

$$0.\underbrace{0000\dots00001}_{1996 \text{ zeros}},$$

where there are 1996 zeros after the decimal point. Which of the following expressions represents the largest number?

- (A) $3 + x$ (B) $3 - x$ (C) $3 \cdot x$ (D) $3/x$ (E) $x/3$
12. What number should be removed from the list

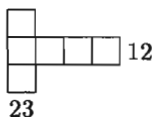
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

so that the average of the remaining numbers is 6.1?

- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8
13. In the fall of 1996, a total of 800 students participated in an annual school clean-up day. The organizers of the event expect that in each of the years 1997, 1998, and 1999, participation will increase by 50% over the previous year. The number of participants the organizers expect in the fall of 1999 is
(A) 1200 (B) 1500 (C) 2000 (D) 2400 (E) 2700

14. Six different digits from the set

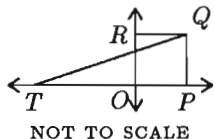
$$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$



are placed in the squares in the figure shown so that the sum of the entries in the vertical column is 23 and the sum of the entries in the horizontal row is 12. The sum of the six digits used is

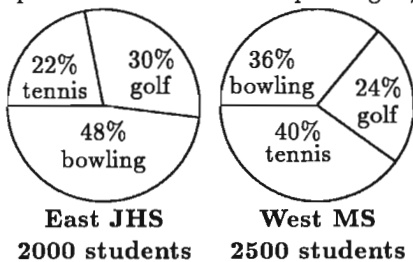
- (A) 27 (B) 29 (C) 31 (D) 33 (E) 35
15. The remainder when the product $1492 \cdot 1776 \cdot 1812 \cdot 1996$ is divided by 5 is
 (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
16. $1 - 2 - 3 + 4 + 5 - 6 - 7 + 8 + 9 - 10 - 11 + 12 + 13 - \dots$
 $\dots + 1992 + 1993 - 1994 - 1995 + 1996 =$
 (A) -998 (B) -1 (C) 0 (D) 1 (E) 998

17. Figure $OPQR$ is a square. Point O is the origin, and point Q has coordinates $(2, 2)$. What are the coordinates for T so that the area of triangle PQT equals the area of square $OPQR$?



- (A) $(-6, 0)$ (B) $(-4, 0)$ (C) $(-2, 0)$
 (D) $(2, 0)$ (E) $(4, 0)$
18. Ana's monthly salary was \$2000 in May. In June she received a 20% raise. In July she received a 20% pay cut. After the two changes in June and July, Ana's monthly salary was
 (A) \$1920 (B) \$1980 (C) \$2000 (D) \$2020 (E) \$2040

19. The pie charts at the right indicate the percent of students who prefer golf, bowling, or tennis at East Junior High School and West Middle School. The total number of students at East is 2000 and at West, 2500. In the two schools combined, the percent of students who prefer tennis is



- (A) 30% (B) 31%
 (C) 32% (D) 33% (E) 34%

20. Suppose there is a special key on a calculator that replaces the number x currently displayed with the number given by the formula $1/(1-x)$. For example, if the calculator is displaying 2 and the special key is pressed, then the calculator will display -1 since $1/(1-2) = -1$. Now suppose that the calculator is displaying 5. After the special key is pressed 100 times in a row, the calculator will display

(A) -0.25 (B) 0 (C) 0.8 (D) 1.25 (E) 5

21. How many subsets containing three different numbers can be selected from the set

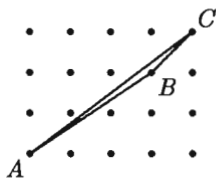
$$\{89, 95, 99, 132, 166, 173\}$$

so that the sum of the three numbers is even?

(A) 6 (B) 8 (C) 10 (D) 12 (E) 24

22. The horizontal and vertical distances between adjacent points equal 1 unit. The area of triangle ABC is

(A) $1/4$ (B) $1/2$ (C) $3/4$
 (D) 1 (E) $5/4$

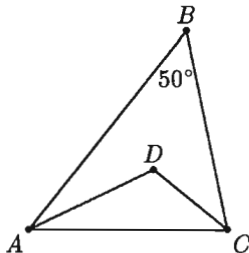


23. The manager of a company planned to distribute a \$50 bonus to each employee from the company fund, but the fund contained \$5 less than what was needed. Instead the manager gave each employee a \$45 bonus and kept the remaining \$95 in the company fund. The amount of money in the company fund before any bonuses were paid was

(A) \$945 (B) \$950 (C) \$955 (D) \$990 (E) \$995

24. The measure of angle ABC is 50° , \overline{AD} bisects angle BAC , and \overline{DC} bisects angle BCA . The measure of angle ADC is

(A) 90° (B) 100° (C) 115°
 (D) 122.5° (E) 125°



25. A point is chosen at random from within a circular region. What is the probability that the point is closer to the center of the region than it is to the boundary of the region?

(A) $1/4$ (B) $1/3$ (C) $1/2$ (D) $2/3$ (E) $3/4$

SOLUTIONS

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WRITE TO US!

Correspondence about the problems and solutions for this AJHSME should be addressed to:

Mr Bruce Brombacher, AJHSME Chairman
Jones Middle School
Upper Arlington, OH 43221

Comments about administrative arrangements should be addressed to:

Prof Walter E Mientka, AMC Executive Director
Department of Mathematics and Statistics, University of Nebraska
Lincoln, NE 68588-0658; Phone: 402-472-2257; Fax: 402-472-6087

1997 AHSME

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1996
American Junior High School Mathematics Examination
(AJHSME)

DO NOT OPEN UNTIL
THURSDAY, NOVEMBER 21, 1996

*****Administration On An Earlier Date Will Disqualify
Your School's Results*****

1. All information (Rules and Instructions) needed to administer the AJHSME is contained in the AJHSME TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE NOVEMBER 21.** Nothing is needed from inside this package until November 21.
2. Your PRINCIPAL or VICE PRINCIPAL must verify on the AJHSME CERTIFICATION Form that all rules associated with the conduct of the examination were followed.
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AMERICAN MATHEMATICS COMPETITIONS
13th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL
MATHEMATICS EXAMINATION
(AJHSME)
THURSDAY, NOVEMBER 20, 1997

Sponsored by

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6. Figures are not necessarily drawn to scale.
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1. $\frac{1}{10} + \frac{9}{100} + \frac{9}{1000} + \frac{7}{10000} =$
(A) 0.0026 (B) 0.0197 (C) 0.1997 (D) 0.26 (E) 1.997
2. Ahn chooses a two-digit number, subtracts it from 200, and doubles the result. What is the largest number Ahn can get?
(A) 200 (B) 202 (C) 220 (D) 380 (E) 398
3. Which of the following numbers is the largest?
(A) 0.97 (B) 0.979 (C) 0.9709 (D) 0.907 (E) 0.9089
4. Julie is preparing a speech for her class. Her speech must last between one-half hour and three-quarters of an hour. The ideal rate of speech is 150 words per minute. If Julie speaks at the ideal rate, which of the following number of words would be an appropriate length for her speech?
(A) 2250 (B) 3000 (C) 4200 (D) 4350 (E) 5650
5. There are many two-digit multiples of 7, but only two of the multiples have a digit sum of 10. The sum of these two multiples of 7 is
(A) 119 (B) 126 (C) 140 (D) 175 (E) 189
6. In the number 74982.1035 the value of the *place* occupied by the digit 9 is how many times as great as the value of the *place* occupied by the digit 3?
(A) 1,000 (B) 10,000 (C) 100,000 (D) 1,000,000 (E) 10,000,000
7. The area of the smallest square that will contain a circle of radius 4 is
(A) 8 (B) 16 (C) 32 (D) 64 (E) 128
8. Walter gets up at 6:30 a.m., catches the school bus at 7:30 a.m., has 6 classes that last 50 minutes each, has 30 minutes for lunch, and has 2 hours additional time at school. He takes the bus home and arrives at 4:00 p.m. How many minutes has he spent on the bus?
(A) 30 (B) 60 (C) 75 (D) 90 (E) 120

9. Three students, with different names, line up single file. What is the probability that they are in alphabetical order from front-to-back?

(A) $\frac{1}{12}$ (B) $\frac{1}{9}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

10. What fraction of this square region is shaded? Stripes are equal in width, and the figure is drawn to scale.

(A) $\frac{5}{12}$ (B) $\frac{1}{2}$ (C) $\frac{7}{12}$ (D) $\frac{2}{3}$ (E) $\frac{5}{6}$



11. Let \boxed{N} mean the number of whole number divisors of N . For example, $\boxed{3} = 2$, because 3 has two divisors, 1 and 3. Find the value of

$$\boxed{11} \times \boxed{20}$$

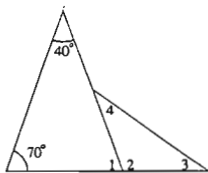
- (A) 6 (B) 8 (C) 12 (D) 16 (E) 24

12. $\angle 1 + \angle 2 = 180^\circ$

$$\angle 3 = \angle 4$$

Find $\angle 4$

(A) 20° (B) 25° (C) 30° (D) 35° (E) 40°



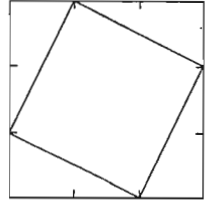
13. Three bags of jelly beans contain 26, 28, and 30 beans. The ratios of yellow beans to all beans in each of these bags are 50%, 25%, and 20%, respectively. All three bags of candy are dumped into one bowl. Which of the following is closest to the ratio of yellow jelly beans to all beans in the bowl?

(A) 31% (B) 32% (C) 33% (D) 35% (E) 95%

14. There is a set of five positive integers whose average (mean) is 5, whose median is 5, and whose only mode is 8. What is the difference between the largest and smallest integers in the set?

(A) 3 (B) 5 (C) 6 (D) 7 (E) 8

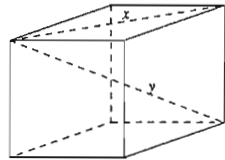
15. Each side of the large square in the figure is trisected (divided into three equal parts). The corners of an inscribed square are at these trisection points, as shown. The ratio of the area of the inscribed square to the area of the large square is



- (A) $\frac{\sqrt{3}}{3}$ (B) $\frac{5}{9}$ (C) $\frac{2}{3}$ (D) $\frac{\sqrt{5}}{3}$ (E) $\frac{7}{9}$
16. Penni Precisely buys \$100 worth of stock in each of three companies: Alabama Almonds, Boston Beans, and California Cauliflower. After one year, AA was up 20%, BB was down 25%, and CC was unchanged. For the second year, AA was down 20% from the previous year, BB was up 25% from the previous year, and CC was unchanged. If A , B , and C are the final values of the stock, then

- (A) $A = B = C$ (B) $A = B < C$ (C) $C < B = A$
 (D) $A < B < C$ (E) $B < A < C$

17. A cube has eight vertices (corners) and twelve edges. A segment, such as x , which joins two vertices not joined by an edge is called a diagonal. Segment y is also a diagonal. How many diagonals does a cube have?



- (A) 6 (B) 8 (C) 12 (D) 14 (E) 16
18. At the grocery store last week, small boxes of facial tissue were priced at 4 boxes for \$5. This week they are on sale at 5 boxes for \$4. The percent decrease in the price per box during the sale was closest to

- (A) 30% (B) 35% (C) 40% (D) 45% (E) 65%

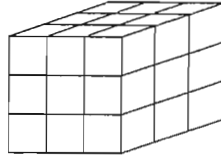
19. If the product $\frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} \cdot \frac{6}{5} \cdot \dots \cdot \frac{a}{b} = 9$, what is the sum of a and b ?

- (A) 11 (B) 13 (C) 17 (D) 35 (E) 37

20. A pair of 8-sided dice have sides numbered 1 through 8. Each side has the same probability (chance) of landing face up. The probability that the product of the two numbers on the sides that land face-up exceeds 36 is

(A) $\frac{5}{32}$ (B) $\frac{11}{64}$ (C) $\frac{3}{16}$ (D) $\frac{1}{4}$ (E) $\frac{1}{2}$

21. Each corner cube is removed from this 3 cm x 3 cm x 3 cm cube. The surface area of the remaining figure is



- (A) 19 sq.cm (B) 24 sq.cm (C) 30 sq.cm (D) 54 sq.cm (E) 72 sq.cm
22. A two-inch cube ($2 \times 2 \times 2$) of silver weighs 3 pounds and is worth \$200. How much is a three-inch cube of silver worth?
- (A) \$300 (B) \$375 (C) \$450 (D) \$560 (E) \$675

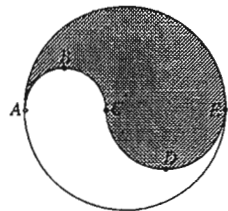
23. There are positive integers that have these properties:

- I. the sum of the squares of their digits is 50, and
 II. each digit is larger than the one to its left.

The product of the digits of the largest integer with both properties is

- (A) 7 (B) 25 (C) 36 (D) 48 (E) 60

24. Diameter ACE is divided at C in the ratio 2:3. The two semicircles, ABC and CDE, divide the circular region into an upper (shaded) region and a lower region. The ratio of the area of the upper region to that of the lower region is



- (A) 2:3 (B) 1:1 (C) 3:2 (D) 9:4 (E) 5:2

25. All of the even numbers from 2 to 98 inclusive, except those ending in 0, are multiplied together. What is the rightmost digit (the units digit) of the product?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

SOLUTIONS

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WRITE TO US!

Correspondence about the problems and solutions for this AJHSME should be addressed to:

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Comments about administrative arrangements should be addressed to:

Professor Walter E Mientka, AMC Executive Director
Department of Mathematics and Statistics, University of Nebraska
Lincoln, NE 68588-0658; Phone: 402-472-2257; Fax: 402-472-6087
eMail: walter@amc.unl.edu

1998 AHSME

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1997

American Junior High School Mathematics Examination
(AJHSME)

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AMERICAN MATHEMATICS COMPETITIONS
14th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL
MATHEMATICS EXAMINATION
(AJHSME)
TUESDAY, NOVEMBER 17, 1998

Sponsored by

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Casualty Actuarial Society American Statistical Association
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1. For $x = 7$, which of the following is smallest?

(A) $\frac{6}{x}$ (B) $\frac{6}{x+1}$ (C) $\frac{6}{x-1}$ (D) $\frac{x}{6}$ (E) $\frac{x+1}{6}$

2. If $\frac{a}{c} \mid \frac{b}{d} = a \cdot d - b \cdot c$, what is the value of $\frac{3 \mid 4}{1 \mid 2}$?

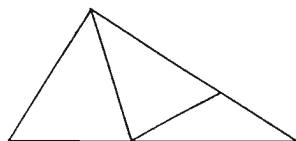
(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

3. $\frac{\frac{3}{8} + \frac{7}{8}}{\frac{4}{5}} =$

(A) 1 (B) $\frac{25}{16}$ (C) 2 (D) $\frac{43}{20}$ (E) $\frac{47}{16}$

4. How many triangles are in this figure?
(Some triangles may overlap other triangles.)

(A) 9 (B) 8 (C) 7 (D) 6 (E) 5

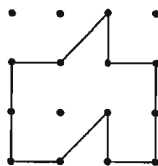


5. Which of the following numbers is largest?

(A) 9.12344 (B) $9.123\overline{4}$ (C) $9.12\overline{34}$ (D) $9.1\overline{234}$ (E) $9.\overline{1234}$

6. Dots are spaced one unit apart, horizontally and vertically. The number of square units enclosed by the polygon is

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9



7. $100 \times 19.98 \times 1.998 \times 1000 =$

(A) $(1.998)^2$ (B) $(19.98)^2$ (C) $(199.8)^2$ (D) $(1998)^2$ (E) $(19980)^2$

8. A child's wading pool contains 200 gallons of water. If water evaporates at the rate of 0.5 gallons per day and no other water is added or removed, how many gallons of water will be in the pool after 30 days?
- (A) 140 (B) 170 (C) 185 (D) 198.5 (E) 199.85
9. For a sale, a store owner reduces the price of a \$10 scarf by 20%. Later the price is lowered again, this time by one-half the reduced price. The price is now
- (A) \$2.00 (B) \$3.75 (C) \$4.00 (D) \$4.90 (E) \$6.40
10. Each of the letters W, X, Y, and Z represents a different integer in the set {1, 2, 3, 4}, but not necessarily in that order. If $\frac{W}{X} - \frac{Y}{Z} = 1$, then the sum of W and Y is
- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7
11. Harry has 3 sisters and 5 brothers. His sister Harriet has S sisters and B brothers. What is the product of S and B?
- (A) 8 (B) 10 (C) 12 (D) 15 (E) 18
12. $2(1 - \frac{1}{2}) + 3(1 - \frac{1}{3}) + 4(1 - \frac{1}{4}) + \dots + 10(1 - \frac{1}{10}) =$
- (A) 45 (B) 49 (C) 50 (D) 54 (E) 55
13. What is the ratio of the area of the shaded square to the area of the large square? (The figure is drawn to scale.)
- (A) $\frac{1}{6}$ (B) $\frac{1}{7}$ (C) $\frac{1}{8}$ (D) $\frac{1}{12}$ (E) $\frac{1}{16}$



14. At Annville Junior High School, 30% of the students in the Math Club are in the Science Club, and 80% of the students in the Science Club are in the Math Club. There are 15 students in the Science Club. How many students are in the Math Club?
- (A) 12 (B) 15 (C) 30 (D) 36 (E) 40

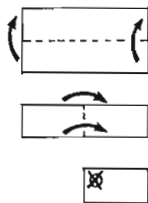
Problems 15, 16, and 17 all refer to the following:

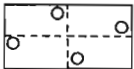
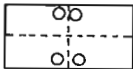

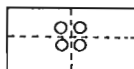
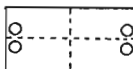
Don't Crowd The Isles

In the very center of the Irenic Sea lie the beautiful Nisos Isles. In 1998 the number of people on these islands is only 200, but the population triples every 25 years. Queen Irene has decreed that there must be at least 1.5 square miles for every person living in the Isles. The total area of the Nisos Isles is 24,900 square miles.

15. Estimate the population of Nisos in the year 2050.
 (A) 600 (B) 800 (C) 1000 (D) 2000 (E) 3000
16. Estimate the year in which the population of Nisos will be approximately 6,000.
 (A) 2050 (B) 2075 (C) 2100 (D) 2125 (E) 2150
17. In how many years, approximately, from 1998 will the population of Nisos be as much as Queen Irene has proclaimed that the islands can support?
 (A) 50 yrs. (B) 75 yrs. (C) 100 yrs. (D) 125 yrs. (E) 150 yrs.

18. As indicated by the diagram at the right, a rectangular piece of paper is folded bottom to top, then left to right, and finally, a hole is punched at X. What does the paper look like when unfolded?

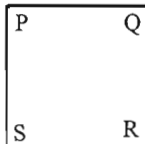


- (A) 
- (B) 
- (C) 
- (D) 
- (E) 

19. Tamika selects two different numbers at random from the set $\{8, 9, 10\}$ and adds them. Carlos takes two different numbers at random from the set $\{3, 5, 6\}$ and multiplies them. What is the probability that Tamika's result is greater than Carlos' result?

(A) $\frac{4}{9}$ (B) $\frac{5}{9}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

20. Let PQRS be a square piece of paper. P is folded onto R and then Q is folded onto S. The area of the resulting figure is 9 square inches. Find the perimeter of square PQRS.



(A) 9 (B) 16 (C) 18 (D) 24 (E) 36

21. A $4 \times 4 \times 4$ cubical box contains 64 identical small cubes that exactly fill the box. How many of these small cubes touch a side or the bottom of the box?

(A) 48 (B) 52 (C) 60 (D) 64 (E) 80

22. Terri produces a sequence of positive integers by following three rules. She starts with a positive integer, then applies the appropriate rule to the result, and continues in this fashion.

Rule 1: If the integer is less than 10, multiply it by 9.

Rule 2: If the integer is even and greater than 9, divide it by 2.

Rule 3: If the integer is odd and greater than 9, subtract 5 from it.

A sample sequence: 23, 18, 9, 81, 76, ...

Find the 98th term of the sequence that begins 98, 49, ...

(A) 6 (B) 11 (C) 22 (D) 27 (E) 54

23. If the pattern in the diagram continues, what fraction of the interior would be shaded in the eighth triangle?



(A) $\frac{3}{8}$ (B) $\frac{5}{27}$ (C) $\frac{7}{16}$ (D) $\frac{9}{16}$ (E) $\frac{11}{45}$

24. A rectangular board of 8 columns has squares numbered beginning in the upper left corner and moving left to right so row one is numbered 1 through 8, row two is 9 through 16, and so on. A student shades square 1, then skips one square and shades square 3, skips two squares and shades square 6, skips 3 squares and shades square 10, and continues in this way until there is at least one shaded square in each column. What is the number of the shaded square that first achieves this result?

(A) 36 (B) 64 (C) 78 (D) 91 (E) 120

| | | | | | | | |
|---|---|----|----|----|----|---|----|
| | 2 | | 4 | 5 | | 7 | 8 |
| 9 | | 11 | 12 | 13 | 14 | | 16 |
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25. Three generous friends, each with some cash, redistribute their money as follows: Ami gives enough money to Jan and Toy to double the amount that each has. Jan then gives enough to Ami and Toy to double their amounts. Finally, Toy gives Ami and Jan enough to double their amounts. If Toy has \$36 when they begin and \$36 when they end, what is the total amount that all three friends have?

(A) \$108 (B) \$180 (C) \$216 (D) \$252 (E) \$288

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1998

**American Junior High School Mathematics Examination
(AJHSME)**

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