MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

63rd Annual

AMC 12 A

American Mathematics Contest 12 A

Tuesday, February 7, 2012

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

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Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 30th annual American Invitational Mathematics Examination (AIME) on Thursday, March 15, 2012 or Wednesday, March 28, 2012. More details about the AIME and other information are on the back page of this test booklet.

2012

AMC 12 A

DO NOT OPEN UNTIL TUESDAY, FEBRUARY 7, 2012

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3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

The American Mathematics Competitions

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1. A bug crawls along a number line, starting at -2. It crawls to -6, then turns around and crawls to 5. How many units does the bug crawl altogether?

   (A) 9 **(B)** 11 **(C)** 13 **(D)** 14 **(E)** 15

2. Cagney can frost a cupcake every 20 seconds and Lacey can frost a cupcake every 30 seconds. Working together, how many cupcakes can they frost in 5 minutes?

   (A) 10 **(B)** 15 **(C)** 20 **(D)** 25 **(E)** 30

3. A box 2 centimeters high, 3 centimeters wide, and 5 centimeters long can hold 40 grams of clay. A second box with twice the height, three times the width, and the same length as the first box can hold n grams of clay. What is n?

   (A) 120 **(B)** 160 **(C)** 200 **(D)** 240 **(E)** 280

4. In a bag of marbles, $\frac{3}{5}$ of the marbles are blue and the rest are red. If the number of red marbles is doubled and the number of blue marbles stays the same, what fraction of the marbles will be red?

   (A) $\dfrac{2}{5}$ **(B)** $\dfrac{3}{7}$ **(C)** $\dfrac{4}{7}$ **(D)** $\dfrac{3}{5}$ **(E)** $\dfrac{4}{5}$

5. A fruit salad consists of blueberries, raspberries, grapes, and cherries. The fruit salad has a total of 280 pieces of fruit. There are twice as many raspberries as blueberries, three times as many grapes as cherries, and four times as many cherries as raspberries. How many cherries are there in the fruit salad?

   (A) 8 **(B)** 16 **(C)** 25 **(D)** 64 **(E)** 96

6. The sums of three whole numbers taken in pairs are 12, 17, and 19. What is the middle number?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

7. Mary divides a circle into 12 sectors. The central angles of these sectors, measured in degrees, are all integers and they form an arithmetic sequence. What is the degree measure of the smallest possible sector angle?

   (A) 5 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

8. An *iterative average* of the numbers 1, 2, 3, 4, and 5 is computed in the following way. Arrange the five numbers in some order. Find the mean of the first two numbers, then find the mean of that with the third number, then the mean of that with the fourth number, and finally the mean of that with the fifth number. What is the difference between the largest and smallest possible values that can be obtained using this procedure?

   (A) $\frac{31}{16}$ **(B)** 2 **(C)** $\frac{17}{8}$ **(D)** 3 **(E)** $\frac{65}{16}$

9. A year is a leap year if and only if the year number is divisible by 400 (such as 2000) or is divisible by 4 but not by 100 (such as 2012). The 200th anniversary of the birth of novelist Charles Dickens was celebrated on February 7, 2012, a Tuesday. On what day of the week was Dickens born?

   (A) Friday **(B)** Saturday **(C)** Sunday **(D)** Monday **(E)** Tuesday

10. A triangle has area 30, one side of length 10, and the median to that side of length 9. Let θ be the acute angle formed by that side and the median. What is $\sin\theta$?

    (A) $\frac{3}{10}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{9}{20}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{9}{10}$

11. Alex, Mel, and Chelsea play a game that has 6 rounds. In each round there is a single winner, and the outcomes of the rounds are independent. For each round the probability that Alex wins is $\frac{1}{2}$, and Mel is twice as likely to win as Chelsea. What is the probability that Alex wins three rounds, Mel wins two rounds, and Chelsea wins one round?

    (A) $\frac{5}{72}$ **(B)** $\frac{5}{36}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{3}$ **(E)** 1

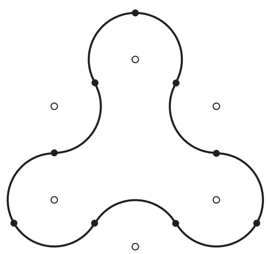
12. A square region $ABCD$ is externally tangent to the circle with equation $x^2+y^2=1$ at the point $(0,1)$ on the side CD. Vertices A and B are on the circle with equation $x^2+y^2=4$. What is the side length of this square?

    (A) $\frac{\sqrt{10}+5}{10}$ **(B)** $\frac{2\sqrt{5}}{5}$ **(C)** $\frac{2\sqrt{2}}{3}$ **(D)** $\frac{2\sqrt{19}-4}{5}$ **(E)** $\frac{9-\sqrt{17}}{5}$

13. Paula the painter and her two helpers each paint at constant, but different, rates. They always start at 8:00 AM and all three always take the same amount of time to eat lunch. On Monday the three of them painted 50% of a house, quitting at 4:00 PM. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at 2:12 PM. On Wednesday Paula worked by herself and finished the house by working until 7:12 PM. How long, in minutes, was each day's lunch break?

(A) 30 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 60

14. The closed curve in the figure is made up of 9 congruent circular arcs each of length $\frac{2\pi}{3}$, where each of the centers of the corresponding circles is among the vertices of a regular hexagon of side 2. What is the area enclosed by the curve?



(A) $2\pi+6$ **(B)** $2\pi+4\sqrt{3}$ **(C)** $3\pi+4$ **(D)** $2\pi+3\sqrt{3}+2$
(E) $\pi+6\sqrt{3}$

15. A 3×3 square is partitioned into 9 unit squares. Each unit square is painted either white or black with each color being equally likely, chosen independently and at random. The square is then rotated 90° clockwise about its center, and every white square in a position formerly occupied by a black square is painted black. The colors of all other squares are left unchanged. What is the probability that the grid is now entirely black?

(A) $\frac{49}{512}$ **(B)** $\frac{7}{64}$ **(C)** $\frac{121}{1024}$ **(D)** $\frac{81}{512}$ **(E)** $\frac{9}{32}$

16. Circle C_1 has its center O lying on circle C_2. The two circles meet at X and Y. Point Z in the exterior of C_1 lies on circle C_2 and $XZ=13$, $OZ=11$, and $YZ=7$. What is the radius of circle C_1?

(A) 5 **(B)** $\sqrt{26}$ **(C)** $3\sqrt{3}$ **(D)** $2\sqrt{7}$ **(E)** $\sqrt{30}$

17. Let S be a subset of $\{1, 2, 3, \ldots, 30\}$ with the property that no pair of distinct elements in S has a sum divisible by 5. What is the largest possible size of S?

(A) 10 **(B)** 13 **(C)** 15 **(D)** 16 **(E)** 18

18. Triangle ABC has $AB = 27$, $AC = 26$, and $BC = 25$. Let I denote the intersection of the internal angle bisectors of $\triangle ABC$. What is BI?

(A) 15 **(B)** $5 + \sqrt{26} + 3\sqrt{3}$ **(C)** $3\sqrt{26}$ **(D)** $\frac{2}{3}\sqrt{546}$ **(E)** $9\sqrt{3}$

19. Adam, Benin, Chiang, Deshawn, Esther, and Fiona have internet accounts. Some, but not all, of them are internet friends with each other, and none of them has an internet friend outside this group. Each of them has the same number of internet friends. In how many different ways can this happen?

(A) 60 **(B)** 170 **(C)** 290 **(D)** 320 **(E)** 660

20. Consider the polynomial

$$P(x) = \prod_{k=0}^{10}(x^{2^k} + 2^k) = (x+1)(x^2+2)(x^4+4)\cdots(x^{1024}+1024).$$

The coefficient of x^{2012} is equal to 2^a. What is a?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 10 **(E)** 24

21. Let a, b, and c be positive integers with $a \geq b \geq c$ such that

$$a^2 - b^2 - c^2 + ab = 2011 \text{ and}$$
$$a^2 + 3b^2 + 3c^2 - 3ab - 2ac - 2bc = -1997.$$

What is a?

(A) 249 **(B)** 250 **(C)** 251 **(D)** 252 **(E)** 253

22. Distinct planes $p_1, p_2, \ldots, p_k$ intersect the interior of a cube Q. Let S be the union of the faces of Q and let $P = \bigcup_{j=1}^{k} p_j$. The intersection of P and S consists of the union of all segments joining the midpoints of every pair of edges belonging to the same face of Q. What is the difference between the maximum and the minimum possible values of k?

(A) 8 **(B)** 12 **(C)** 20 **(D)** 23 **(E)** 24

23. Let S be the square one of whose diagonals has endpoints $(0.1, 0.7)$ and $(-0.1, -0.7)$. A point $v = (x, y)$ is chosen uniformly at random over all pairs of real numbers x and y such that $0 \le x \le 2012$ and $0 \le y \le 2012$. Let $T(v)$ be a translated copy of S centered at v. What is the probability that the square region determined by $T(v)$ contains exactly two points with integer coordinates in its interior?

(A) 0.125 **(B)** 0.14 **(C)** 0.16 **(D)** 0.25 **(E)** 0.32

24. Let $\{a_k\}_{k=1}^{2011}$ be the sequence of real numbers defined by

$$a_1 = 0.201, \quad a_2 = (0.2011)^{a_1}, \quad a_3 = (0.20101)^{a_2}, \quad a_4 = (0.201011)^{a_3},$$

and more generally

$$a_k = \begin{cases} (0.\underbrace{20101\ldots0101}_{k+2 \text{ digits}})^{a_{k-1}}, & \text{if } k \text{ is odd,} \\ (0.\underbrace{20101\ldots01011}_{k+2 \text{ digits}})^{a_{k-1}}, & \text{if } k \text{ is even.} \end{cases}$$

Rearranging the numbers in the sequence $\{a_k\}_{k=1}^{2011}$ in decreasing order produces a new sequence $\{b_k\}_{k=1}^{2011}$. What is the sum of all the integers k, $1 \le k \le 2011$, such that $a_k = b_k$?

(A) 671 **(B)** 1006 **(C)** 1341 **(D)** 2011 **(E)** 2012

25. Let $f(x) = |2\{x\} - 1|$ where $\{x\}$ denotes the fractional part of x. The number n is the smallest positive integer such that the equation

$$nf(xf(x)) = x$$

has at least 2012 real solutions x. What is n?

Note: the fractional part of x is a real number $y = \{x\}$, such that $0 \le y < 1$ and $x - y$ is an integer.

(A) 30 **(B)** 31 **(C)** 32 **(D)** 62 **(E)** 64

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American Mathematics Competitions

WRITE TO US!

Correspondence about the problems and solutions for this AMC 12
and orders for publications should be addressed to:

American Mathematics Competitions
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Lincoln, NE 68501-1606
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The problems and solutions for this AMC 12 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 12 Subcommittee Chair:

Prof. Bernardo M. Abrego

2012 AIME

The 30th annual AIME will be held on Thursday, March 15, with the alternate on Wednesday, March 28. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 41st Annual USA Mathematical Olympiad (USAMO) on April 24-25, 2012. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
amc.maa.org

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

63rd Annual

AMC 12 B

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Wednesday, February 22, 2012

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2012

AMC 12 B

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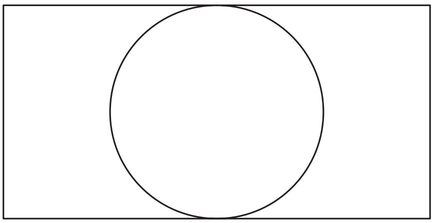
1. Each third-grade classroom at Pearl Creek Elementary has 18 students and 2 pet rabbits. How many more students than rabbits are there in all 4 of the third-grade classrooms?

   (A) 48 **(B)** 56 **(C)** 64 **(D)** 72 **(E)** 80

2. A circle of radius 5 is inscribed in a rectangle as shown. The ratio of the length of the rectangle to its width is 2 : 1. What is the area of the rectangle?



   (A) 50 **(B)** 100 **(C)** 125 **(D)** 150 **(E)** 200

3. For a science project, Sammy observed a chipmunk and a squirrel stashing acorns in holes. The chipmunk hid 3 acorns in each of the holes it dug. The squirrel hid 4 acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed 4 fewer holes. How many acorns did the chipmunk hide?

   (A) 30 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 54

4. Suppose that the euro is worth 1.30 dollars. If Diana has 500 dollars and Étienne has 400 euros, by what percent is the value of Étienne's money greater than the value of Diana's money?

   (A) 2 **(B)** 4 **(C)** 6.5 **(D)** 8 **(E)** 13

5. Two integers have a sum of 26. When two more integers are added to the first two integers the sum is 41. Finally when two more integers are added to the sum of the previous four integers the sum is 57. What is the minimum number of even integers among the 6 integers?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

6. In order to estimate the value of $x - y$ where x and y are real numbers with $x > y > 0$, Xiaoli rounded x up by a small amount, rounded y down by the same amount, and then subtracted her rounded values. Which of the following statements is necessarily correct?

   (A) Her estimate is larger than $x - y$.

   (B) Her estimate is smaller than $x - y$.

   (C) Her estimate equals $x - y$.

   (D) Her estimate equals $y - x$.

   (E) Her estimate is 0.

7. Small lights are hung on a string 6 inches apart in the order red, red, green, green, green, red, red, green, green, green, and so on continuing this pattern of 2 red lights followed by 3 green lights. How many feet separate the 3rd red light and the 21st red light?

   Note: 1 foot is equal to 12 inches.

   (A) 18 **(B)** 18.5 **(C)** 20 **(D)** 20.5 **(E)** 22.5

8. A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

   (A) 729 **(B)** 972 **(C)** 1024 **(D)** 2187 **(E)** 2304

9. It takes Clea 60 seconds to walk down an escalator when it is not operating, and only 24 seconds to walk down the escalator when it is operating. How many seconds does it take Clea to ride down the operating escalator when she just stands on it?

   (A) 36 **(B)** 40 **(C)** 42 **(D)** 48 **(E)** 52

10. What is the area of the polygon whose vertices are the points of intersection of the curves $x^2 + y^2 = 25$ and $(x - 4)^2 + 9y^2 = 81$?

    (A) 24 **(B)** 27 **(C)** 36 **(D)** 37.5 **(E)** 42

11. In the equation below, A and B are consecutive positive integers, and A, B, and $A + B$ represent number bases:

$$132_A + 43_B = 69_{A+B}.$$

What is $A + B$?

(A) 9 **(B)** 11 **(C)** 13 **(D)** 15 **(E)** 17

12. How many sequences of zeros and/or ones of length 20 have all the zeros consecutive, or all the ones consecutive, or both?

(A) 190 **(B)** 192 **(C)** 211 **(D)** 380 **(E)** 382

13. Two parabolas have equations $y = x^2 + ax + b$ and $y = x^2 + cx + d$, where a, b, c, and d are integers (not necessarily different), each chosen independently by rolling a fair six-sided die. What is the probability that the parabolas have at least one point in common?

(A) $\frac{1}{2}$ **(B)** $\frac{25}{36}$ **(C)** $\frac{5}{6}$ **(D)** $\frac{31}{36}$ **(E)** 1

14. Bernardo and Silvia play the following game. An integer between 0 and 999, inclusive, is selected and given to Bernardo. Whenever Bernardo receives a number, he doubles it and passes the result to Silvia. Whenever Silvia receives a number, she adds 50 to it and passes the result to Bernardo. The winner is the last person who produces a number less than 1000. Let N be the smallest initial number that results in a win for Bernardo. What is the sum of the digits of N?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

15. Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger?

(A) $\frac{1}{8}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{\sqrt{10}}{10}$ **(D)** $\frac{\sqrt{5}}{6}$ **(E)** $\frac{\sqrt{10}}{5}$

16. Amy, Beth, and Jo listen to four different songs and discuss which ones they like. No song is liked by all three. Furthermore, for each of the three pairs of the girls, there is at least one song liked by those two girls but disliked by the third. In how many different ways is this possible?

(A) 108 **(B)** 132 **(C)** 671 **(D)** 846 **(E)** 1105

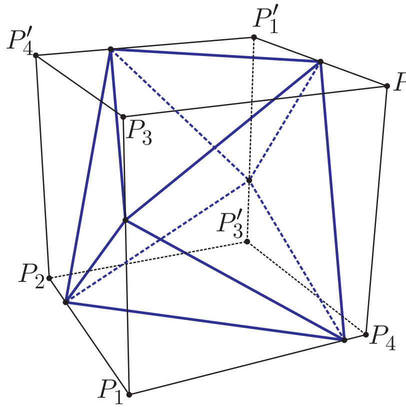
17. Square $PQRS$ lies in the first quadrant. Points $(3,0)$, $(5,0)$, $(7,0)$, and $(13,0)$ lie on lines SP, RQ, PQ, and SR, respectively. What is the sum of the coordinates of the center of the square $PQRS$?

(A) 6 **(B)** 6.2 **(C)** 6.4 **(D)** 6.6 **(E)** 6.8

18. Let $(a_1, a_2, \ldots, a_{10})$ be a list of the first 10 positive integers such that for each $2 \le i \le 10$ either $a_i + 1$ or $a_i - 1$ or both appear somewhere before a_i in the list. How many such lists are there?

(A) 120 **(B)** 512 **(C)** 1024 **(D)** 181,440 **(E)** 362,880

19. A unit cube has vertices P_1, P_2, P_3, P_4, P_1', P_2', P_3', and P_4'. Vertices P_2, P_3, and P_4 are adjacent to P_1, and for $1 \le i \le 4$, vertices P_i and P_i' are opposite to each other. A regular octahedron has one vertex in each of the segments P_1P_2, P_1P_3, P_1P_4, $P_1'P_2'$, $P_1'P_3'$, and $P_1'P_4'$. What is the octahedron's side length?



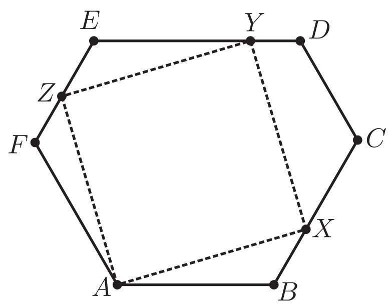
(A) $\frac{3\sqrt{2}}{4}$ **(B)** $\frac{7\sqrt{6}}{16}$ **(C)** $\frac{\sqrt{5}}{2}$ **(D)** $\frac{2\sqrt{3}}{3}$ **(E)** $\frac{\sqrt{6}}{2}$

20. A trapezoid has side lengths 3, 5, 7, and 11. The sum of all the possible areas of the trapezoid can be written in the form of $r_1\sqrt{n_1}+r_2\sqrt{n_2}+r_3$, where r_1, r_2, and r_3 are rational numbers and n_1 and n_2 are positive integers not divisible by the square of a prime. What is the greatest integer less than or equal to

$$r_1 + r_2 + r_3 + n_1 + n_2\,?$$

(A) 57 **(B)** 59 **(C)** 61 **(D)** 63 **(E)** 65

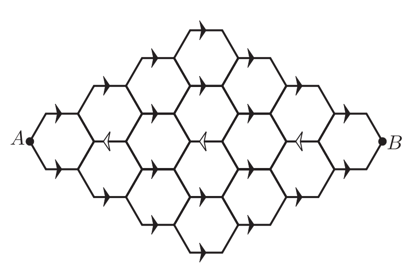
21. Square $AXYZ$ is inscribed in equiangular hexagon $ABCDEF$ with X on $\overline{BC}$, Y on $\overline{DE}$, and Z on $\overline{EF}$. Suppose that $AB = 40$ and $EF = 41(\sqrt{3}-1)$. What is the side-length of the square?



(A) $29\sqrt{3}$ **(B)** $\frac{21}{2}\sqrt{2}+\frac{41}{2}\sqrt{3}$ **(C)** $20\sqrt{3}+16$ **(D)** $20\sqrt{2}+13\sqrt{3}$

(E) $21\sqrt{6}$

22. A bug travels from A to B along the segments in the hexagonal lattice pictured below. The segments marked with an arrow can be traveled only in the direction of the arrow, and the bug never travels the same segment more than once. How many different paths are there?



(A) 2112 **(B)** 2304 **(C)** 2368 **(D)** 2384 **(E)** 2400

23. Consider all polynomials of a complex variable, $P(z) = 4z^4 + az^3 + bz^2 + cz + d$, where $a, b, c,$ and d are integers, $0 \le d \le c \le b \le a \le 4$, and the polynomial has a zero z_0 with $|z_0| = 1$. What is the sum of all values $P(1)$ over all the polynomials with these properties?

(A) 84 **(B)** 92 **(C)** 100 **(D)** 108 **(E)** 120

24. Define the function f_1 on the positive integers by setting $f_1(1) = 1$ and if $n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$ is the prime factorization of $n > 1$, then

$$f_1(n) = (p_1 + 1)^{e_1 - 1}(p_2 + 1)^{e_2 - 1} \cdots (p_k + 1)^{e_k - 1}.$$

For every $m \ge 2$, let $f_m(n) = f_1(f_{m-1}(n))$. For how many N in the range $1 \le N \le 400$ is the sequence $(f_1(N), f_2(N), f_3(N), \ldots)$ unbounded?

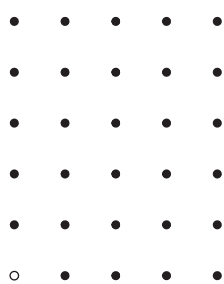
Note: a sequence of positive numbers is unbounded if for every integer B, there is a member of the sequence greater than B.

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

25. Let $S = \{(x, y) : x \in \{0, 1, 2, 3, 4\}, y \in \{0, 1, 2, 3, 4, 5\}, \text{ and } (x, y) \ne (0, 0)\}$. Let T be the set of all right triangles whose vertices are in S. For every right triangle $t = \triangle ABC$ with vertices A, B, and C in counter-clockwise order and right angle at A, let $f(t) = \tan(\angle CBA)$. What is

$$\prod_{t \in T} f(t)\,?$$

S



(A) 1 **(B)** $\frac{625}{144}$ **(C)** $\frac{125}{24}$ **(D)** 6 **(E)** $\frac{625}{24}$

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

WRITE TO US!

Correspondence about the problems and solutions for this AMC 12 and orders for publications should be addressed to:

American Mathematics Competitions
University of Nebraska, P.O. Box 81606
Lincoln, NE 68501-1606
Phone 402-472-2257 | Fax 402-472-6087 | amcinfo@maa.org

The problems and solutions for this AMC 12 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 12 Subcommittee Chair:

Prof. Bernardo M. Abrego

2012 AIME

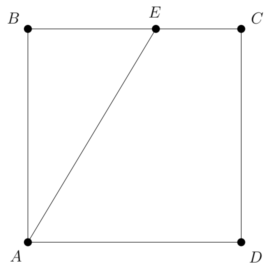
The 30th annual AIME will be held on Thursday, March 15, with the alternate on Wednesday, March 28. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 41st Annual USA Mathematical Olympiad (USAMO) on April 24-25, 2012. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: amc.maa.org

1. Square $ABCD$ has side length 10. Point E is on $\overline{BC}$, and the area of $\triangle ABE$ is 40. What is BE?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8



2. A softball team played ten games, scoring 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 runs. They lost by one run in exactly five games. In each of their other games, they scored twice as many runs as their opponent. How many total runs did their opponents score?

   (A) 35 **(B)** 40 **(C)** 45 **(D)** 50 **(E)** 55

3. A flower bouquet contains pink roses, red roses, pink carnations, and red carnations. One third of the pink flowers are roses, three fourths of the red flowers are carnations, and six tenths of the flowers are pink. What percent of the flowers are carnations?

   (A) 15 **(B)** 30 **(C)** 40 **(D)** 60 **(E)** 70

4. What is the value of
$$\frac{2^{2014}+2^{2012}}{2^{2014}-2^{2012}}\,?$$

   (A) -1 **(B)** 1 **(C)** $\frac{5}{3}$ **(D)** 2013 **(E)** 2^{4024}

5. Tom, Dorothy, and Sammy went on a vacation and agreed to split the costs evenly. During their trip Tom paid \$105, Dorothy paid \$125, and Sammy paid \$175. In order to share the costs equally, Tom gave Sammy t dollars, and Dorothy gave Sammy d dollars. What is $t-d$?

   (A) 15 **(B)** 20 **(C)** 25 **(D)** 30 **(E)** 35

6. In a recent basketball game, Shenille attempted only three-point shots and two-point shots. She was successful on 20% of her three-point shots and 30% of her two-point shots. Shenille attempted 30 shots. How many points did she score?

(A) 12 **(B)** 18 **(C)** 24 **(D)** 30 **(E)** 36

7. The sequence $S_1, S_2, S_3, \ldots, S_{10}$ has the property that every term beginning with the third is the sum of the previous two. That is,

$$S_n = S_{n-2} + S_{n-1} \text{ for } n \geq 3.$$

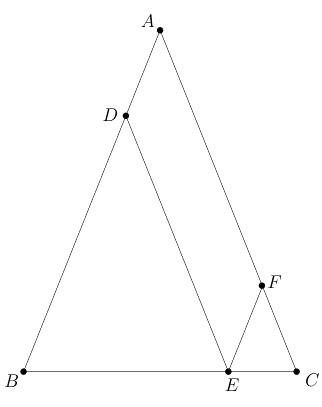
Suppose that $S_9 = 110$ and $S_7 = 42$. What is S_4?

(A) 4 **(B)** 6 **(C)** 10 **(D)** 12 **(E)** 16

8. Given that x and y are distinct nonzero real numbers such that $x + \frac{2}{x} = y + \frac{2}{y}$, what is xy?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** 4

9. In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points D, E, and F are on sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively, such that $\overline{DE}$ and $\overline{EF}$ are parallel to $\overline{AC}$ and $\overline{AB}$, respectively. What is the perimeter of parallelogram $ADEF$?

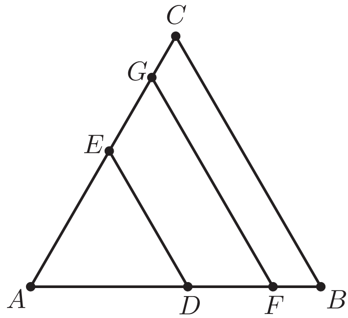


(A) 48 **(B)** 52 **(C)** 56 **(D)** 60 **(E)** 72

10. Let S be the set of positive integers n for which $\frac{1}{n}$ has the repeating decimal representation $0.\overline{ab} = 0.ababab\ldots$, with a and b different digits. What is the sum of the elements of S?

(A) 11 **(B)** 44 **(C)** 110 **(D)** 143 **(E)** 155

11. Triangle ABC is equilateral with $AB = 1$. Points E and G are on $\overline{AC}$ and points D and F are on $\overline{AB}$ such that both $\overline{DE}$ and $\overline{FG}$ are parallel to $\overline{BC}$. Furthermore, triangle ADE and trapezoids $DFGE$ and $FBCG$ all have the same perimeter. What is $DE + FG$?



(A) 1 **(B)** $\frac{3}{2}$ **(C)** $\frac{21}{13}$ **(D)** $\frac{13}{8}$ **(E)** $\frac{5}{3}$

12. The angles in a particular triangle are in arithmetic progression, and the side lengths are 4, 5, and x. The sum of the possible values of x equals $a + \sqrt{b} + \sqrt{c}$, where a, b, and c are positive integers. What is $a + b + c$?

(A) 36 **(B)** 38 **(C)** 40 **(D)** 42 **(E)** 44

13. Let points $A = (0, 0)$, $B = (1, 2)$, $C = (3, 3)$, and $D = (4, 0)$. Quadrilateral $ABCD$ is cut into equal area pieces by a line passing through A. This line intersects $\overline{CD}$ at point $(\frac{p}{q}, \frac{r}{s})$, where these fractions are in lowest terms. What is $p + q + r + s$?

(A) 54 **(B)** 58 **(C)** 62 **(D)** 70 **(E)** 75

14. The sequence

$$\log_{12} 162,\ \log_{12} x,\ \log_{12} y,\ \log_{12} z,\ \log_{12} 1250$$

is an arithmetic progression. What is x?

(A) $125\sqrt{3}$ **(B)** 270 **(C)** $162\sqrt{5}$ **(D)** 434 **(E)** $225\sqrt{6}$

15. Rabbits Peter and Pauline have three offspring—Flopsie, Mopsie, and Cotton-tail. These five rabbits are to be distributed to four different pet stores so that no store gets both a parent and a child. It is not required that every store gets a rabbit. In how many different ways can this be done?

(A) 96 **(B)** 108 **(C)** 156 **(D)** 204 **(E)** 372

16. A, B, and C are three piles of rocks. The mean weight of the rocks in A is 40 pounds, the mean weight of the rocks in B is 50 pounds, the mean weight of the rocks in the combined piles A and B is 43 pounds, and the mean weight of the rocks in the combined piles A and C is 44 pounds. What is the greatest possible integer value for the mean in pounds of the rocks in the combined piles B and C?

    (A) 55 **(B)** 56 **(C)** 57 **(D)** 58 **(E)** 59

17. A group of 12 pirates agree to divide a treasure chest of gold coins among themselves as follows. The k^{th} pirate to take a share takes $\frac{k}{12}$ of the coins that remain in the chest. The number of coins initially in the chest is the smallest number for which this arrangement will allow each pirate to receive a positive whole number of coins. How many coins does the 12^{th} pirate receive?

    (A) 720 **(B)** 1296 **(C)** 1728 **(D)** 1925 **(E)** 3850

18. Six spheres of radius 1 are positioned so that their centers are at the vertices of a regular hexagon of side length 2. The six spheres are internally tangent to a larger sphere whose center is the center of the hexagon. An eighth sphere is externally tangent to the six smaller spheres and internally tangent to the larger sphere. What is the radius of this eighth sphere?

    (A) $\sqrt{2}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{5}{3}$ **(D)** $\sqrt{3}$ **(E)** 2

19. In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

    (A) 11 **(B)** 28 **(C)** 33 **(D)** 61 **(E)** 72

20. Let S be the set $\{1, 2, 3, \ldots, 19\}$. For $a, b \in S$, define $a \succ b$ to mean that either $0 < a - b \le 9$ or $b - a > 9$. How many ordered triples (x, y, z) of elements of S have the property that $x \succ y$, $y \succ z$, and $z \succ x$?

    (A) 810 **(B)** 855 **(C)** 900 **(D)** 950 **(E)** 988

21. Consider

$$A = \log(2013 + \log(2012 + \log(2011 + \log(\cdots + \log(3 + \log 2)\cdots)))).$$

Which of the following intervals contains A?

(A) $(\log 2016, \log 2017)$

(B) $(\log 2017, \log 2018)$

(C) $(\log 2018, \log 2019)$

(D) $(\log 2019, \log 2020)$

(E) $(\log 2020, \log 2021)$

22. A palindrome is a nonnegative integer number that reads the same forwards and backwards when written in base 10 with no leading zeros. A 6-digit palindrome n is chosen uniformly at random. What is the probability that $\frac{n}{11}$ is also a palindrome?

(A) $\frac{8}{25}$ **(B)** $\frac{33}{100}$ **(C)** $\frac{7}{20}$ **(D)** $\frac{9}{25}$ **(E)** $\frac{11}{30}$

23. $ABCD$ is a square of side length $\sqrt{3}+1$. Point P is on $\overline{AC}$ such that $AP = \sqrt{2}$. The square region bounded by $ABCD$ is rotated 90° counterclockwise with center P, sweeping out a region whose area is $\frac{1}{c}(a\pi + b)$, where a, b, and c are positive integers and $\gcd(a, b, c) = 1$. What is $a + b + c$?

(A) 15 **(B)** 17 **(C)** 19 **(D)** 21 **(E)** 23

24. Three distinct segments are chosen at random among the segments whose end-points are the vertices of a regular 12-gon. What is the probability that the lengths of these three segments are the three side lengths of a triangle with positive area?

(A) $\frac{553}{715}$ **(B)** $\frac{443}{572}$ **(C)** $\frac{111}{143}$ **(D)** $\frac{81}{104}$ **(E)** $\frac{223}{286}$

25. Let $f : \mathbb{C} \to \mathbb{C}$ be defined by $f(z) = z^2 + iz + 1$. How many complex numbers z are there such that $\text{Im}(z) > 0$ and both the real and the imaginary parts of $f(z)$ are integers with absolute value at most 10?

(A) 399 **(B)** 401 **(C)** 413 **(D)** 431 **(E)** 441

1. On a particular January day, the high temperature in Lincoln, Nebraska, was 16 degrees higher than the low temperature, and the average of the high and low temperatures was $3°$. In degrees, what was the low temperature in Lincoln that day?

   (A) -13 **(B)** -8 **(C)** -5 **(D)** -3 **(E)** 11

2. Mr. Green measures his rectangular garden by walking two of the sides and finds that it is 15 steps by 20 steps. Each of Mr. Green's steps is 2 feet long. Mr. Green expects a half a pound of potatoes per square foot from his garden. How many pounds of potatoes does Mr. Green expect from his garden?

   (A) 600 **(B)** 800 **(C)** 1000 **(D)** 1200 **(E)** 1400

3. When counting from 3 to 201, 53 is the 51^{st} number counted. When counting backwards from 201 to 3, 53 is the n^{th} number counted. What is n?

   (A) 146 **(B)** 147 **(C)** 148 **(D)** 149 **(E)** 150

4. Ray's car averages 40 miles per gallon of gasoline, and Tom's car averages 10 miles per gallon of gasoline. Ray and Tom each drive the same number of miles. What is the cars' combined rate of miles per gallon of gasoline?

   (A) 10 **(B)** 16 **(C)** 25 **(D)** 30 **(E)** 40

5. The average age of 33 fifth-graders is 11. The average age of 55 of their parents is 33. What is the average age of all of these parents and fifth-graders?

   (A) 22 **(B)** 23.25 **(C)** 24.75 **(D)** 26.25 **(E)** 28

6. Real numbers x and y satisfy the equation $x^2 + y^2 = 10x - 6y - 34$. What is $x + y$?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 8

7. Jo and Blair take turns counting from 1 to one more than the last number said by the other person. Jo starts by saying "1", so Blair follows by saying "1, 2". Jo then says "1, 2, 3", and so on. What is the 53^{rd} number said?

   (A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 8

8. Line ℓ_1 has equation $3x - 2y = 1$ and goes through $A = (-1, -2)$. Line ℓ_2 has equation $y = 1$ and meets line ℓ_1 at point B. Line ℓ_3 has positive slope, goes through point A, and meets ℓ_2 at point C. The area of $\triangle ABC$ is 3. What is the slope of ℓ_3?

   (A) $\frac{2}{3}$ **(B)** $\frac{3}{4}$ **(C)** 1 **(D)** $\frac{4}{3}$ **(E)** $\frac{3}{2}$

9. What is the sum of the exponents of the prime factors of the square root of the largest perfect square that divides $12!$?

   (A) 5 **(B)** 7 **(C)** 8 **(D)** 10 **(E)** 12

10. Alex has 75 red tokens and 75 blue tokens. There is a booth where Alex can give two red tokens and receive in return a silver token and a blue token, and another booth where Alex can give three blue tokens and receive in return a silver token and a red token. Alex continues to exchange tokens until no more exchanges are possible. How many silver tokens will Alex have at the end?

    (A) 62 **(B)** 82 **(C)** 83 **(D)** 102 **(E)** 103

11. Two bees start at the same spot and fly at the same rate in the following directions. Bee A travels 1 foot north, then 1 foot east, then 1 foot upwards, and then continues to repeat this pattern. Bee B travels 1 foot south, then 1 foot west, and then continues to repeat this pattern. In what directions are the bees traveling when they are exactly 10 feet away from each other?

    (A) A east, B west

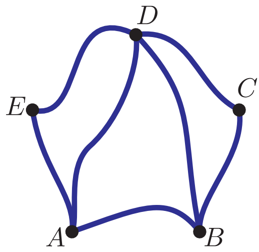
    (B) A north, B south

    (C) A north, B west

    (D) A up, B south

    (E) A up, B west

12. Cities A, B, C, D, and E are connected by roads $\widetilde{AB}$, $\widetilde{AD}$, $\widetilde{AE}$, $\widetilde{BC}$, $\widetilde{BD}$, $\widetilde{CD}$, and $\widetilde{DE}$. How many different routes are there from A to B that use each road exactly once? (Such a route will necessarily visit some cities more than once.)



(A) 7 **(B)** 9 **(C)** 12 **(D)** 16 **(E)** 18

13. The internal angles of quadrilateral $ABCD$ form an arithmetic progression. Triangles ABD and DCB are similar with $\angle DBA = \angle DCB$ and $\angle ADB = \angle CBD$. Moreover, the angles in each of these two triangles also form an arithmetic progression. In degrees, what is the largest possible sum of the two largest angles of $ABCD$?

(A) 210 **(B)** 220 **(C)** 230 **(D)** 240 **(E)** 250

14. Two non-decreasing sequences of nonnegative integers have different first terms. Each sequence has the property that each term beginning with the third is the sum of the previous two terms, and the seventh term of each sequence is N. What is the smallest possible value of N?

(A) 55 **(B)** 89 **(C)** 104 **(D)** 144 **(E)** 273

15. The number 2013 is expressed in the form

$$2013 = \frac{a_1!a_2!\cdots a_m!}{b_1!b_2!\cdots b_n!},$$

where $a_1 \geq a_2 \geq \cdots \geq a_m$ and $b_1 \geq b_2 \geq \cdots \geq b_n$ are positive integers and $a_1 + b_1$ is as small as possible. What is $|a_1 - b_1|$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

16. Let $ABCDE$ be an equiangular convex pentagon of perimeter 1. The pairwise intersections of the lines that extend the sides of the pentagon determine a five-pointed star polygon. Let s be the perimeter of this star. What is the difference between the maximum and the minimum possible values of s?

(A) 0 **(B)** $\frac{1}{2}$ **(C)** $\frac{\sqrt{5}-1}{2}$ **(D)** $\frac{\sqrt{5}+1}{2}$ **(E)** $\sqrt{5}$

17. Let a, b, and c be real numbers such that

$$\begin{cases} a+b+c=2, \text{ and} \\ a^2+b^2+c^2=12. \end{cases}$$

What is the difference between the maximum and minimum possible values of c?

(A) 2 **(B)** $\frac{10}{3}$ **(C)** 4 **(D)** $\frac{16}{3}$ **(E)** $\frac{20}{3}$

18. Barbara and Jenna play the following game, in which they take turns. A number of coins lie on a table. When it is Barbara's turn, she must remove 2 or 4 coins, unless only one coin remains, in which case she loses her turn. When it is Jenna's turn, she must remove 1 or 3 coins. A coin flip determines who goes first. Whoever removes the last coin wins the game. Assume both players use their best strategy. Who will win when the game starts with 2013 coins and when the game starts with 2014 coins?

(A) Barbara will win with 2013 coins, and Jenna will win with 2014 coins.

(B) Jenna will win with 2013 coins, and whoever goes first will win with 2014 coins.

(C) Barbara will win with 2013 coins, and whoever goes second will win with 2014 coins.

(D) Jenna will win with 2013 coins, and Barbara will win with 2014 coins.

(E) Whoever goes first will win with 2013 coins, and whoever goes second will win with 2014 coins.

19. In triangle ABC, $AB = 13$, $BC = 14$, and $CA = 15$. Distinct points D, E, and F lie on segments $\overline{BC}$, $\overline{CA}$, and $\overline{DE}$, respectively, such that $\overline{AD} \perp \overline{BC}$, $\overline{DE} \perp \overline{AC}$, and $\overline{AF} \perp \overline{BF}$. The length of segment $\overline{DF}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

(A) 18 **(B)** 21 **(C)** 24 **(D)** 27 **(E)** 30

20. For $135^\circ < x < 180^\circ$, points $P = (\cos x, \cos^2 x)$, $Q = (\cot x, \cot^2 x)$, $R = (\sin x, \sin^2 x)$, and $S = (\tan x, \tan^2 x)$ are the vertices of a trapezoid. What is $\sin(2x)$?

(A) $2 - 2\sqrt{2}$ **(B)** $3\sqrt{3} - 6$ **(C)** $3\sqrt{2} - 5$ **(D)** $-\frac{3}{4}$ **(E)** $1 - \sqrt{3}$

21. Consider the set of 30 parabolas defined as follows: all parabolas have as focus the point $(0, 0)$ and the directrix lines have the form $y = ax + b$ with a and b integers such that $a \in \{-2, -1, 0, 1, 2\}$ and $b \in \{-3, -2, -1, 1, 2, 3\}$. No three of these parabolas have a common point. How many points in the plane are on two of these parabolas?

(A) 720 **(B)** 760 **(C)** 810 **(D)** 840 **(E)** 870

22. Let $m > 1$ and $n > 1$ be integers. Suppose that the product of the solutions for x of the equation

$$8(\log_n x)(\log_m x) - 7\log_n x - 6\log_m x - 2013 = 0$$

is the smallest possible integer. What is $m + n$?

(A) 12 **(B)** 20 **(C)** 24 **(D)** 48 **(E)** 272

23. Bernardo chooses a three-digit positive integer N and writes both its base-5 and base-6 representations on a blackboard. Later LeRoy sees the two numbers Bernardo has written. Treating the two numbers as base-10 integers, he adds them to obtain an integer S. For example, if $N = 749$, Bernardo writes the numbers 10,444 and 3,245, and LeRoy obtains the sum $S = 13{,}689$. For how many choices of N are the two rightmost digits of S, in order, the same as those of $2N$?

(A) 5 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 25

24. Let ABC be a triangle where M is the midpoint of $\overline{AC}$, and $\overline{CN}$ is the angle bisector of $\angle ACB$ with N on $\overline{AB}$. Let X be the intersection of the median $\overline{BM}$ and the bisector $\overline{CN}$. In addition $\triangle BXN$ is equilateral and $AC = 2$. What is BN^2?

(A) $\frac{10 - 6\sqrt{2}}{7}$ **(B)** $\frac{2}{9}$ **(C)** $\frac{5\sqrt{2} - 3\sqrt{3}}{8}$ **(D)** $\frac{\sqrt{2}}{6}$ **(E)** $\frac{3\sqrt{3} - 4}{5}$

25. Let G be the set of polynomials of the form

$$P(z) = z^n + c_{n-1}z^{n-1} + \cdots + c_2z^2 + c_1z + 50,$$

where $c_1, c_2, \ldots, c_{n-1}$ are integers and $P(z)$ has n distinct roots of the form $a + ib$ with a and b integers. How many polynomials are in G?

(A) 288 **(B)** 528 **(C)** 576 **(D)** 992 **(E)** 1056

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

65th Annual

AMC 12 A

American Mathematics Contest 12 A

Tuesday, February 4, 2014

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 32nd annual American Invitational Mathematics Examination (AIME) on Thursday, March 13, 2014 or Wednesday, March 26, 2014. More details about the AIME and other information are on the back page of this test booklet.

2014

AMC 12 A

DO NOT OPEN UNTIL TUESDAY, FEBRUARY 4, 2014

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 4, 2014. Nothing is needed from inside this package until February 4.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 12 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

WRITE TO US!

Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

American Mathematics Competitions
1740 Vine Street
Lincoln, NE 68508
Phone 402-472-2257 | Fax 402-472-6087 | amcinfo@maa.org

The problems and solutions for this AMC 2 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 12 Subcommittee Chair:

Prof. Bernardo M. Abrego

2014 AIME

The 32nd annual AIME will be held on Thursday, March 13, with the alternate on Wednesday, March 26. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 43rd Annual USA Mathematical Olympiad (USAMO) on April 29-30, 2014. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: maa.org/math-competitions

1. What is $10 \cdot (\frac{1}{2} + \frac{1}{5} + \frac{1}{10})^{-1}$?

   (A) 3 **(B)** 8 **(C)** $\frac{25}{2}$ **(D)** $\frac{170}{3}$ **(E)** 170

2. At the theater children get in for half price. The price for 5 adult tickets and 4 child tickets is \$24.50. How much would 8 adult tickets and 6 child tickets cost?

   (A) \$35 **(B)** \$38.50 **(C)** \$40 **(D)** \$42 **(E)** \$42.50

3. Walking down Jane Street, Ralph passed four houses in a row, each painted a different color. He passed the orange house before the red house, and he passed the blue house before the yellow house. The blue house was not next to the yellow house. How many orderings of the colored houses are possible?

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

4. Suppose that a cows give b gallons of milk in c days. At this rate, how many gallons of milk will d cows give in e days?

   (A) $\frac{bde}{ac}$ **(B)** $\frac{ac}{bde}$ **(C)** $\frac{abde}{c}$ **(D)** $\frac{bcde}{a}$ **(E)** $\frac{abc}{de}$

5. On an algebra quiz, 10% of the students scored 70 points, 35% scored 80 points, 30% scored 90 points, and the rest scored 100 points. What is the difference between the mean and the median of the students' scores on this quiz?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

6. The difference between a two-digit number and the number obtained by reversing its digits is 5 times the sum of the digits of either number. What is the sum of the two-digit number and its reverse?

   (A) 44 **(B)** 55 **(C)** 77 **(D)** 99 **(E)** 110

7. The first three terms of a geometric progression are $\sqrt{3}$, $\sqrt[3]{3}$, and $\sqrt[6]{3}$. What is the fourth term?

   (A) 1 **(B)** $\sqrt[7]{3}$ **(C)** $\sqrt[8]{3}$ **(D)** $\sqrt[9]{3}$ **(E)** $\sqrt[10]{3}$

8. A customer who intends to purchase an appliance has three coupons, only one of which may be used:

   Coupon 1: 10% off the listed price if the listed price is at least \$50

   Coupon 2: \$20 off the listed price if the listed price is at least \$100

   Coupon 3: 18% off the amount by which the listed price exceeds \$100

   For which of the following listed prices will coupon 1 offer a greater price reduction than either coupon 2 or coupon 3?

   (A) \$179.95 **(B)** \$199.95 **(C)** \$219.95 **(D)** \$239.95 **(E)** \$259.95

9. Five positive consecutive integers starting with a have average b. What is the average of 5 consecutive integers that start with b?

   (A) $a+3$ **(B)** $a+4$ **(C)** $a+5$ **(D)** $a+6$ **(E)** $a+7$

10. Three congruent isosceles triangles are constructed with their bases on the sides of an equilateral triangle of side length 1. The sum of the areas of the three isosceles triangles is the same as the area of the equilateral triangle. What is the length of one of the two congruent sides of one of the isosceles triangles?

    (A) $\frac{\sqrt{3}}{4}$ **(B)** $\frac{\sqrt{3}}{3}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{\sqrt{2}}{2}$ **(E)** $\frac{\sqrt{3}}{2}$

11. David drives from his home to the airport to catch a flight. He drives 35 miles in the first hour, but realizes that he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way to the airport and arrives 30 minutes early. How many miles is the airport from his home?

    (A) 140 **(B)** 175 **(C)** 210 **(D)** 245 **(E)** 280

12. Two circles intersect at points A and B. The minor arcs AB measure 30° on one circle and 60° on the other circle. What is the ratio of the area of the larger circle to the area of the smaller circle?

    (A) 2 **(B)** $1+\sqrt{3}$ **(C)** 3 **(D)** $2+\sqrt{3}$ **(E)** 4

13. A fancy bed and breakfast inn has 5 rooms, each with a distinctive color-coded decor. One day 5 friends arrive to spend the night. There are no other guests that night. The friends can room in any combination they wish, but with no more than 2 friends per room. In how many ways can the innkeeper assign the guests to the rooms?

    (A) 2100 **(B)** 2220 **(C)** 3000 **(D)** 3120 **(E)** 3125

24. Let $f_0(x) = x + |x - 100| - |x + 100|$, and for $n \geq 1$, let $f_n(x) = |f_{n-1}(x)| - 1$. For how many values of x is $f_{100}(x) = 0$?

(A) 299 **(B)** 300 **(C)** 301 **(D)** 302 **(E)** 303

25. The parabola P has focus $(0, 0)$ and goes through the points $(4, 3)$ and $(-4, -3)$. For how many points $(x, y) \in P$ with integer coordinates is it true that $|4x + 3y| \leq 1000$?

(A) 38 **(B)** 40 **(C)** 42 **(D)** 44 **(E)** 46

18. The domain of the function $f(x) = \log_{\frac{1}{2}}(\log_4(\log_{\frac{1}{4}}(\log_{16}(\log_{\frac{1}{16}} x))))$ is an interval of length $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 19 **(B)** 31 **(C)** 271 **(D)** 319 **(E)** 511

19. There are exactly N distinct rational numbers k such that $|k| < 200$ and

$$5x^2 + kx + 12 = 0$$

has at least one integer solution for x. What is N?

(A) 6 **(B)** 12 **(C)** 24 **(D)** 48 **(E)** 78

20. In $\triangle BAC$, $\angle BAC = 40°$, $AB = 10$, and $AC = 6$. Points D and E lie on $\overline{AB}$ and $\overline{AC}$, respectively. What is the minimum possible value of $BE + DE + CD$?

(A) $6\sqrt{3} + 3$ **(B)** $\frac{27}{2}$ **(C)** $8\sqrt{3}$ **(D)** 14 **(E)** $3\sqrt{3} + 9$

21. For every real number x, let $\lfloor x \rfloor$ denote the greatest integer not exceeding x, and let

$$f(x) = \lfloor x \rfloor (2014^{x - \lfloor x \rfloor} - 1).$$

The set of all numbers x such that $1 \le x < 2014$ and $f(x) \le 1$ is a union of disjoint intervals. What is the sum of the lengths of those intervals?

(A) 1 **(B)** $\frac{\log 2015}{\log 2014}$ **(C)** $\frac{\log 2014}{\log 2013}$ **(D)** $\frac{2014}{2013}$ **(E)** $2014^{\frac{1}{2014}}$

22. The number 5^{867} is between 2^{2013} and 2^{2014}. How many pairs of integers (m, n) are there such that $1 \le m \le 2012$ and

$$5^n < 2^m < 2^{m+2} < 5^{n+1}\,?$$

(A) 278 **(B)** 279 **(C)** 280 **(D)** 281 **(E)** 282

23. The fraction

$$\frac{1}{99^2} = 0.\overline{b_{n-1}b_{n-2}\ldots b_2b_1b_0},$$

where n is the length of the period of the repeating decimal expansion. What is the sum $b_0 + b_1 + \cdots + b_{n-1}$?

(A) 874 **(B)** 883 **(C)** 887 **(D)** 891 **(E)** 892

14. Let $a < b < c$ be three integers such that a, b, c is an arithmetic progression and a, c, b is a geometric progression. What is the smallest possible value for c?

(A) -2 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 6

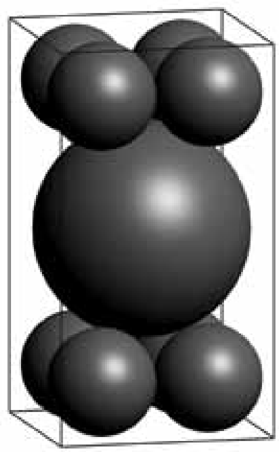
15. A five-digit palindrome is a positive integer with respective digits $abcba$, where a is not zero. Let S be the sum of all five-digit palindromes. What is the sum of the digits of S?

(A) 9 **(B)** 18 **(C)** 27 **(D)** 36 **(E)** 45

16. The product $(8)(888\ldots8)$, where the second factor has k digits, is an integer whose digits have a sum of 1000. What is k?

(A) 901 **(B)** 911 **(C)** 919 **(D)** 991 **(E)** 999

17. A $4 \times 4 \times h$ rectangular box contains a sphere of radius 2 and eight smaller spheres of radius 1. The smaller spheres are each tangent to three sides of the box, and the larger sphere is tangent to each of the smaller spheres. What is h?



(A) $2 + 2\sqrt{7}$ **(B)** $3 + 2\sqrt{5}$ **(C)** $4 + 2\sqrt{7}$ **(D)** $4\sqrt{5}$ **(E)** $4\sqrt{7}$

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

65th Annual

AMC 12 B

American Mathematics Contest 12 B

Wednesday, February 19, 2014

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 32nd annual American Invitational Mathematics Examination (AIME) on Thursday, March 13, 2014 or Wednesday, March 26, 2014. More details about the AIME and other information are on the back page of this test booklet.

2014

AMC 12 B

DO NOT OPEN UNTIL WEDNESDAY, FEBRUARY 19, 2014

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 19, 2014. Nothing is needed from inside this package until February 19.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 12 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

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1. Leah has 13 coins, all of which are pennies and nickels. If she had one more nickel than she has now, then she would have the same number of pennies and nickels. In cents, how much are Leah's coins worth?

   (A) 33 **(B)** 35 **(C)** 37 **(D)** 39 **(E)** 41

2. Orvin went to the store with just enough money to buy 30 balloons. When he arrived he discovered that the store had a special sale on balloons: buy 1 balloon at the regular price and get a second at $\frac{1}{3}$ off the regular price. What is the greatest number of balloons Orvin could buy?

   (A) 33 **(B)** 34 **(C)** 36 **(D)** 38 **(E)** 39

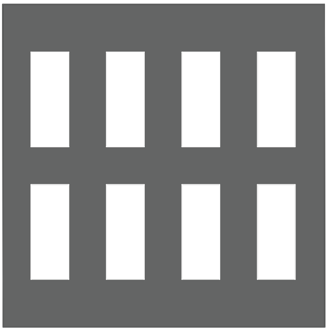
3. Randy drove the first third of his trip on a gravel road, the next 20 miles on pavement, and the remaining one-fifth on a dirt road. In miles, how long was Randy's trip?

   (A) 30 **(B)** $\dfrac{400}{11}$ **(C)** $\dfrac{75}{2}$ **(D)** 40 **(E)** $\dfrac{300}{7}$

4. Susie pays for 4 muffins and 3 bananas. Calvin spends twice as much paying for 2 muffins and 16 bananas. A muffin is how many times as expensive as a banana?

   (A) $\dfrac{3}{2}$ **(B)** $\dfrac{5}{3}$ **(C)** $\dfrac{7}{4}$ **(D)** 2 **(E)** $\dfrac{13}{4}$

5. Doug constructs a square window using 8 equal-size panes of glass, as shown. The ratio of the height to width for each pane is 5 : 2, and the borders around and between the panes are 2 inches wide. In inches, what is the side length of the square window?



   (A) 26 **(B)** 28 **(C)** 30 **(D)** 32 **(E)** 34

6. Ed and Ann both have lemonade with their lunch. Ed orders the regular size. Ann gets the large lemonade, which is 50% more than the regular. After both consume $\frac{3}{4}$ of their drinks, Ann gives Ed a third of what she has left, and 2 additional ounces. When they finish their lemonades they realize that they both drank the same amount. How many ounces of lemonade did they drink together?

   (A) 30 **(B)** 32 **(C)** 36 **(D)** 40 **(E)** 50

7. For how many positive integers n is $\frac{n}{30-n}$ also a positive integer?

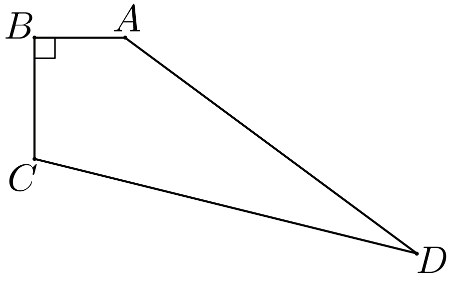
   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

8. In the addition shown below A, B, C, and D are distinct digits. How many different values are possible for D?

$$\begin{array}{r} ABBCB \\ +\quad BCADA \\ \hline DBDDD \end{array}$$

   (A) 2 **(B)** 4 **(C)** 7 **(D)** 8 **(E)** 9

9. Convex quadrilateral $ABCD$ has $AB = 3$, $BC = 4$, $CD = 13$, $AD = 12$, and $\angle ABC = 90^\circ$, as shown. What is the area of the quadrilateral?



   (A) 30 **(B)** 36 **(C)** 40 **(D)** 48 **(E)** 58.5

10. Danica drove her new car on a trip for a whole number of hours, averaging 55 miles per hour. At the beginning of the trip, abc miles was displayed on the odometer, where abc is a 3-digit number with $a \geq 1$ and $a + b + c \leq 7$. At the end of the trip, the odometer showed cba miles. What is $a^2 + b^2 + c^2$?

(A) 26 **(B)** 27 **(C)** 36 **(D)** 37 **(E)** 41

11. A list of 11 positive integers has a mean of 10, a median of 9, and a unique mode of 8. What is the largest possible value of an integer in the list?

(A) 24 **(B)** 30 **(C)** 31 **(D)** 33 **(E)** 35

12. A set S consists of triangles whose sides have integer lengths less than 5, and no two elements of S are congruent or similar. What is the largest number of elements that S can have?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

13. Real numbers a and b are chosen with $1 < a < b$ such that no triangle with positive area has side lengths 1, a, and b or $\frac{1}{b}$, $\frac{1}{a}$, and 1. What is the smallest possible value of b ?

(A) $\dfrac{3+\sqrt{3}}{2}$ **(B)** $\dfrac{5}{2}$ **(C)** $\dfrac{3+\sqrt{5}}{2}$ **(D)** $\dfrac{3+\sqrt{6}}{2}$ **(E)** 3

14. A rectangular box has a total surface area of 94 square inches. The sum of the lengths of all its edges is 48 inches. What is the sum of the lengths in inches of all of its interior diagonals?

(A) $8\sqrt{3}$ **(B)** $10\sqrt{2}$ **(C)** $16\sqrt{3}$ **(D)** $20\sqrt{2}$ **(E)** $40\sqrt{2}$

15. When $p = \sum_{k=1}^{6} k \ln k$, the number e^p is an integer. What is the largest power of 2 that is a factor of e^p ?

(A) 2^{12} **(B)** 2^{14} **(C)** 2^{16} **(D)** 2^{18} **(E)** 2^{20}

16. Let P be a cubic polynomial with $P(0) = k$, $P(1) = 2k$, and $P(-1) = 3k$. What is $P(2) + P(-2)$?

(A) 0 **(B)** k **(C)** $6k$ **(D)** $7k$ **(E)** $14k$

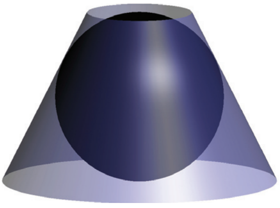
17. Let $\mathcal{P}$ be the parabola with equation $y = x^2$ and let $Q = (20, 14)$. There are real numbers r and s such that the line through Q with slope m does not intersect $\mathcal{P}$ if and only if $r < m < s$. What is $r + s$?

(A) 1 **(B)** 26 **(C)** 40 **(D)** 52 **(E)** 80

18. The numbers 1, 2, 3, 4, 5 are to be arranged in a circle. An arrangement is *bad* if it is not true that for every n from 1 to 15 one can find a subset of the numbers that appear consecutively on the circle that sum to n. Arrangements that differ only by a rotation or a reflection are considered the same. How many different bad arrangements are there?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

19. A sphere is inscribed in a truncated right circular cone as shown. The volume of the truncated cone is twice that of the sphere. What is the ratio of the radius of the bottom base of the truncated cone to the radius of the top base of the truncated cone?

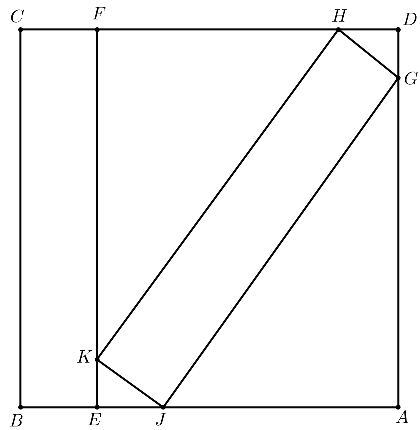


(A) $\frac{3}{2}$ **(B)** $\frac{1+\sqrt{5}}{2}$ **(C)** $\sqrt{3}$ **(D)** 2 **(E)** $\frac{3+\sqrt{5}}{2}$

20. For how many positive integers x is $\log_{10}(x - 40) + \log_{10}(60 - x) < 2$?

(A) 10 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** infinitely many

21. In the figure, $ABCD$ is a square of side length 1. The rectangles $JKHG$ and $EBCF$ are congruent. What is BE?



(A) $\frac{1}{2}(\sqrt{6}-2)$ **(B)** $\frac{1}{4}$ **(C)** $2-\sqrt{3}$ **(D)** $\frac{\sqrt{3}}{6}$ **(E)** $1-\frac{\sqrt{2}}{2}$

22. In a small pond there are eleven lily pads in a row labeled 0 through 10. A frog is sitting on pad 1. When the frog is on pad N, $0 < N < 10$, it will jump to pad $N-1$ with probability $\frac{N}{10}$ and to pad $N+1$ with probability $1-\frac{N}{10}$. Each jump is independent of the previous jumps. If the frog reaches pad 0 it will be eaten by a patiently waiting snake. If the frog reaches pad 10 it will exit the pond, never to return. What is the probability that the frog will escape being eaten by the snake?

(A) $\frac{32}{79}$ **(B)** $\frac{161}{384}$ **(C)** $\frac{63}{146}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{1}{2}$

23. The number 2017 is prime. Let $S=\sum_{k=0}^{62}\binom{2014}{k}$. What is the remainder when S is divided by 2017?

(A) 32 **(B)** 684 **(C)** 1024 **(D)** 1576 **(E)** 2016

24. Let $ABCDE$ be a pentagon inscribed in a circle such that $AB=CD=3$, $BC=DE=10$, and $AE=14$. The sum of the lengths of all diagonals of $ABCDE$ is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

(A) 129 **(B)** 247 **(C)** 353 **(D)** 391 **(E)** 421

25. What is the sum of all positive real solutions x to the equation

$$2\cos(2x)\left(\cos(2x)-\cos\left(\frac{2014\pi^2}{x}\right)\right)=\cos(4x)-1\,?$$

(A) π **(B)** 810π **(C)** 1008π **(D)** 1080π **(E)** 1800π

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

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Correspondence about the problems and solutions for this AMC 12 and orders for publications should be addressed to:

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The problems and solutions for this AMC 12 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 12 Subcommittee Chair:

Prof. Bernardo M. Abrego

2014 AIME

The 32nd annual AIME will be held on Thursday, March 13, with the alternate on Wednesday, March 26. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 43rd Annual USA Mathematical Olympiad (USAMO) on April 29-30, 2014. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: maa.org/math-competitions

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

66th Annual

AMC 12 A

American Mathematics Contest 12 A

Tuesday, February 3, 2015

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
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2015

AMC 12 A

DO NOT OPEN UNTIL TUESDAY, FEBRUARY 3, 2015

****Administration On An Earlier Date Will Disqualify Your School's Results****

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The problems and solutions for this AMC 12 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 12 Subcommittee Chair:

Jerrold W. Grossman

2015 AIME

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PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: maa.org/math-competitions

1. What is the value of $(2^0 - 1 + 5^2 + 0)^{-1} \times 5$?

   (A) -125 **(B)** -120 **(C)** $\frac{1}{5}$ **(D)** $\frac{5}{24}$ **(E)** 25

2. Two of the three sides of a triangle are 20 and 15. Which of the following numbers is not a possible perimeter of the triangle?

   (A) 52 **(B)** 57 **(C)** 62 **(D)** 67 **(E)** 72

3. Mr. Patrick teaches math to 15 students. He was grading tests and found that when he graded everyone's test except Payton's, the average grade for the class was 80. After he graded Payton's test, the class average became 81. What was Payton's score on the test?

   (A) 81 **(B)** 85 **(C)** 91 **(D)** 94 **(E)** 95

4. The sum of two positive numbers is 5 times their difference. What is the ratio of the larger number to the smaller?

   (A) $\frac{5}{4}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{9}{5}$ **(D)** 2 **(E)** $\frac{5}{2}$

5. Amelia needs to estimate the quantity $\frac{a}{b} - c$, where a, b, and c are large positive integers. She rounds each of the integers so that the calculation will be easier to do mentally. In which of these situations will her answer necessarily be greater than the exact value of $\frac{a}{b} - c$?

   (A) She rounds all three numbers up.

   (B) She rounds a and b up, and she rounds c down.

   (C) She rounds a and c up, and she rounds b down.

   (D) She rounds a up, and she rounds b and c down.

   (E) She rounds c up, and she rounds a and b down.

6. Two years ago Pete was three times as old as his cousin Claire. Two years before that, Pete was four times as old as Claire. In how many years will the ratio of their ages be $2:1$?

   (A) 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

7. Two right circular cylinders have the same volume. The radius of the second cylinder is 10% more than the radius of the first. What is the relationship between the heights of the two cylinders?

   (A) The second height is 10% less than the first

   (B) The first height is 10% more than the second

   (C) The second height is 21% less than the first

   (D) The first height is 21% more than the second

   (E) The second height is 80% of the first

8. The ratio of the length to the width of a rectangle is 4 : 3. If the rectangle has diagonal of length d, then the area may be expressed as kd^2 for some constant k. What is k?

   (A) $\frac{2}{7}$ **(B)** $\frac{3}{7}$ **(C)** $\frac{12}{25}$ **(D)** $\frac{16}{25}$ **(E)** $\frac{3}{4}$

9. A box contains 2 red marbles, 2 green marbles, and 2 yellow marbles. Carol takes 2 marbles from the box at random; then Claudia takes 2 of the remaining marbles at random; and then Cheryl takes the last 2 marbles. What is the probability that Cheryl gets 2 marbles of the same color?

   (A) $\frac{1}{10}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

10. Integers x and y with $x > y > 0$ satisfy $x + y + xy = 80$. What is x?

    (A) 8 **(B)** 10 **(C)** 15 **(D)** 18 **(E)** 26

11. On a sheet of paper, Isabella draws a circle of radius 2, a circle of radius 3, and all possible lines simultaneously tangent to both circles. Isabella notices that she has drawn exactly $k \geq 0$ lines. How many different values of k are possible?

    (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

12. The parabolas $y = ax^2 \quad 2$ and $y = 4 - bx^2$ intersect the coordinate axes in exactly four points, and these four points are the vertices of a kite of area 12 What is $a + b$?

    (A) 1 **(B)** 1.5 **(C)** 2 **(D)** 2.5 **(E)** 3

13. A league with 12 teams holds a round-robin tournament, with each team playing every other team exactly once. Games either end with one team victorious or else end in a draw. A team scores 2 points for every game it wins and 1 point for every game it draws. Which of the following is *not* a true statement about the list of 12 scores?

(A) There must be an even number of odd scores.

(B) There must be an even number of even scores.

(C) There cannot be two scores of 0.

(D) The sum of the scores must be at least 100.

(E) The highest score must be at least 12.

14. What is the value of a for which $\frac{1}{\log_2 a} + \frac{1}{\log_3 a} + \frac{1}{\log_4 a} = 1$?

(A) 9 **(B)** 12 **(C)** 18 **(D)** 24 **(E)** 36

15. What is the minimum number of digits to the right of the decimal point needed to express the fraction $\frac{123\,456\,789}{2^{26} \cdot 5^4}$ as a decimal?

(A) 4 **(B)** 22 **(C)** 26 **(D)** 30 **(E)** 104

16. Tetrahedron $ABCD$ has $AB = 5$, $AC = 3$, $BC = 4$, $BD = 4$, $AD = 3$, and $CD = \frac{12}{5}\sqrt{2}$. What is the volume of the tetrahedron?

(A) $3\sqrt{2}$ **(B)** $2\sqrt{5}$ **(C)** $\frac{24}{5}$ **(D)** $3\sqrt{3}$ **(E)** $\frac{24}{5}\sqrt{2}$

17. Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?

(A) $\frac{47}{256}$ **(B)** $\frac{3}{16}$ **(C)** $\frac{49}{256}$ **(D)** $\frac{25}{128}$ **(E)** $\frac{51}{256}$

18. The zeros of the function $f(x) = x^2 - ax + 2a$ are integers. What is the sum of the possible values of a?

(A) 7 **(B)** 8 **(C)** 16 **(D)** 17 **(E)** 18

19. For some positive integers p, there is a quadrilateral $ABCD$ with positive integer side lengths, perimeter p, right angles at B and C, $AB = 2$, and $CD = AD$. How many different values of $p < 2015$ are possible?

(A) 30 **(B)** 31 **(C)** 61 **(D)** 62 **(E)** 63

20. Isosceles triangles T and T' are not congruent but have the same area and the same perimeter. The sides of T have lengths of 5, 5, and 8, while those of T' have lengths a, a, and b. Which of the following numbers is closest to b?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

21. A circle of radius r passes through both foci of, and exactly four points on, the ellipse with equation $x^2 + 16y^2 = 16$. The set of all possible values of r is an interval $[a, b)$. What is $a + b$?

(A) $5\sqrt{2}+4$ **(B)** $\sqrt{17}+7$ **(C)** $6\sqrt{2}+3$ **(D)** $\sqrt{15}+8$ **(E)** 12

22. For each positive integer n, let $S(n)$ be the number of sequences of length n consisting solely of the letters A and B, with no more than three As in a row and no more than three Bs in a row. What is the remainder when $S(2015)$ is divided by 12?

(A) 0 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

23. Let S be a square of side length 1. Two points are chosen independently at random on the sides of S. The probability that the straight-line distance between the points is at least $\frac{1}{2}$ is $\frac{a-b\pi}{c}$, where a, b, and c are positive integers and $\gcd(a, b, c) = 1$. What is $a + b + c$?

(A) 59 **(B)** 60 **(C)** 61 **(D)** 62 **(E)** 63

24. Rational numbers a and b are chosen at random among all rational numbers in the interval $[0, 2)$ that can be written as fractions $\frac{n}{d}$ where n and d are integers with $1 \le d \le 5$. What is the probability that

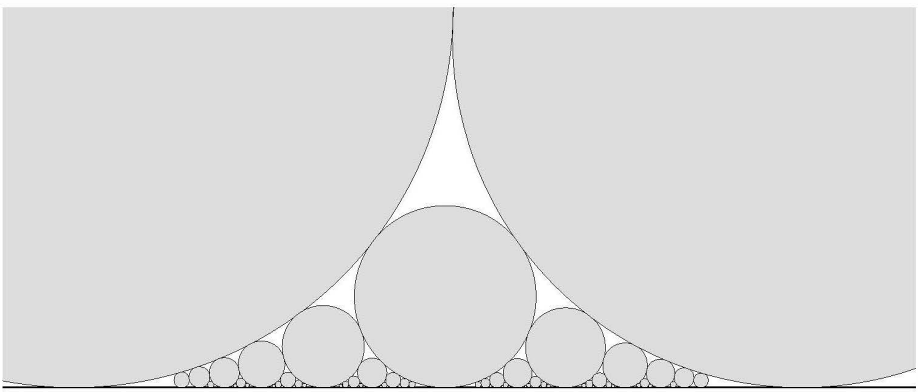
$$(\cos(a\pi) + i\sin(b\pi))^4$$

is a real number?

(A) $\frac{3}{50}$ **(B)** $\frac{4}{25}$ **(C)** $\frac{41}{200}$ **(D)** $\frac{6}{25}$ **(E)** $\frac{13}{50}$

25. A collection of circles in the upper half-plane, all tangent to the x-axis, is constructed in layers as follows. Layer L_0 consists of two circles of radii 70^2 and 73^2 that are externally tangent. For $k \geq 1$, the circles in $\bigcup_{j=0}^{k-1} L_j$ are ordered according to their points of tangency with the x-axis. For every pair of consecutive circles in this order, a new circle is constructed externally tangent to each of the two circles in the pair. Layer L_k consists of the 2^{k-1} circles constructed in this way. Let $S = \bigcup_{j=0}^{6} L_j$, and for every circle C denote by $r(C)$ its radius. What is

$$\sum_{C \in S} \frac{1}{\sqrt{r(C)}}?$$



(A) $\frac{286}{35}$ **(B)** $\frac{583}{70}$ **(C)** $\frac{715}{73}$ **(D)** $\frac{143}{14}$ **(E)** $\frac{1573}{146}$

1. What is the value of $2-(-2)^{-2}$?

   (A) -2 **(B)** $\frac{1}{16}$ **(C)** $\frac{7}{4}$ **(D)** $\frac{9}{4}$ **(E)** 6

2. Marie does three equally time-consuming tasks in a row without taking breaks. She begins the first task at 1:00 PM and finishes the second task at 2:40 PM. When does she finish the third task?

   (A) 3:10 PM **(B)** 3:30 PM **(C)** 4:00 PM **(D)** 4:10 PM **(E)** 4:30 PM

3. Isaac has written down one integer two times and another integer three times. The sum of the five numbers is 100, and one of the numbers is 28. What is the other number?

   (A) 8 **(B)** 11 **(C)** 14 **(D)** 15 **(E)** 18

4. David, Hikmet, Jack, Marta, Rand, and Todd were in a 12-person race with 6 other people. Rand finished 6 places ahead of Hikmet. Marta finished 1 place behind Jack. David finished 2 places behind Hikmet. Jack finished 2 places behind Todd. Todd finished 1 place behind Rand. Marta finished in 6th place. Who finished in 8th place?

   (A) David **(B)** Hikmet **(C)** Jack **(D)** Rand **(E)** Todd

5. The Tigers beat the Sharks 2 out of the first 3 times they played. They then played N more times, and the Sharks ended up winning at least 95% of all the games played. What is the minimum possible value for N?

   (A) 35 **(B)** 37 **(C)** 39 **(D)** 41 **(E)** 43

6. Back in 1930, Tillie had to memorize her multiplication facts from 0×0 through 12×12. The multiplication table she was given had rows and columns labeled with the factors, and the products formed the body of the table. To the nearest hundredth, what fraction of the numbers in the body of the table are odd?

   (A) 0.21 **(B)** 0.25 **(C)** 0.46 **(D)** 0.50 **(E)** 0.75

7. A regular 15-gon has L lines of symmetry, and the smallest positive angle for which it has rotational symmetry is R degrees. What is $L+R$?

   (A) 24 **(B)** 27 **(C)** 32 **(D)** 39 **(E)** 54

8. What is the value of $\left(625^{\log_5 2015}\right)^{\frac{1}{4}}$?

(A) 5 **(B)** $\sqrt[4]{2015}$ **(C)** 625 **(D)** 2015 **(E)** $\sqrt[4]{5^{2015}}$

9. Larry and Julius are playing a game, taking turns throwing a ball at a bottle sitting on a ledge. Larry throws first. The winner is the first person to knock the bottle off the ledge. At each turn the probability that a player knocks the bottle off the ledge is $\frac{1}{2}$, independently of what has happened before. What is the probability that Larry wins the game?

(A) $\dfrac{1}{2}$ **(B)** $\dfrac{3}{5}$ **(C)** $\dfrac{2}{3}$ **(D)** $\dfrac{3}{4}$ **(E)** $\dfrac{4}{5}$

10. How many noncongruent integer-sided triangles with positive area and perimeter less than 15 are neither equilateral, isosceles, nor right triangles?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

11. The line $12x + 5y = 60$ forms a triangle with the coordinate axes. What is the sum of the lengths of the altitudes of this triangle?

(A) 20 **(B)** $\dfrac{360}{17}$ **(C)** $\dfrac{107}{5}$ **(D)** $\dfrac{43}{2}$ **(E)** $\dfrac{281}{13}$

12. Let a, b, and c be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation $(x-a)(x-b)+(x-b)(x-c)=0$?

(A) 15 **(B)** 15.5 **(C)** 16 **(D)** 16.5 **(E)** 17

13. Quadrilateral $ABCD$ is inscribed in a circle with $\angle BAC = 70^\circ$, $\angle ADB = 40^\circ$, $AD = 4$, and $BC = 6$. What is AC ?

(A) $3+\sqrt{5}$ **(B)** 6 **(C)** $\dfrac{9}{2}\sqrt{2}$ **(D)** $8-\sqrt{2}$ **(E)** 7

14. A circle of radius 2 is centered at A. An equilateral triangle with side 4 has a vertex at A. What is the difference between the area of the region that lies inside the circle but outside the triangle and the area of the region that lies inside the triangle but outside the circle?

(A) $8-\pi$ **(B)** $\pi+2$ **(C)** $2\pi-\dfrac{\sqrt{2}}{2}$ **(D)** $4(\pi-\sqrt{3})$ **(E)** $2\pi+\dfrac{\sqrt{3}}{2}$

15. At Rachelle's school an A counts 4 points, a B 3 points, a C 2 points, and a D 1 point. Her GPA on the four classes she is taking is computed as the total sum of points divided by 4. She is certain that she will get As in both Mathematics and Science, and at least a C in each of English and History. She thinks she has a $\frac{1}{6}$ chance of getting an A in English, and a $\frac{1}{4}$ chance of getting a B. In History, she has a $\frac{1}{4}$ chance of getting an A, and a $\frac{1}{3}$ chance of getting a B, independently of what she gets in English. What is the probability that Rachelle will get a GPA of at least 3.5 ?

(A) $\frac{11}{72}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{3}{16}$ **(D)** $\frac{11}{24}$ **(E)** $\frac{1}{2}$

16. A regular hexagon with sides of length 6 has an isosceles triangle attached to each side. Each of these triangles has two sides of length 8. The isosceles triangles are folded to make a pyramid with the hexagon as the base of the pyramid. What is the volume of the pyramid?

(A) 18 **(B)** 162 **(C)** $36\sqrt{21}$ **(D)** $18\sqrt{138}$ **(E)** $54\sqrt{21}$

17. An unfair coin lands on heads with a probability of $\frac{1}{4}$. When tossed n times, the probability of exactly two heads is the same as the probability of exactly three heads. What is the value of n ?

(A) 5 **(B)** 8 **(C)** 10 **(D)** 11 **(E)** 13

18. For every composite positive integer n, define $r(n)$ to be the sum of the factors in the prime factorization of n. For example, $r(50) = 12$ because the prime factorization of 50 is $2 \cdot 5^2$, and $2 + 5 + 5 = 12$. What is the range of the function r, $\{r(n) : n \text{ is a composite positive integer}\}$?

(A) the set of positive integers

(B) the set of composite positive integers

(C) the set of even positive integers

(D) the set of integers greater than 3

(E) the set of integers greater than 4

19. In $\triangle ABC$, $\angle C = 90°$ and $AB = 12$. Squares $ABXY$ and $ACWZ$ are constructed outside of the triangle. The points X, Y, Z, and W lie on a circle. What is the perimeter of the triangle?

(A) $12 + 9\sqrt{3}$ **(B)** $18 + 6\sqrt{3}$ **(C)** $12 + 12\sqrt{2}$ **(D)** 30 **(E)** 32

20. For every positive integer n, let $\text{mod}_5(n)$ be the remainder obtained when n is divided by 5. Define a function $f : \{0, 1, 2, 3, \ldots\} \times \{0, 1, 2, 3, 4\} \to \{0, 1, 2, 3, 4\}$ recursively as follows:

$$f(i,j) = \begin{cases} \text{mod}_5(j+1) & \text{if } i = 0 \text{ and } 0 \le j \le 4, \\ f(i-1, 1) & \text{if } i \ge 1 \text{ and } j = 0, \text{ and} \\ f(i-1, f(i, j-1)) & \text{if } i \ge 1 \text{ and } 1 \le j \le 4. \end{cases}$$

What is $f(2015, 2)$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

21. Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. However, instead of walking up the steps one at a time, both Cozy and Dash jump. Cozy goes two steps up with each jump (though if necessary, he will just jump the last step). Dash goes five steps up with each jump (though if necessary, he will just jump the last steps if there are fewer than 5 steps left). Suppose that Dash takes 19 fewer jumps than Cozy to reach the top of the staircase. Let s denote the sum of all possible numbers of steps this staircase can have. What is the sum of the digits of s?

(A) 9 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 15

22. Six chairs are evenly spaced around a circular table. One person is seated in each chair. Each person gets up and sits down in a chair that is not the same chair and is not adjacent to the chair he or she originally occupied, so that again one person is seated in each chair. In how many ways can this be done?

(A) 14 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 24

23. A rectangular box measures $a \times b \times c$, where a, b, and c are integers and $1 \le a \le b \le c$. The volume and the surface area of the box are numerically equal. How many ordered triples (a, b, c) are possible?

(A) 4 **(B)** 10 **(C)** 12 **(D)** 21 **(E)** 26

24. Four circles, no two of which are congruent, have centers at A, B, C, and D, and points P and Q lie on all four circles. The radius of circle A is $\frac{5}{8}$ times the radius of circle B, and the radius of circle C is $\frac{5}{8}$ times the radius of circle D. Furthermore, $AB = CD = 39$ and $PQ = 48$. Let R be the midpoint of $\overline{PQ}$. What is $AR + BR + CR + DR$?

(A) 180 **(B)** 184 **(C)** 188 **(D)** 192 **(E)** 196

25. A bee starts flying from point P_0. She flies 1 inch due east to point P_1. For $j \geq 1$, once the bee reaches point P_j, she turns 30° counterclockwise and then flies $j+1$ inches straight to point P_{j+1}. When the bee reaches P_{2015} she is exactly $a\sqrt{b} + c\sqrt{d}$ inches away from P_0, where a, b, c, and d are positive integers and b and d are not divisible by the square of any prime. What is $a + b + c + d$?

(A) 2016 **(B)** 2024 **(C)** 2032 **(D)** 2040 **(E)** 2048

MATHEMATICAL ASSOCIATION OF AMERICA
CELEBRATING A CENTURY OF ADVANCING MATHEMATICS

American Mathematics Competitions

67th Annual

AMC 12A

American Mathematics Contest 12A
Tuesday, February 2, 2016

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded. **No copies.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score well on this AMC 12 will be invited to take the 34th annual American Invitational Mathematics Examination (AIME) on Thursday, March 3, 2016 or Wednesday, March 16, 2016. More details about the AIME are on the back page of this test booklet.

2016

AMC 12A

DO NOT OPEN UNTIL TUESDAY, FEBRUARY 2, 2016

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL. PLEASE READ THE MANUAL BEFORE FEBRUARY 2, 2016.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 12 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.*

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The problems and solutions for this AMC 12 were prepared by MAA's Subcommittee on the AMC10/AMC12 Exams, under the direction of the co-chairs Jerrold W. Grossman and Silvia Fernandez.

2015 AIME

The 34th annual AIME will be held on Thursday, March 3, 2016, with the alternate on Wednesday, March 16, 2016. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate if you achieve a high score on this contest. Top-scoring students on the AMC 10/12/AIME will be selected to take the 45th Annual USA Mathematical Olympiad (USAMO) on April 19–20, 2016. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:

www.maa.org/amc

1. What is the value of $\frac{11!-10!}{9!}$?

   (A) 99 **(B)** 100 **(C)** 110 **(D)** 121 **(E)** 132

2. For what value of x does $10^x \cdot 100^{2x} = 1000^5$?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

3. The remainder function can be defined for all real numbers x and y with $y \neq 0$ by

$$\text{rem}(x, y) = x - y \left\lfloor \frac{x}{y} \right\rfloor,$$

   where $\left\lfloor \frac{x}{y} \right\rfloor$ denotes the greatest integer less than or equal to $\frac{x}{y}$. What is the value of $\text{rem}\left(\frac{3}{8}, -\frac{2}{5}\right)$?

   (A) $-\frac{3}{8}$ **(B)** $-\frac{1}{40}$ **(C)** 0 **(D)** $\frac{3}{8}$ **(E)** $\frac{31}{40}$

4. The mean, median, and mode of the 7 data values $60, 100, x, 40, 50, 200, 90$ are all equal to x. What is the value of x?

   (A) 50 **(B)** 60 **(C)** 75 **(D)** 90 **(E)** 100

5. Goldbach's conjecture states that every even integer greater than 2 can be written as the sum of two prime numbers (for example, $2016 = 13 + 2003$). So far, no one has been able to prove that the conjecture is true, and no one has found a counterexample to show that the conjecture is false. What would a counterexample consist of?

   (A) an odd integer greater than 2 that can be written as the sum of two prime numbers

   (B) an odd integer greater than 2 that cannot be written as the sum of two prime numbers

   (C) an even integer greater than 2 that can be written as the sum of two numbers that are not prime

   (D) an even integer greater than 2 that can be written as the sum of two prime numbers

   (E) an even integer greater than 2 that cannot be written as the sum of two prime numbers

6. A triangular array of 2016 coins has 1 coin in the first row, 2 coins in the second row, 3 coins in the third row, and so on up to N coins in the Nth row. What is the sum of the digits of N?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

7. Which of these describes the graph of $x^2(x+y+1) = y^2(x+y+1)$?

(A) two parallel lines

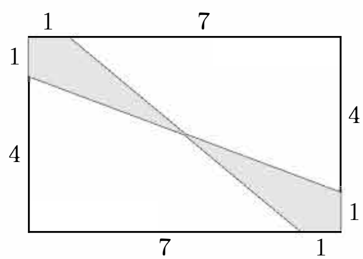
(B) two intersecting lines

(C) three lines that all pass through a common point

(D) three lines that do not all pass through a common point

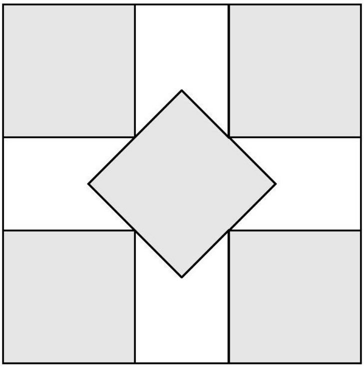
(E) a line and a parabola

8. What is the area of the shaded region of the given 8×5 rectangle?



(A) $4\frac{3}{4}$ **(B)** 5 **(C)** $5\frac{1}{4}$ **(D)** $6\frac{1}{2}$ **(E)** 8

9. The five small shaded squares inside this unit square are congruent and have disjoint interiors. The midpoint of each side of the middle square coincides with one of the vertices of the other four small squares as shown. The common side length is $\frac{a-\sqrt{2}}{b}$, where a and b are positive integers. What is $a+b$?



(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

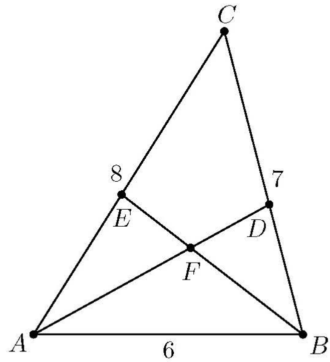
10. Five friends sat in a movie theater in a row containing 5 seats, numbered 1 to 5 from left to right. (The directions "left" and "right" are from the point of view of the people as they sit in the seats.) During the movie Ada went to the lobby to get some popcorn. When she returned, she found that Bea had moved two seats to the right, Ceci had moved one seat to the left, and Dee and Edie had switched seats, leaving an end seat for Ada. In which seat had Ada been sitting before she got up?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

11. Each of the 100 students in a certain summer camp can either sing, dance, or act. Some students have more than one talent, but no student has all three talents. There are 42 students who cannot sing, 65 students who cannot dance, and 29 students who cannot act. How many students have two of these talents?

(A) 16 **(B)** 25 **(C)** 36 **(D)** 49 **(E)** 64

12. In $\triangle ABC$, $AB = 6$, $BC = 7$, and $CA = 8$. Point D lies on $\overline{BC}$, and $\overline{AD}$ bisects $\angle BAC$. Point E lies on $\overline{AC}$, and $\overline{BE}$ bisects $\angle ABC$. The bisectors intersect at F. What is the ratio $AF : FD$?



(A) $3:2$ **(B)** $5:3$ **(C)** $2:1$ **(D)** $7:3$ **(E)** $5:2$

13. Let N be a positive multiple of 5. One red ball and N green balls are arranged in a line in random order. Let $P(N)$ be the probability that at least $\frac{3}{5}$ of the green balls are on the same side of the red ball. Observe that $P(5) = 1$ and that $P(N)$ approaches $\frac{4}{5}$ as N grows large. What is the sum of the digits of the least value of N such that $P(N) < \frac{321}{400}$?

(A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

14. Each vertex of a cube is to be labeled with an integer from 1 through 8, with each integer being used once, in such a way that the sum of the four numbers on the vertices of a face is the same for each face. Arrangements that can be obtained from each other through rotations of the cube are considered to be the same. How many different arrangements are possible?

(A) 1 **(B)** 3 **(C)** 6 **(D)** 12 **(E)** 24

15. Circles with centers P, Q, and R, having radii 1, 2, and 3, respectively, lie on the same side of line l and are tangent to l at P', Q', and R', respectively, with Q' between P' and R'. The circle with center Q is externally tangent to each of the other two circles. What is the area of $\triangle PQR$?

(A) 0 **(B)** $\sqrt{\frac{2}{3}}$ **(C)** 1 **(D)** $\sqrt{6}-\sqrt{2}$ **(E)** $\sqrt{\frac{3}{2}}$

16. The graphs of $y = \log_3 x$, $y = \log_x 3$, $y = \log_{\frac{1}{3}} x$, and $y = \log_x \frac{1}{3}$ are plotted on the same set of axes. How many points in the plane with positive x-coordinates lie on two or more of the graphs?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

17. Let $ABCD$ be a square. Let E, F, G, and H be the centers, respectively, of equilateral triangles with bases $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, each exterior to the square. What is the ratio of the area of square $EFGH$ to the area of square $ABCD$?

(A) 1 **(B)** $\dfrac{2+\sqrt{3}}{3}$ **(C)** $\sqrt{2}$ **(D)** $\dfrac{\sqrt{2}+\sqrt{3}}{2}$ **(E)** $\sqrt{3}$

18. For some positive integer n, the number $110n^3$ has 110 positive integer divisors, including 1 and the number $110n^3$. How many positive integer divisors does the number $81n^4$ have?

(A) 110 **(B)** 191 **(C)** 261 **(D)** 325 **(E)** 425

19. Jerry starts at 0 on the real number line. He tosses a fair coin 8 times. When he gets heads, he moves 1 unit in the positive direction; when he gets tails, he moves 1 unit in the negative direction. The probability that he reaches 4 at some time during this process is $\frac{a}{b}$, where a and b are relatively prime positive integers. What is $a + b$? (For example, he succeeds if his sequence of tosses is HTHHHHHH.)

(A) 69 **(B)** 151 **(C)** 257 **(D)** 293 **(E)** 313

20. A binary operation $\diamondsuit$ has the properties that $a \diamondsuit (b \diamondsuit c) = (a \diamondsuit b) \cdot c$ and that $a \diamondsuit a = 1$ for all nonzero real numbers a, b, and c. (Here the dot $\cdot$ represents the usual multiplication operation.) The solution to the equation $2016 \diamondsuit (6 \diamondsuit x) = 100$ can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p + q$?

(A) 109 **(B)** 201 **(C)** 301 **(D)** 3049 **(E)** 33,601

21. A quadrilateral is inscribed in a circle of radius $200\sqrt{2}$. Three of the sides of this quadrilateral have length 200. What is the length of its fourth side?

(A) 200 **(B)** $200\sqrt{2}$ **(C)** $200\sqrt{3}$ **(D)** $300\sqrt{2}$ **(E)** 500

22. How many ordered triples (x, y, z) of positive integers satisfy $\text{lcm}(x, y) = 72$, $\text{lcm}(x, z) = 600$, and $\text{lcm}(y, z) = 900$?

(A) 15 **(B)** 16 **(C)** 24 **(D)** 27 **(E)** 64

23. Three numbers in the interval $[0, 1]$ are chosen independently and at random. What is the probability that the chosen numbers are the side lengths of a triangle with positive area?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{5}{6}$

24. There is a smallest positive real number a such that there exists a positive real number b such that all the roots of the polynomial $x^3 - ax^2 + bx - a$ are real. In fact, for this value of a the value of b is unique. What is this value of b?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

25. Let k be a positive integer. Bernardo and Silvia take turns writing and erasing numbers on a blackboard as follows: Bernardo starts by writing the smallest perfect square with $k + 1$ digits. Every time Bernardo writes a number, Silvia erases the last k digits of it. Bernardo then writes the next perfect square, Silvia erases the last k digits of it, and this process continues until the last two numbers that remain on the board differ by at least 2. Let $f(k)$ be the smallest positive integer not written on the board. For example, if $k = 1$, then the numbers that Bernardo writes are $16, 25, 36, 49$, and 64, and the numbers showing on the board after Silvia erases are $1, 2, 3, 4$, and 6, and thus $f(1) = 5$. What is the sum of the digits of $f(2) + f(4) + f(6) + \cdots + f(2016)$?

(A) 7986 **(B)** 8002 **(C)** 8030 **(D)** 8048 **(E)** 8064

MAA 100

MATHEMATICAL ASSOCIATION OF AMERICA

CELEBRATING A CENTURY OF ADVANCING MATHEMATICS

American Mathematics Competitions

67th Annual

AMC 12B

American Mathematics Contest 12B

Wednesday, February 17, 2016

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded. **No copies.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

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Students who score well on this AMC 12 will be invited to take the 34th annual American Invitational Mathematics Examination (AIME) on Thursday, March 3, 2016 or Wednesday, March 16, 2016. More details about the AIME are on the back page of this test booklet.

The publication, reproduction or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.

2016

AMC 12B

DO NOT OPEN UNTIL WEDNESDAY, FEBRUARY 17, 2016

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL. PLEASE READ THE MANUAL BEFORE FEBRUARY 17, 2016.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 12 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.*

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American Mathematics Competitions

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Correspondence about the problems and solutions for this AMC 12 and orders for publications should be addressed to:

MAA American Mathematics Competitions
PO Box 471
Annapolis Junction, MD 20701
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The problems and solutions for this AMC 12 were prepared by MAA's Subcommittee on the AMC10/AMC12 Exams, under the direction of the co-chairs Jerrold W. Grossman and Silvia Fernandez.

2016 AIME

The 34th annual AIME will be held on Thursday, March 3, 2016, with the alternate on Wednesday, March 16, 2016. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate if you achieve a high score on this test. Top-scoring students on the AMC 10/12/AIME will be selected to take the 45th Annual USA Mathematical Olympiad (USAMO) on April 19–20, 2016. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
www.maa.org/amc

1. What is the value of

$$\frac{2a^{-1}+\frac{a^{-1}}{2}}{a}$$

when $a=\frac{1}{2}$?

(A) 1 **(B)** 2 **(C)** $\frac{5}{2}$ **(D)** 10 **(E)** 20

2. The harmonic mean of two numbers can be computed as twice their product divided by their sum. The harmonic mean of 1 and 2016 is closest to which integer?

(A) 2 **(B)** 45 **(C)** 504 **(D)** 1008 **(E)** 2015

3. Let $x=-2016$. What is the value of $\Bigg\vert\Big\vert |x|-x\Big\vert-|x|\Bigg\vert-x$?

(A) -2016 **(B)** 0 **(C)** 2016 **(D)** 4032 **(E)** 6048

4. The ratio of the measures of two acute angles is $5:4$, and the complement of one of these two angles is twice as large as the complement of the other. What is the sum of the degree measures of the two angles?

(A) 75 **(B)** 90 **(C)** 135 **(D)** 150 **(E)** 270

5. The War of 1812 started with a declaration of war on Thursday, June 18, 1812. The peace treaty to end the war was signed 919 days later, on December 24, 1814. On what day of the week was the treaty signed?

(A) Friday **(B)** Saturday **(C)** Sunday **(D)** Monday **(E)** Tuesday

6. All three vertices of $\triangle ABC$ lie on the parabola defined by $y=x^2$, with A at the origin and $\overline{BC}$ parallel to the x-axis. The area of the triangle is 64. What is the length BC?

(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 16

7. Josh writes the numbers $1,2,3,\ldots,99,100$. He marks out 1, skips the next number (2), marks out 3, and continues skipping and marking out the next number to the end of his list. Then he goes back to the start of his list, marks out the first remaining number (2), skips the next number (4), marks out 6, skips 8, marks out 10, and so on to the end. Josh continues in this manner until only one number remains. What is that number?

(A) 13 **(B)** 32 **(C)** 56 **(D)** 64 **(E)** 96

8. A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 ounces. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length 5 inches. Which of the following is closest to the weight, in ounces, of the second piece?

(A) 14.0 **(B)** 16.0 **(C)** 20.0 **(D)** 33.3 **(E)** 55.6

9. Carl decided to fence in his rectangular garden. He bought 20 fence posts, placed one on each of the four corners, and spaced out the rest evenly along the edges of the garden, leaving exactly 4 yards between neighboring posts. The longer side of his garden, including the corners, has twice as many posts as the shorter side, including the corners. What is the area, in square yards, of Carl's garden?

(A) 256 **(B)** 336 **(C)** 384 **(D)** 448 **(E)** 512

10. A quadrilateral has vertices $P(a, b)$, $Q(b, a)$, $R(-a, -b)$, and $S(-b, -a)$, where a and b are integers with $a > b > 0$. The area of $PQRS$ is 16. What is $a + b$?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 12 **(E)** 13

11. How many squares whose sides are parallel to the axes and whose vertices have coordinates that are integers lie entirely within the region bounded by the line $y = \pi x$, the line $y = -0.1$, and the line $x = 5.1$?

(A) 30 **(B)** 41 **(C)** 45 **(D)** 50 **(E)** 57

12. All the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ are written in a 3×3 array of squares, one number in each square, in such a way that if two numbers are consecutive then they occupy squares that share an edge. The numbers in the four corners add up to 18. What number is in the center?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

13. Alice and Bob live 10 miles apart. One day Alice looks due north from her house and sees an airplane. At the same time Bob looks due west from his house and sees the same airplane. The angle of elevation of the airplane is 30° from Alice's position and 60° from Bob's position. Which of the following is closest to the airplane's altitude, in miles?

(A) 3.5 **(B)** 4 **(C)** 4.5 **(D)** 5 **(E)** 5.5

14. The sum of an infinite geometric series is a positive number S, and the second term in the series is 1. What is the smallest possible value of S?

(A) $\frac{1+\sqrt{5}}{2}$ **(B)** 2 **(C)** $\sqrt{5}$ **(D)** 3 **(E)** 4

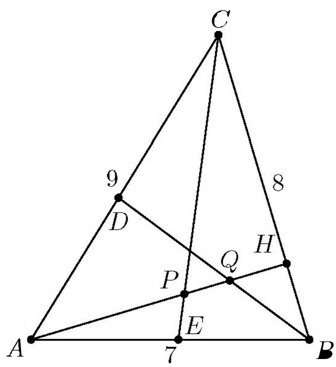
15. All the numbers $2, 3, 4, 5, 6, 7$ are assigned to the six faces of a cube, one number to each face. For each of the eight vertices of the cube, a product of three numbers is computed, where the three numbers are the numbers assigned to the three faces that include that vertex. What is the greatest possible value of the sum of these eight products?

(A) 312 **(B)** 343 **(C)** 625 **(D)** 729 **(E)** 1680

16. In how many ways can 345 be written as the sum of an increasing sequence of two or more consecutive positive integers?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 7

17. In $\triangle ABC$ shown in the figure, $AB = 7$, $BC = 8$, $CA = 9$, and $\overline{AH}$ is an altitude. Points D and E lie on sides $\overline{AC}$ and $\overline{AB}$, respectively, so that $\overline{BD}$ and $\overline{CE}$ are angle bisectors, intersecting $\overline{AH}$ at Q and P, respectively. What is PQ?



(A) 1 **(B)** $\frac{5}{8}\sqrt{3}$ **(C)** $\frac{4}{5}\sqrt{2}$ **(D)** $\frac{8}{15}\sqrt{5}$ **(E)** $\frac{6}{5}$

18. What is the area of the region enclosed by the graph of the equation $x^2 + y^2 = |x| + |y|$?

(A) $\pi + \sqrt{2}$ **(B)** $\pi + 2$ **(C)** $\pi + 2\sqrt{2}$ **(D)** $2\pi + \sqrt{2}$ **(E)** $2\pi + 2\sqrt{2}$

19. Tom, Dick, and Harry are playing a game. Starting at the same time, each of them flips a fair coin repeatedly until he gets his first head, at which point he stops. What is the probability that all three flip their coins the same number of times?

(A) $\frac{1}{8}$ **(B)** $\frac{1}{7}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{1}{3}$

20. A set of teams held a round-robin tournament in which every team played every other team exactly once. Every team won 10 games and lost 10 games; there were no ties. How many sets of three teams $\{A, B, C\}$ were there in which A beat B, B beat C, and C beat A?

(A) 385 **(B)** 665 **(C)** 945 **(D)** 1140 **(E)** 1330

21. Let $ABCD$ be a unit square. Let Q_1 be the midpoint of $\overline{CD}$. For $i = 1, 2, \ldots,$ let P_i be the intersection of $\overline{AQ_i}$ and $\overline{BD}$, and let Q_{i+1} be the foot of the perpendicular from P_i to $\overline{CD}$. What is

$$\sum_{i=1}^{\infty} \text{Area of } \triangle DQ_iP_i\,?$$

(A) $\frac{1}{6}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** 1

22. For a certain positive integer n less than 1000, the decimal equivalent of $\frac{1}{n}$ is $0.\overline{abcdef}$, a repeating decimal of period 6, and the decimal equivalent of $\frac{1}{n+6}$ is $0.\overline{wxyz}$, a repeating decimal of period 4. In which interval does n lie?

(A) $[1, 200]$ **(B)** $[201, 400]$ **(C)** $[401, 600]$ **(D)** $[601, 800]$ **(E)** $[801, 999]$

23. What is the volume of the region in three-dimensional space defined by the inequalities $|x| + |y| + |z| \le 1$ and $|x| + |y| + |z - 1| \le 1$?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** 1

24. There are exactly 77,000 ordered quadruples (a, b, c, d) such that $\gcd(a, b, c, d) = 77$ and $\operatorname{lcm}(a, b, c, d) = n$. What is the smallest possible value of n?

(A) 13,860 **(B)** 20,790 **(C)** 21,560 **(D)** 27,720 **(E)** 41,580

25. The sequence (a_n) is defined recursively by $a_0 = 1$, $a_1 = \sqrt[19]{2}$, and $a_n = a_{n-1}a_{n-2}^2$ for $n \ge 2$. What is the smallest positive integer k such that the product $a_1 a_2 \cdots a_k$ is an integer?

(A) 17 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** 21

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

68th Annual

AMC 12A

American Mathematics Competition 12A

Tuesday, February 7, 2017

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded. **No copies.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score well on this AMC 10 will be invited to take the 35th annual American Invitational Mathematics Examination (AIME) on Thursday, March 7, 2017 or Wednesday, March 22, 2017. More details about the AIME are on the back page of this test booklet.

The publication, reproduction or communication of the problems or solutions for this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.

2017

AMC 12A

DO NOT OPEN UNTIL TUESDAY, February 7, 2017

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the Teachers' Manual. PLEASE READ THE MANUAL BEFORE FEBRUARY 7, 2017.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 12 CERTIFICATION FORM (found on amc.maa.org under 'AMC 10/12') that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.*

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The problems and solutions for this AMC 12 were prepared by MAA's Subcommittee on the AMC10/AMC12 Exams, under the direction of the co-chairs Jerrold W. Grossman and Carl Yerger.

2017 AIME

The 35th annual AIME will be held on Thursday, March 7, 2017 with the alternate on Wednesday, March 22, 2017. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate if you achieve a high score on this competition. Top-scoring students on the AMC 10/12/AIME will be selected to take the 46th Annual USA Mathematical Olympiad (USAMO) on April 19–20, 2017. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
www.maa.org/amc

1. Pablo buys popsicles for his friends. The store sells single popsicles for \$1 each, 3-popsicle boxes for \$2, and 5-popsicle boxes for \$3. What is the greatest number of popsicles that Pablo can buy with \$8 ?

   (A) 8 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 15

2. The sum of two nonzero real numbers is 4 times their product. What is the sum of the reciprocals of the two numbers?

   (A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 12

3. Ms. Carroll promised that anyone who got all the multiple choice questions right on the upcoming exam would receive an A on the exam. Which one of these statements necessarily follows logically?

   (A) If Lewis did not receive an A, then he got all of the multiple choice questions wrong.

   (B) If Lewis did not receive an A, then he got at least one of the multiple choice questions wrong.

   (C) If Lewis got at least one of the multiple choice questions wrong, then he did not receive an A.

   (D) If Lewis received an A, then he got all of the multiple choice questions right.

   (E) If Lewis received an A, then he got at least one of the multiple choice questions right.

4. Jerry and Silvia wanted to go from the southwest corner of a square field to the northeast corner. Jerry walked due east and then due north to reach the goal, but Silvia headed northeast and reached the goal walking in a straight line. Which of the following is closest to how much shorter Silvia's trip was, compared to Jerry's trip?

   (A) 30% **(B)** 40% **(C)** 50% **(D)** 60% **(E)** 70%

5. At a gathering of 30 people, there are 20 people who all know each other and 10 people who know no one. People who know each other hug, and people who do not know each other shake hands. How many handshakes occur?

   (A) 240 **(B)** 245 **(C)** 290 **(D)** 480 **(E)** 490

6. Joy has 30 thin rods, one each of every integer length from 1 cm through 30 cm. She places the rods with lengths 3 cm, 7 cm, and 15 cm on a table. She then wants to choose a fourth rod that she can put with these three to form a quadrilateral with positive area. How many of the remaining rods can she choose as the fourth rod?

   (A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

7. Define a function on the positive integers recursively by $f(1) = 2$, $f(n) = f(n-1) + 1$ if n is even, and $f(n) = f(n-2) + 2$ if n is odd and greater than 1. What is $f(2017)$?

   (A) 2017 **(B)** 2018 **(C)** 4034 **(D)** 4035 **(E)** 4036

8. The region consisting of all points in three-dimensional space within 3 units of line segment $\overline{AB}$ has volume 216π. What is the length AB?

   (A) 6 **(B)** 12 **(C)** 18 **(D)** 20 **(E)** 24

9. Let S be the set of points (x, y) in the coordinate plane such that two of the three quantities 3, $x + 2$, and $y - 4$ are equal and the third of the three quantities is no greater than this common value. Which of the following is a correct description of S?

   (A) a single point **(B)** two intersecting lines

   (C) three lines whose pairwise intersections are three distinct points

   (D) a triangle **(E)** three rays with a common endpoint

10. Chloé chooses a real number uniformly at random from the interval $[0, 2017]$. Independently, Laurent chooses a real number uniformly at random from the interval $[0, 4034]$. What is the probability that Laurent's number is greater than Chloé's number?

    (A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{5}{6}$ **(E)** $\frac{7}{8}$

11. Claire adds the degree measures of the interior angles of a convex polygon and arrives at a sum of 2017. She then discovers that she forgot to include one angle. What is the degree measure of the forgotten angle?

    (A) 37 **(B)** 63 **(C)** 117 **(D)** 143 **(E)** 163

24. Quadrilateral $ABCD$ is inscribed in circle O and has sides $AB = 3$, $BC = 2$, $CD = 6$, and $DA = 8$. Let X and Y be points on $\overline{BD}$ such that

$$\frac{DX}{BD} = \frac{1}{4} \quad \text{and} \quad \frac{BY}{BD} = \frac{11}{36}.$$

Let E be the intersection of line AX and the line through Y parallel to $\overline{AD}$. Let F be the intersection of line CX and the line through E parallel to $\overline{AC}$. Let G be the point on circle O other than C that lies on line CX. What is $XF \cdot XG$?

(A) 17 **(B)** $\dfrac{59 - 5\sqrt{2}}{3}$ **(C)** $\dfrac{91 - 12\sqrt{3}}{4}$ **(D)** $\dfrac{67 - 10\sqrt{2}}{3}$

(E) 18

25. The vertices V of a centrally symmetric hexagon in the complex plane are given by

$$V = \left\{ \sqrt{2}i, -\sqrt{2}i, \frac{1}{\sqrt{8}}(1+i), \frac{1}{\sqrt{8}}(-1+i), \frac{1}{\sqrt{8}}(1-i), \frac{1}{\sqrt{8}}(-1-i) \right\}.$$

For each j, $1 \le j \le 12$, an element z_j is chosen from V at random, independently of the other choices. Let $P = \prod_{j=1}^{12} z_j$ be the product of the 12 numbers selected. What is the probability that $P = -1$?

(A) $\dfrac{5 \cdot 11}{3^{10}}$ **(B)** $\dfrac{5^2 \cdot 11}{2 \cdot 3^{10}}$ **(C)** $\dfrac{5 \cdot 11}{3^9}$ **(D)** $\dfrac{5 \cdot 7 \cdot 11}{2 \cdot 3^{10}}$

(E) $\dfrac{2^2 \cdot 5 \cdot 11}{3^{10}}$

21. A set S is constructed as follows. To begin, $S = \{0, 10\}$. Repeatedly, as long as possible, if x is an integer root of some polynomial $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ for some $n \geq 1$, all of whose coefficients a_i are elements of S, then x is put into S. When no more elements can be added to S, how many elements does S have?

(A) 4 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 11

22. A square is drawn in the Cartesian coordinate plane with vertices at $(2, 2)$, $(-2, 2)$, $(-2, -2)$, and $(2, -2)$. A particle starts at $(0, 0)$. Every second it moves with equal probability to one of the eight lattice points (points with integer coordinates) closest to its current position, independently of its previous moves. In other words, the probability is $\frac{1}{8}$ that the particle will move from (x, y) to each of $(x, y + 1)$, $(x + 1, y + 1)$, $(x + 1, y)$, $(x + 1, y - 1)$, $(x, y - 1)$, $(x - 1, y - 1)$, $(x - 1, y)$, or $(x - 1, y + 1)$. The particle will eventually hit the square for the first time, either at one of the 4 corners of the square or at one of the 12 lattice points in the interior of one of the sides of the square. The probability that it will hit at a corner rather than at an interior point of a side is $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 4 **(B)** 5 **(C)** 7 **(D)** 15 **(E)** 39

23. For certain real numbers a, b, and c, the polynomial

$$g(x) = x^3 + ax^2 + x + 10$$

has three distinct roots, and each root of $g(x)$ is also a root of the polynomial

$$f(x) = x^4 + x^3 + bx^2 + 100x + c.$$

What is $f(1)$?

(A) -9009 **(B)** -8008 **(C)** -7007 **(D)** -6006 **(E)** -5005

12. There are 10 horses, named Horse 1, Horse 2, ..., Horse 10. They get their names from how many minutes it takes them to run one lap around a circular race track: Horse k runs one lap in exactly k minutes. At time 0 all the horses are together at the starting point on the track. The horses start running in the same direction, and they keep running around the circular track at their constant speeds. The least time $S > 0$, in minutes, at which all 10 horses will again simultaneously be at the starting point is $S = 2520$. Let $T > 0$ be the least time, in minutes, such that at least 5 of the horses are again at the starting point. What is the sum of the digits of T?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

13. Driving at a constant speed, Sharon usually takes 180 minutes to drive from her house to her mother's house. One day Sharon begins the drive at her usual speed, but after driving $\frac{1}{3}$ of the way, she hits a bad snowstorm and reduces her speed by 20 miles per hour. This time the trip takes her a total of 276 minutes. How many miles is the drive from Sharon's house to her mother's house?

(A) 132 **(B)** 135 **(C)** 138 **(D)** 141 **(E)** 144

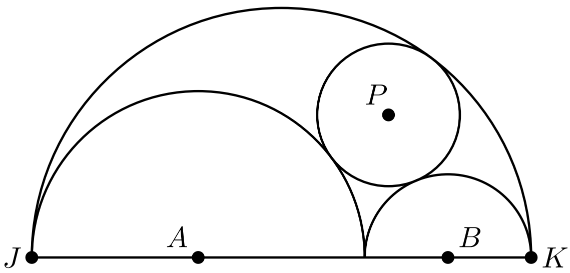
14. Alice refuses to sit next to either Bob or Carla. Derek refuses to sit next to Eric. How many ways are there for the five of them to sit in a row of 5 chairs under these conditions?

(A) 12 **(B)** 16 **(C)** 28 **(D)** 32 **(E)** 40

15. Let $f(x) = \sin x + 2\cos x + 3\tan x$, using radian measure for the variable x. In what interval does the smallest positive value of x for which $f(x) = 0$ lie?

(A) $(0,1)$ **(B)** $(1,2)$ **(C)** $(2,3)$ **(D)** $(3,4)$ **(E)** $(4,5)$

16. In the figure below, semicircles with centers at A and B and with radii 2 and 1, respectively, are drawn in the interior of, and sharing bases with, a semicircle with diameter $\overline{JK}$. The two smaller semicircles are externally tangent to each other and internally tangent to the largest semicircle. A circle centered at P is drawn externally tangent to the two smaller semicircles and internally tangent to the largest semicircle. What is the radius of the circle centered at P?



(A) $\frac{3}{4}$ **(B)** $\frac{6}{7}$ **(C)** $\frac{1}{2}\sqrt{3}$ **(D)** $\frac{5}{8}\sqrt{2}$ **(E)** $\frac{11}{12}$

17. There are 24 different complex numbers z such that $z^{24} = 1$. For how many of these is z^6 a real number?

(A) 0 **(B)** 4 **(C)** 6 **(D)** 12 **(E)** 24

18. Let $S(n)$ equal the sum of the digits of positive integer n. For example, $S(1507) = 13$. For a particular positive integer n, $S(n) = 1274$. Which of the following could be the value of $S(n+1)$?

(A) 1 **(B)** 3 **(C)** 12 **(D)** 1239 **(E)** 1265

19. A square with side length x is inscribed in a right triangle with sides of length 3, 4, and 5 so that one vertex of the square coincides with the right-angle vertex of the triangle. A square with side length y is inscribed in another right triangle with sides of length 3, 4, and 5 so that one side of the square lies on the hypotenuse of the triangle. What is $\frac{x}{y}$?

(A) $\frac{12}{13}$ **(B)** $\frac{35}{37}$ **(C)** 1 **(D)** $\frac{37}{35}$ **(E)** $\frac{13}{12}$

20. How many ordered pairs (a, b) such that a is a positive real number and b is an integer between 2 and 200, inclusive, satisfy the equation $(\log_b a)^{2017} = \log_b(a^{2017})$?

(A) 198 **(B)** 199 **(C)** 398 **(D)** 399 **(E)** 597

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

68th Annual

AMC 12B

American Mathematics Competition 12B

Wednesday, February 15, 2017

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded. **No copies.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score well on this AMC 10 will be invited to take the 35th annual American Invitational Mathematics Examination (AIME) on Thursday, March 7, 2017 or Wednesday, March 22, 2017. More details about the AIME are on the back page of this test booklet.

2017

AMC 12B

DO NOT OPEN UNTIL WEDNESDAY, February 15, 2017

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the Teachers' Manual. PLEASE READ THE MANUAL BEFORE FEBRUARY 15, 2017.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 12 CERTIFICATION FORM (found on amc.maa.org under 'AMC 10/12') that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.*

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MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

WRITE TO US!

Correspondence about the problems and solutions for this AMC 12 and orders for publications should be addressed to:

MAA American Mathematics Competitions
PO Box 471
Annapolis Junction, MD 20701
Phone 800.527.3690 | Fax 240.396.5647 | amcinfo@maa.org

The problems and solutions for this AMC 12 were prepared by MAA's Subcommittee on the AMC10/AMC12 Exams, under the direction of the co-chairs Jerrold W. Grossman and Carl Yerger.

2017 AIME

The 35th annual AIME will be held on Thursday, March 7, 2017 with the alternate on Wednesday, March 22, 2017. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate if you achieve a high score on this competition. Top-scoring students on the AMC 10/12/AIME will be selected to take the 46th Annual USA Mathematical Olympiad (USAMO) on April 19–20, 2017. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
www.maa.org/amc

1. Kymbrea's comic book collection currently has 30 comic books in it, and she is adding to her collection at the rate of 2 comic books per month. LaShawn's collection currently has 10 comic books in it, and he is adding to his collection at the rate of 6 comic books per month. After how many months will LaShawn's collection have twice as many comic books as Kymbrea's?

   (A) 1 **(B)** 4 **(C)** 5 **(D)** 20 **(E)** 25

2. Real numbers x, y, and z satisfy the inequalities

   $$0 < x < 1, \qquad -1 < y < 0, \qquad \text{and} \qquad 1 < z < 2.$$

   Which of the following numbers is necessarily positive?

   (A) $y + x^2$ **(B)** $y + xz$ **(C)** $y + y^2$ **(D)** $y + 2y^2$
   (E) $y + z$

3. Suppose that x and y are nonzero real numbers such that

   $$\frac{3x + y}{x - 3y} = -2.$$

   What is the value of

   $$\frac{x + 3y}{3x - y}?$$

   (A) -3 **(B)** -1 **(C)** 1 **(D)** 2 **(E)** 3

4. Samia set off on her bicycle to visit her friend, traveling at an average speed of 17 kilometers per hour. When she had gone half the distance to her friend's house, a tire went flat, and she walked the rest of the way at 5 kilometers per hour. In all it took her 44 minutes to reach her friend's house. In kilometers rounded to the nearest tenth, how far did Samia walk?

   (A) 2.0 **(B)** 2.2 **(C)** 2.8 **(D)** 3.4 **(E)** 4.4

5. The data set [6, 19, 33, 33, 39, 41, 41, 43, 51, 57] has median $Q_2 = 40$, first quartile $Q_1 = 33$, and third quartile $Q_3 = 43$. An outlier in a data set is a value that is more than 1.5 times the interquartile range below the first quartile (Q_1) or more than 1.5 times the interquartile range above the third quartile (Q_3), where the interquartile range is defined as $Q_3 - Q_1$. How many outliers does this data set have?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

6. The circle having $(0, 0)$ and $(8, 6)$ as the endpoints of a diameter intersects the x-axis at a second point. What is the x-coordinate of this point?

   (A) $4\sqrt{2}$ **(B)** 6 **(C)** $5\sqrt{2}$ **(D)** 8 **(E)** $6\sqrt{2}$

7. The functions $\sin(x)$ and $\cos(x)$ are periodic with least period 2π. What is the least period of the function $\cos(\sin(x))$?

   (A) $\frac{\pi}{2}$ **(B)** π **(C)** 2π **(D)** 4π **(E)** It's not periodic.

8. The ratio of the short side of a certain rectangle to the long side is equal to the ratio of the long side to the diagonal. What is the square of the ratio of the short side to the long side of this rectangle?

   (A) $\frac{\sqrt{3}-1}{2}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{\sqrt{5}-1}{2}$ **(D)** $\frac{\sqrt{2}}{2}$ **(E)** $\frac{\sqrt{6}-1}{2}$

9. A circle has center $(-10, -4)$ and radius 13. Another circle has center $(3, 9)$ and radius $\sqrt{65}$. The line passing through the two points of intersection of the two circles has equation $x + y = c$. What is c?

   (A) 3 **(B)** $3\sqrt{3}$ **(C)** $4\sqrt{2}$ **(D)** 6 **(E)** $\frac{13}{2}$

10. At Typico High School, 60% of the students like dancing, and the rest dislike it. Of those who like dancing, 80% say that they like it, and the rest say that they dislike it. Of those who dislike dancing, 90% say that they dislike it, and the rest say that they like it. What fraction of students who say they dislike dancing actually like it?

    (A) 10% **(B)** 12% **(C)** 20% **(D)** 25% **(E)** $33\frac{1}{3}\%$

11. Call a positive integer *monotonous* if it is a one-digit number or its digits, when read from left to right, form either a strictly increasing or a strictly decreasing sequence. For example, 3, 23578, and 987620 are monotonous, but 88, 7434, and 23557 are not. How many monotonous positive integers are there?

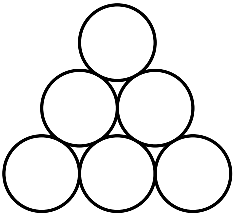
    (A) 1024 **(B)** 1524 **(C)** 1533 **(D)** 1536 **(E)** 2048

12. What is the sum of the roots of $z^{12} = 64$ that have a positive real part?

    (A) 2 **(B)** 4 **(C)** $\sqrt{2} + 2\sqrt{3}$ **(D)** $2\sqrt{2} + \sqrt{6}$

    (E) $(1 + \sqrt{3}) + (1 + \sqrt{3})i$

13. In the figure below, 3 of the 6 disks are to be painted blue, 2 are to be painted red, and 1 is to be painted green. Two paintings that can be obtained from one another by a rotation or a reflection of the entire figure are considered the same. How many different paintings are possible?



(A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 15

14. An ice-cream novelty item consists of a cup in the shape of a 4-inch-tall frustum of a right circular cone, with a 2-inch-diameter base at the bottom and a 4-inch-diameter base at the top, packed solid with ice cream, together with a solid cone of ice cream of height 4 inches, whose base, at the bottom, is the top base of the frustum. What is the total volume of the ice cream, in cubic inches?

(A) 8π **(B)** $\frac{28\pi}{3}$ **(C)** 12π **(D)** 14π **(E)** $\frac{44\pi}{3}$

15. Let ABC be an equilateral triangle. Extend side $\overline{AB}$ beyond B to a point B' so that $BB' = 3AB$. Similarly, extend side $\overline{BC}$ beyond C to a point C' so that $CC' = 3BC$, and extend side $\overline{CA}$ beyond A to a point A' so that $AA' = 3CA$. What is the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$?

(A) $9:1$ **(B)** $16:1$ **(C)** $25:1$ **(D)** $36:1$ **(E)** $37:1$

16. The number $21! = 51{,}090{,}942{,}171{,}709{,}440{,}000$ has over 60,000 positive integer divisors. One of them is chosen at random. What is the probability that it is odd?

(A) $\frac{1}{21}$ **(B)** $\frac{1}{19}$ **(C)** $\frac{1}{18}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{11}{21}$

17. A coin is biased in such a way that on each toss the probability of heads is $\frac{2}{3}$ and the probability of tails is $\frac{1}{3}$. The outcomes of the tosses are independent. A player has the choice of playing Game A or Game B. In Game A she tosses the coin three times and wins if all three outcomes are the same. In Game B she tosses the coin four times and wins if both the outcomes of the first and second tosses are the same and the outcomes of the third and fourth tosses are the same. How do the chances of winning Game A compare to the chances of winning Game B?

    (A) The probability of winning Game A is $\frac{4}{81}$ less than the probability of winning Game B.

    (B) The probability of winning Game A is $\frac{2}{81}$ less than the probability of winning Game B.

    (C) The probabilities are the same.

    (D) The probability of winning Game A is $\frac{2}{81}$ greater than the probability of winning Game B.

    (E) The probability of winning Game A is $\frac{4}{81}$ greater than the probability of winning Game B.

18. The diameter $\overline{AB}$ of a circle of radius 2 is extended to a point D outside the circle so that $BD = 3$. Point E is chosen so that $ED = 5$ and line ED is perpendicular to line AD. Segment $\overline{AE}$ intersects the circle at a point C between A and E. What is the area of $\triangle ABC$?

    (A) $\dfrac{120}{37}$ **(B)** $\dfrac{140}{39}$ **(C)** $\dfrac{145}{39}$ **(D)** $\dfrac{140}{37}$ **(E)** $\dfrac{120}{31}$

19. Let $N = 123456789101112\dots4344$ be the 79-digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when N is divided by 45?

    (A) 1 **(B)** 4 **(C)** 9 **(D)** 18 **(E)** 44

20. Real numbers x and y are chosen independently and uniformly at random from the interval $(0, 1)$. What is the probability that $\lfloor \log_2 x \rfloor = \lfloor \log_2 y \rfloor$, where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to the real number r?

    (A) $\dfrac{1}{8}$ **(B)** $\dfrac{1}{6}$ **(C)** $\dfrac{1}{4}$ **(D)** $\dfrac{1}{3}$ **(E)** $\dfrac{1}{2}$

21. Last year Isabella took 7 math tests and received 7 different scores, each an integer between 91 and 100, inclusive. After each test she noticed that the average of her test scores was an integer. Her score on the seventh test was 95. What was her score on the sixth test?

(A) 92 **(B)** 94 **(C)** 96 **(D)** 98 **(E)** 100

22. Abby, Bernardo, Carl, and Debra play a game in which each of them starts with four coins. The game consists of four rounds. In each round, four balls are placed in an urn—one green, one red, and two white. The players each draw a ball at random without replacement. Whoever gets the green ball gives one coin to whoever gets the red ball. What is the probability that, at the end of the fourth round, each of the players has four coins?

(A) $\frac{7}{576}$ **(B)** $\frac{5}{192}$ **(C)** $\frac{1}{36}$ **(D)** $\frac{5}{144}$ **(E)** $\frac{7}{48}$

23. The graph of $y = f(x)$, where $f(x)$ is a polynomial of degree 3, contains points $A(2,4)$, $B(3,9)$, and $C(4,16)$. Lines AB, AC, and BC intersect the graph again at points D, E, and F, respectively, and the sum of the x-coordinates of D, E, and F is 24. What is $f(0)$?

(A) -2 **(B)** 0 **(C)** 2 **(D)** $\frac{24}{5}$ **(E)** 8

24. Quadrilateral $ABCD$ has right angles at B and C, $\triangle ABC \sim \triangle BCD$, and $AB > BC$. There is a point E in the interior of $ABCD$ such that $\triangle ABC \sim \triangle CEB$ and the area of $\triangle AED$ is 17 times the area of $\triangle CEB$. What is $\frac{AB}{BC}$?

(A) $1+\sqrt{2}$ **(B)** $2+\sqrt{2}$ **(C)** $\sqrt{17}$ **(D)** $2+\sqrt{5}$ **(E)** $1+2\sqrt{3}$

25. A set of n people participate in an online video basketball tournament. Each person may be a member of any number of 5-player teams, but no two teams may have exactly the same 5 members. The site statistics show a curious fact: The average, over all subsets of size 9 of the set of n participants, of the number of complete teams whose members are among those 9 people is equal to the reciprocal of the average, over all subsets of size 8 of the set of n participants, of the number of complete teams whose members are among those 8 people. How many values n, $9 \le n \le 2017$, can be the number of participants?

(A) 477 **(B)** 482 **(C)** 487 **(D)** 557 **(E)** 562

MAA AMC

American Mathematics Competitions

American Mathematics Competitions

69th Annual

AMC 12A

American Mathematics Competition 12A

Wednesday, February 7, 2018

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR COMPETITION MANAGER TELLS YOU.
2. This is a 25-question multiple-choice exam. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Sheet with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer sheet will be graded. **You must use and submit the original answer sheets provided by the MAA AMC. Photocopies will not be scored.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only scratch paper, graph paper, rulers, compasses, protractors, and erasers are allowed as aids. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the exam will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the exam, your competition manager will ask you to record certain information on the answer form.
8. When your competition manager gives the signal, begin working on the problems. You will have **75 minutes** to complete the exam.
9. When you finish the exam, *sign your name* in the space provided at the top of the Answer Sheet.

The Committee on the American Mathematics Competitions reserves the right to disqualify scores from a school if it determines that the required security procedures were not followed.

Students who score well on this AMC 12 will be invited to take the 36th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 6, 2018 or Wednesday, March 21, 2018. More details about the AIME are on the back page of this test booklet.

1. A large urn contains 100 balls, of which 36% are red and the rest are blue. How many of the blue balls must be removed so that the percentage of red balls in the urn will be 72% ? (No red balls are to be removed.)

   (A) 28 **(B)** 32 **(C)** 36 **(D)** 50 **(E)** 64

2. While exploring a cave, Carl comes across a collection of 5-pound rocks worth \$14 each, 4-pound rocks worth \$11 each, and 1-pound rocks worth \$2 each. There are at least 20 of each size. He can carry at most 18 pounds. What is the maximum value, in dollars, of the rocks he can carry out of the cave?

   (A) 48 **(B)** 49 **(C)** 50 **(D)** 51 **(E)** 52

3. How many ways can a student schedule 3 mathematics courses—algebra, geometry, and number theory—in a 6-period day if no two mathematics courses can be taken in consecutive periods? (What courses the student takes during the other 3 periods is of no concern here.)

   (A) 3 **(B)** 6 **(C)** 12 **(D)** 18 **(E)** 24

4. Alice, Bob, and Charlie were on a hike and were wondering how far away the nearest town was. When Alice said, "We are at least 6 miles away," Bob replied, "We are at most 5 miles away." Charlie then remarked, "Actually the nearest town is at most 4 miles away." It turned out that none of the three statements was true. Let d be the distance in miles to the nearest town. Which of the following intervals is the set of all possible values of d ?

   (A) $(0,4)$ **(B)** $(4,5)$ **(C)** $(4,6)$ **(D)** $(5,6)$ **(E)** $(5,\infty)$

5. What is the sum of all possible values of k for which the polynomials x^2-3x+2 and x^2-5x+k have a root in common?

   (A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 10

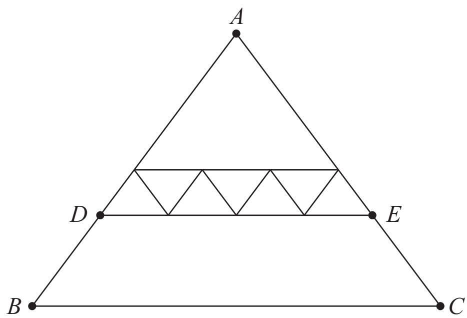
6. For positive integers m and n such that $m+10<n+1$, both the mean and the median of the set $\{m, m+4, m+10, n+1, n+2, 2n\}$ are equal to n. What is $m+n$?

   (A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

7. For how many (not necessarily positive) integer values of n is the value of $4000 \cdot \left(\tfrac{2}{5}\right)^n$ an integer?

(A) 3 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 9

8. All of the triangles in the diagram below are similar to isosceles triangle ABC, in which $AB = AC$. Each of the 7 smallest triangles has area 1, and $\triangle ABC$ has area 40. What is the area of trapezoid $DBCE$?



(A) 16 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24

9. Which of the following describes the largest subset of values of y within the closed interval $[0, \pi]$ for which

$$\sin(x + y) \leq \sin(x) + \sin(y)$$

for every x between 0 and π, inclusive?

(A) $y = 0$ **(B)** $0 \leq y \leq \frac{\pi}{4}$ **(C)** $0 \leq y \leq \frac{\pi}{2}$ **(D)** $0 \leq y \leq \frac{3\pi}{4}$

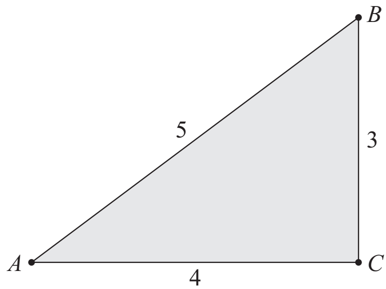
(E) $0 \leq y \leq \pi$

10. How many ordered pairs of real numbers (x, y) satisfy the following system of equations?

$$x + 3y = 3$$
$$\big|\,|x| - |y|\,\big| = 1$$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 8

11. A paper triangle with sides of lengths 3, 4, and 5 inches, as shown, is folded so that point A falls on point B. What is the length in inches of the crease?



(A) $1 + \frac{1}{2}\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** $\frac{7}{4}$ **(D)** $\frac{15}{8}$ **(E)** 2

12. Let S be a set of 6 integers taken from $\{1, 2, \ldots, 12\}$ with the property that if a and b are elements of S with $a < b$, then b is not a multiple of a. What is the least possible value of an element of S?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 7

13. How many nonnegative integers can be written in the form

$$a_7 \cdot 3^7 + a_6 \cdot 3^6 + a_5 \cdot 3^5 + a_4 \cdot 3^4 + a_3 \cdot 3^3 + a_2 \cdot 3^2 + a_1 \cdot 3^1 + a_0 \cdot 3^0,$$

where $a_i \in \{-1, 0, 1\}$ for $0 \le i \le 7$?

(A) 512 **(B)** 729 **(C)** 1094 **(D)** 3281 **(E)** 59,048

14. The solution to the equation $\log_{3x} 4 = \log_{2x} 8$, where x is a positive real number other than $\frac{1}{3}$ or $\frac{1}{2}$, can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p + q$?

(A) 5 **(B)** 13 **(C)** 17 **(D)** 31 **(E)** 35

15. A scanning code consists of a 7×7 grid of squares, with some of its squares colored black and the rest colored white. There must be at least one square of each color in this grid of 49 squares. A scanning code is called *symmetric* if its look does not change when the entire square is rotated by a multiple of 90° counterclockwise around its center, nor when it is reflected across a line joining opposite corners or a line joining midpoints of opposite sides. What is the total number of possible symmetric scanning codes?

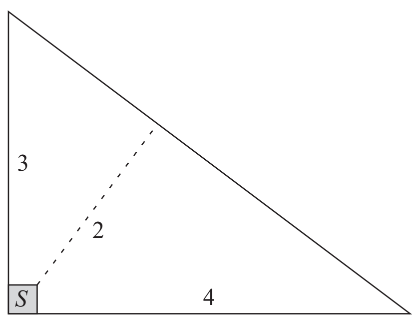
(A) 510 **(B)** 1022 **(C)** 8190 **(D)** 8192 **(E)** 65,534

16. Which of the following describes the set of values of a for which the curves $x^2 + y^2 = a^2$ and $y = x^2 - a$ in the real xy-plane intersect at exactly 3 points?

(A) $a = \frac{1}{4}$ **(B)** $\frac{1}{4} < a < \frac{1}{2}$ **(C)** $a > \frac{1}{4}$ **(D)** $a = \frac{1}{2}$

(E) $a > \frac{1}{2}$

17. Farmer Pythagoras has a field in the shape of a right triangle. The right triangle's legs have lengths of 3 and 4 units. In the corner where those sides meet at a right angle, he leaves a small unplanted square S so that from the air it looks like the right angle symbol. The rest of the field is planted. The shortest distance from S to the hypotenuse is 2 units. What fraction of the field is planted?



(A) $\frac{25}{27}$ **(B)** $\frac{26}{27}$ **(C)** $\frac{73}{75}$ **(D)** $\frac{145}{147}$ **(E)** $\frac{74}{75}$

18. Triangle ABC with $AB = 50$ and $AC = 10$ has area 120. Let D be the midpoint of $\overline{AB}$, and let E be the midpoint of $\overline{AC}$. The angle bisector of $\angle BAC$ intersects $\overline{DE}$ and $\overline{BC}$ at F and G, respectively. What is the area of quadrilateral $FDBG$?

(A) 60 **(B)** 65 **(C)** 70 **(D)** 75 **(E)** 80

19. Let A be the set of positive integers that have no prime factors other than 2, 3, or 5. The infinite sum

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}+\frac{1}{12}+\frac{1}{15}+\frac{1}{16}+\frac{1}{18}+\frac{1}{20}+\cdots$$

of the reciprocals of all the elements of A can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

(A) 16 **(B)** 17 **(C)** 19 **(D)** 23 **(E)** 36

20. Triangle ABC is an isosceles right triangle with $AB = AC = 3$. Let M be the midpoint of hypotenuse $\overline{BC}$. Points I and E lie on sides $\overline{AC}$ and $\overline{AB}$, respectively, so that $AI > AE$ and $AIME$ is a cyclic quadrilateral. Given that triangle EMI has area 2, the length CI can be written as $\frac{a-\sqrt{b}}{c}$, where a, b, and c are positive integers and b is not divisible by the square of any prime. What is the value of $a+b+c$?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

21. Which of the following polynomials has the greatest real root?

(A) $x^{19} + 2018x^{11} + 1$ **(B)** $x^{17} + 2018x^{11} + 1$

(C) $x^{19} + 2018x^{13} + 1$ **(D)** $x^{17} + 2018x^{13} + 1$

(E) $2019x + 2018$

22. The solutions to the equations $z^2 = 4 + 4\sqrt{15}i$ and $z^2 = 2 + 2\sqrt{3}i$, where $i = \sqrt{-1}$, form the vertices of a parallelogram in the complex plane. The area of this parallelogram can be written in the form $p\sqrt{q} - r\sqrt{s}$, where p, q, r, and s are positive integers and neither q nor s is divisible by the square of any prime number. What is $p+q+r+s$?

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

23. In $\triangle PAT$, $\angle P = 36^\circ$, $\angle A = 56^\circ$, and $PA = 10$. Points U and G lie on sides $\overline{TP}$ and $\overline{TA}$, respectively, so that $PU = AG = 1$. Let M and N be the midpoints of segments $\overline{PA}$ and $\overline{UG}$, respectively. What is the degree measure of the acute angle formed by lines MN and PA?

(A) 76 **(B)** 77 **(C)** 78 **(D)** 79 **(E)** 80

24. Alice, Bob, and Carol play a game in which each of them chooses a real number between 0 and 1. The winner of the game is the one whose number is between the numbers chosen by the other two players. Alice announces that she will choose her number uniformly at random from all the numbers between 0 and 1, and Bob announces that he will choose his number uniformly at random from all the numbers between $\frac{1}{2}$ and $\frac{2}{3}$. Armed with this information, what number should Carol choose to maximize her chance of winning?

(A) $\frac{1}{2}$ **(B)** $\frac{13}{24}$ **(C)** $\frac{7}{12}$ **(D)** $\frac{5}{8}$ **(E)** $\frac{2}{3}$

25. For a positive integer n and nonzero digits a, b, and c, let A_n be the n-digit integer each of whose digits is equal to a; let B_n be the n-digit integer each of whose digits is equal to b; and let C_n be the $2n$-digit (not n-digit) integer each of whose digits is equal to c. What is the greatest possible value of $a + b + c$ for which there are at least two values of n such that $C_n - B_n = A_n^2$?

(A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

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2018

AMC 12A

DO NOT OPEN UNTIL WEDNESDAY, February 7, 2018

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All the information needed to administer this exam is contained in the AMC 10/12 Teacher's Manual. PLEASE READ THE MANUAL BEFORE FEBRUARY 7, 2018.
2. Your PRINCIPAL or VICE PRINCIPAL must verify on the AMC 10/12 COMPETITION CERTIFICATION FORM (found on maa.org/amc under 'AMC 10/12') that you followed all rules associated with the administration of the exam.
3. Answer sheets must be returned to the MAA AMC office the day after the competition. Ship with appropriate postage using a trackable method. FedEx or UPS is strongly recommended.
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MAA AMC

American Mathematics Competitions

American Mathematics Competitions

69th Annual

AMC 12B

American Mathematics Competition 12B

Thursday, February 15, 2018

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR COMPETITION MANAGER TELLS YOU.
2. This is a 25-question multiple-choice exam. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Sheet with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer sheet will be graded. **You must use and submit the original answer sheets provided by the MAA AMC. Photocopies will not be scored.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only scratch paper, graph paper, rulers, compasses, protractors, and erasers are allowed as aids. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the exam will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the exam, your competition manager will ask you to record certain information on the answer form.
8. When your competition manager gives the signal, begin working on the problems. You will have **75 minutes** to complete the exam.
9. When you finish the exam, *sign your name* in the space provided at the top of the Answer Sheet.

The Committee on the American Mathematics Competitions reserves the right to disqualify scores from a school if it determines that the required security procedures were not followed.

Students who score well on this AMC 12 will be invited to take the 36th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 6, 2018 or Wednesday, March 21, 2018. More details about the AIME are on the back page of this exam booklet.

1. Kate bakes a 20-inch by 18-inch pan of cornbread. The cornbread is cut into pieces that measure 2 inches by 2 inches. How many pieces of cornbread does the pan contain?

   (A) 90 **(B)** 100 **(C)** 180 **(D)** 200 **(E)** 360

2. Sam drove 96 miles in 90 minutes. His average speed during the first 30 minutes was 60 mph (miles per hour), and his average speed during the second 30 minutes was 65 mph. What was his average speed, in mph, during the last 30 minutes?

   (A) 64 **(B)** 65 **(C)** 66 **(D)** 67 **(E)** 68

3. A line with slope 2 intersects a line with slope 6 at the point $(40, 30)$. What is the distance between the x-intercepts of these two lines?

   (A) 5 **(B)** 10 **(C)** 20 **(D)** 25 **(E)** 50

4. A circle has a chord of length 10, and the distance from the center of the circle to the chord is 5. What is the area of the circle?

   (A) 25π **(B)** 50π **(C)** 75π **(D)** 100π **(E)** 125π

5. How many subsets of $\{2, 3, 4, 5, 6, 7, 8, 9\}$ contain at least one prime number?

   (A) 128 **(B)** 192 **(C)** 224 **(D)** 240 **(E)** 256

6. Suppose S cans of soda can be purchased from a vending machine for Q quarters. Which of the following expressions describes the number of cans of soda that can be purchased for D dollars, where 1 dollar is worth 4 quarters?

   (A) $\frac{4DQ}{S}$ **(B)** $\frac{4DS}{Q}$ **(C)** $\frac{4Q}{DS}$ **(D)** $\frac{DQ}{4S}$ **(E)** $\frac{DS}{4Q}$

7. What is the value of

$$\log_3 7 \cdot \log_5 9 \cdot \log_7 11 \cdot \log_9 13 \cdots \log_{21} 25 \cdot \log_{23} 27\,?$$

   (A) 3 **(B)** $3\log_7 23$ **(C)** 6 **(D)** 9 **(E)** 10

8. Line segment $\overline{AB}$ is a diameter of a circle with $AB = 24$. Point C, not equal to A or B, lies on the circle. As point C moves around the circle, the centroid (center of mass) of $\triangle ABC$ traces out a closed curve missing two points. To the nearest positive integer, what is the area of the region bounded by this curve?

   (A) 25 **(B)** 38 **(C)** 50 **(D)** 63 **(E)** 75

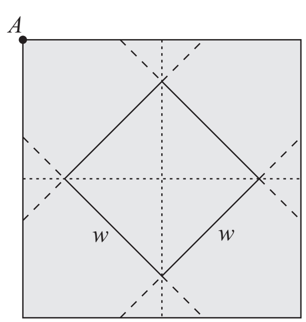
9. What is
$$\sum_{i=1}^{100}\sum_{j=1}^{100}(i+j)?$$

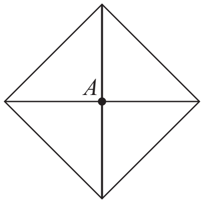
   (A) 100,100 **(B)** 500,500 **(C)** 505,000 **(D)** 1,001,000 **(E)** 1,010,000

10. A list of 2018 positive integers has a unique mode, which occurs exactly 10 times. What is the least number of distinct values that can occur in the list?

   (A) 202 **(B)** 223 **(C)** 224 **(D)** 225 **(E)** 234

11. A closed box with a square base is to be wrapped with a square sheet of wrapping paper. The box is centered on the wrapping paper with the vertices of the base lying on the midlines of the square sheet of paper, as shown in the figure on the left. The four corners of the wrapping paper are to be folded up over the sides and brought together to meet at the center of the top of the box, point A in the figure on the right. The box has base length w and height h. What is the area of the sheet of wrapping paper?



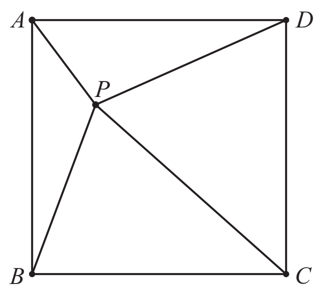


   (A) $2(w+h)^2$ **(B)** $\dfrac{(w+h)^2}{2}$ **(C)** $2w^2+4wh$ **(D)** $2w^2$ **(E)** w^2h

12. Side $\overline{AB}$ of $\triangle ABC$ has length 10. The bisector of angle A meets $\overline{BC}$ at D, and $CD = 3$. The set of all possible values of AC is an open interval (m, n). What is $m + n$?

(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

13. Square $ABCD$ has side length 30. Point P lies inside the square so that $AP = 12$ and $BP = 26$. The centroids of $\triangle ABP$, $\triangle BCP$, $\triangle CDP$, and $\triangle DAP$ are the vertices of a convex quadrilateral. What is the area of that quadrilateral?



(A) $100\sqrt{2}$ **(B)** $100\sqrt{3}$ **(C)** 200 **(D)** $200\sqrt{2}$ **(E)** $200\sqrt{3}$

14. Joey and Chloe and their daughter Zoe all have the same birthday. Joey is 1 year older than Chloe, and Zoe is exactly 1 year old today. Today is the first of the 9 birthdays on which Chloe's age will be an integral multiple of Zoe's age. What will be the sum of the two digits of Joey's age the next time his age is a multiple of Zoe's age?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

15. How many 3-digit positive odd multiples of 3 do not include the digit 3?

(A) 96 **(B)** 97 **(C)** 98 **(D)** 102 **(E)** 120

16. The solutions to the equation $(z+6)^8 = 81$ are connected in the complex plane to form a convex regular polygon, three of whose vertices are labeled A, B, and C. What is the least possible area of $\triangle ABC$?

(A) $\frac{1}{6}\sqrt{6}$ **(B)** $\frac{3}{2}\sqrt{2} - \frac{3}{2}$ **(C)** $2\sqrt{3} - 2\sqrt{2}$ **(D)** $\frac{1}{2}\sqrt{2}$

(E) $\sqrt{3} - 1$

17. Let p and q be positive integers such that

$$\frac{5}{9} < \frac{p}{q} < \frac{4}{7}$$

and q is as small as possible. What is $q - p$?

(A) 7 **(B)** 11 **(C)** 13 **(D)** 17 **(E)** 19

18. A function f is defined recursively by $f(1) = f(2) = 1$ and

$$f(n) = f(n-1) - f(n-2) + n$$

for all integers $n \geq 3$. What is $f(2018)$?

(A) 2016 **(B)** 2017 **(C)** 2018 **(D)** 2019 **(E)** 2020

19. Mary chose an even 4-digit number n. She wrote down all the divisors of n in increasing order from left to right: $1, 2, \ldots, \frac{n}{2}, n$. At some moment Mary wrote 323 as a divisor of n. What is the smallest possible value of the next divisor written to the right of 323?

(A) 324 **(B)** 330 **(C)** 340 **(D)** 361 **(E)** 646

20. Let $ABCDEF$ be a regular hexagon with side length 1. Denote by X, Y, and Z the midpoints of sides $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$, respectively. What is the area of the convex hexagon whose interior is the intersection of the interiors of $\triangle ACE$ and $\triangle XYZ$?

(A) $\frac{3}{8}\sqrt{3}$ **(B)** $\frac{7}{16}\sqrt{3}$ **(C)** $\frac{15}{32}\sqrt{3}$ **(D)** $\frac{1}{2}\sqrt{3}$ **(E)** $\frac{9}{16}\sqrt{3}$

21. In $\triangle ABC$ with side lengths $AB = 13$, $AC = 12$, and $BC = 5$, let O and I denote the circumcenter and incenter, respectively. A circle with center M is tangent to the legs $\overline{AC}$ and $\overline{BC}$ and to the circumcircle of $\triangle ABC$. What is the area of $\triangle MOI$?

(A) $\frac{5}{2}$ **(B)** $\frac{11}{4}$ **(C)** 3 **(D)** $\frac{13}{4}$ **(E)** $\frac{7}{2}$

22. Consider polynomials $P(x)$ of degree at most 3, each of whose coefficients is an element of $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. How many such polynomials satisfy $P(-1) = -9$?

(A) 110 **(B)** 143 **(C)** 165 **(D)** 220 **(E)** 286

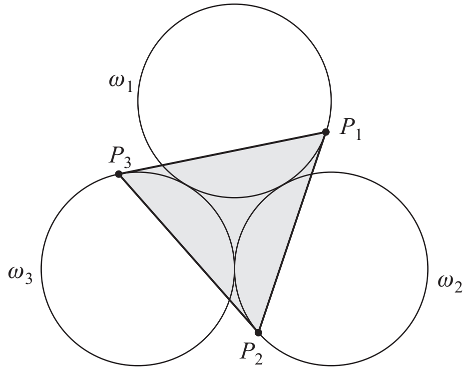
23. Ajay is standing at point A near Pontianak, Indonesia, $0°$ latitude and $110°$ E longitude. Billy is standing at point B near Big Baldy Mountain, Idaho, USA, $45°$ N latitude and $115°$ W longitude. Assume that Earth is a perfect sphere with center C. What is the degree measure of $\angle ACB$?

(A) 105 **(B)** $112\frac{1}{2}$ **(C)** 120 **(D)** 135 **(E)** 150

24. Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. How many real numbers x satisfy the equation $x^2 + 10{,}000\lfloor x \rfloor = 10{,}000x$?

(A) 197 **(B)** 198 **(C)** 199 **(D)** 200 **(E)** 201

25. Circles ω_1, ω_2, and ω_3 each have radius 4 and are placed in the plane so that each circle is externally tangent to the other two. Points P_1, P_2, and P_3 lie on ω_1, ω_2, and ω_3, respectively, so that $P_1P_2 = P_2P_3 = P_3P_1$ and line P_iP_{i+1} is tangent to ω_i for each $i = 1, 2, 3$, where $P_4 = P_1$. See the figure below. The area of $\triangle P_1P_2P_3$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are positive integers. What is $a + b$?



(A) 546 **(B)** 548 **(C)** 550 **(D)** 552 **(E)** 554

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2018

AMC 12B

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