MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

13th Annual

AMC 10 A

American Mathematics Contest 10 A

Tuesday, February 7, 2012

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 120 or above or finish in the top 2.5% on this AMC 10 will be invited to take the 30th annual American Invitational Mathematics Examination (AIME) on Thursday, March 15, 2012 or Wednesday, March 28, 2012. More details about the AIME and other information are on the back page of this test booklet.

2012

AMC 10 A

DO NOT OPEN UNTIL TUESDAY, February 7, 2012

****Administration On An Earlier Date Will Disqualify Your School's Results****

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2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

1. Cagney can frost a cupcake every 20 seconds and Lacey can frost a cupcake every 30 seconds. Working together, how many cupcakes can they frost in 5 minutes?

   (A) 10 **(B)** 15 **(C)** 20 **(D)** 25 **(E)** 30

2. A square with side length 8 is cut in half, creating two congruent rectangles. What are the dimensions of one of these rectangles?

   (A) 2 by 4 **(B)** 2 by 6 **(C)** 2 by 8 **(D)** 4 by 4 **(E)** 4 by 8

3. A bug crawls along a number line, starting at -2. It crawls to -6, then turns around and crawls to 5. How many units does the bug crawl altogether?

   (A) 9 **(B)** 11 **(C)** 13 **(D)** 14 **(E)** 15

4. Let $\angle ABC = 24^\circ$ and $\angle ABD = 20^\circ$. What is the smallest possible degree measure for $\angle CBD$?

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 12

5. Last year 100 adult cats, half of whom were female, were brought into the Smallville Animal Shelter. Half of the adult female cats were accompanied by a litter of kittens. The average number of kittens per litter was 4. What was the total number of cats and kittens received by the shelter last year?

   (A) 150 **(B)** 200 **(C)** 250 **(D)** 300 **(E)** 400

6. The product of two positive numbers is 9. The reciprocal of one of these numbers is 4 times the reciprocal of the other number. What is the sum of the two numbers?

   (A) $\frac{10}{3}$ **(B)** $\frac{20}{3}$ **(C)** 7 **(D)** $\frac{15}{2}$ **(E)** 8

7. In a bag of marbles, $\frac{3}{5}$ of the marbles are blue and the rest are red. If the number of red marbles is doubled and the number of blue marbles stays the same, what fraction of the marbles will be red?

   (A) $\frac{2}{5}$ **(B)** $\frac{3}{7}$ **(C)** $\frac{4}{7}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{4}{5}$

8. The sums of three whole numbers taken in pairs are 12, 17, and 19. What is the middle number?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

9. A pair of six-sided fair dice are labeled so that one die has only even numbers (two each of 2, 4, and 6), and the other die has only odd numbers (two each of 1, 3, and 5). The pair of dice is rolled. What is the probability that the sum of the numbers on the tops of the two dice is 7?

   (A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

10. Mary divides a circle into 12 sectors. The central angles of these sectors, measured in degrees, are all integers and they form an arithmetic sequence. What is the degree measure of the smallest possible sector angle?

    (A) 5 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

11. Externally tangent circles with centers at points A and B have radii of lengths 5 and 3, respectively. A line externally tangent to both circles intersects ray AB at point C. What is BC?

    (A) 4 **(B)** 4.8 **(C)** 10.2 **(D)** 12 **(E)** 14.4

12. A year is a leap year if and only if the year number is divisible by 400 (such as 2000) or is divisible by 4 but not by 100 (such as 2012). The 200th anniversary of the birth of novelist Charles Dickens was celebrated on February 7, 2012, a Tuesday. On what day of the week was Dickens born?

    (A) Friday **(B)** Saturday **(C)** Sunday **(D)** Monday **(E)** Tuesday

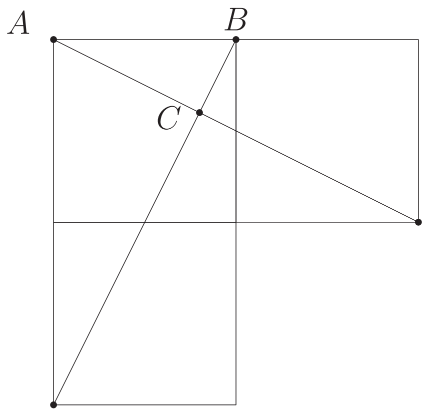
13. An *iterative average* of the numbers 1, 2, 3, 4, and 5 is computed in the following way. Arrange the five numbers in some order. Find the mean of the first two numbers, then find the mean of that with the third number, then the mean of that with the fourth number, and finally the mean of that with the fifth number. What is the difference between the largest and smallest possible values that can be obtained using this procedure?

    (A) $\frac{31}{16}$ **(B)** 2 **(C)** $\frac{17}{8}$ **(D)** 3 **(E)** $\frac{65}{16}$

14. Chubby makes nonstandard checkerboards that have 31 squares on each side. The checkerboards have a black square in every corner and alternate red and black squares along every row and column. How many black squares are there on such a checkerboard?

(A) 480 **(B)** 481 **(C)** 482 **(D)** 483 **(E)** 484

15. Three unit squares and two line segments connecting two pairs of vertices are shown. What is the area of $\triangle ABC$?



(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{2}{9}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{\sqrt{2}}{4}$

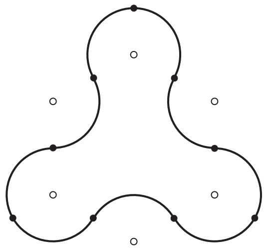
16. Three runners start running simultaneously from the same point on a 500-meter circular track. They each run clockwise around the course maintaining constant speeds of 4.4, 4.8, and 5.0 meters per second. The runners stop once they are all together again somewhere on the circular course. How many seconds do the runners run?

(A) 1,000 **(B)** 1,250 **(C)** 2,500 **(D)** 5,000 **(E)** 10,000

17. Let a and b be relatively prime integers with $a > b > 0$ and $\frac{a^3-b^3}{(a-b)^3} = \frac{73}{3}$. What is $a - b$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

18. The closed curve in the figure is made up of 9 congruent circular arcs each of length $\frac{2\pi}{3}$, where each of the centers of the corresponding circles is among the vertices of a regular hexagon of side 2. What is the area enclosed by the curve?



(A) $2\pi + 6$ **(B)** $2\pi + 4\sqrt{3}$ **(C)** $3\pi + 4$ **(D)** $2\pi + 3\sqrt{3} + 2$
(E) $\pi + 6\sqrt{3}$

19. Paula the painter and her two helpers each paint at constant, but different, rates. They always start at 8:00 AM, and all three always take the same amount of time to eat lunch. On Monday the three of them painted 50% of a house, quitting at 4:00 PM. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at 2:12 PM. On Wednesday Paula worked by herself and finished the house by working until 7:12 PM. How long, in minutes, was each day's lunch break?

(A) 30 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 60

20. A 3×3 square is partitioned into 9 unit squares. Each unit square is painted either white or black with each color being equally likely, chosen independently and at random. The square is then rotated 90° clockwise about its center, and every white square in a position formerly occupied by a black square is painted black. The colors of all other squares are left unchanged. What is the probability that the grid is now entirely black?

(A) $\frac{49}{512}$ **(B)** $\frac{7}{64}$ **(C)** $\frac{121}{1024}$ **(D)** $\frac{81}{512}$ **(E)** $\frac{9}{32}$

21. Let points $A = (0, 0, 0)$, $B = (1, 0, 0)$, $C = (0, 2, 0)$, and $D = (0, 0, 3)$. Points E, F, G, and H are midpoints of line segments $\overline{BD}$, $\overline{AB}$, $\overline{AC}$, and $\overline{DC}$ respectively. What is the area of $EFGH$?

(A) $\sqrt{2}$ **(B)** $\frac{2\sqrt{5}}{3}$ **(C)** $\frac{3\sqrt{5}}{4}$ **(D)** $\sqrt{3}$ **(E)** $\frac{2\sqrt{7}}{3}$

22. The sum of the first m positive odd integers is 212 more than the sum of the first n positive even integers. What is the sum of all possible values of n?

(A) 255 **(B)** 256 **(C)** 257 **(D)** 258 **(E)** 259

23. Adam, Benin, Chiang, Deshawn, Esther, and Fiona have internet accounts. Some, but not all, of them are internet friends with each other, and none of them has an internet friend outside this group. Each of them has the same number of internet friends. In how many different ways can this happen?

(A) 60 **(B)** 170 **(C)** 290 **(D)** 320 **(E)** 660

24. Let a, b, and c be positive integers with $a \geq b \geq c$ such that

$$a^2 - b^2 - c^2 + ab = 2011 \text{ and}$$
$$a^2 + 3b^2 + 3c^2 - 3ab - 2ac - 2bc = -1997.$$

What is a?

(A) 249 **(B)** 250 **(C)** 251 **(D)** 252 **(E)** 253

25. Real numbers x, y, and z are chosen independently and at random from the interval $[0, n]$ for some positive integer n. The probability that no two of x, y, and z are within 1 unit of each other is greater than $\frac{1}{2}$. What is the smallest possible value of n?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

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American Mathematics Competitions
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Lincoln, NE 68501-1606
Phone 402-472-2257 | Fax 402-472-6087 | amcinfo@maa.org

The problems and solutions for this AMC 10 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 10 Subcommittee Chair:

Dr. Leroy Wenstrom

2012 AIME

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PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: amc.maa.org

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

13th Annual

AMC 10 B

American Mathematics Contest 10 B

Wednesday, February 22, 2012

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2012

AMC 10 B

DO NOT OPEN UNTIL WEDNESDAY, FEBRUARY 22, 2012

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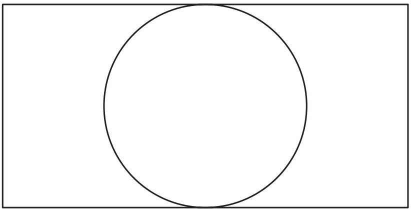
1. Each third-grade classroom at Pearl Creek Elementary has 18 students and 2 pet rabbits. How many more students than rabbits are there in all 4 of the third-grade classrooms?

   (A) 48 **(B)** 56 **(C)** 64 **(D)** 72 **(E)** 80

2. A circle of radius 5 is inscribed in a rectangle as shown. The ratio of the length of the rectangle to its width is $2:1$. What is the area of the rectangle?

   (A) 50 **(B)** 100 **(C)** 125 **(D)** 150 **(E)** 200

3. The point in the xy-plane with coordinates $(1000, 2012)$ is reflected across the line $y = 2000$. What are the coordinates of the reflected point?

   (A) $(998, 2012)$ **(B)** $(1000, 1988)$ **(C)** $(1000, 2024)$

   (D) $(1000, 4012)$ **(E)** $(1012, 2012)$

4. When Ringo places his marbles into bags with 6 marbles per bag, he has 4 marbles left over. When Paul does the same with his marbles, he has 3 marbles left over. Ringo and Paul pool their marbles and place them into as many bags as possible, with 6 marbles per bag. How many marbles will be left over?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

5. Anna enjoys dinner at a restaurant in Washington, D.C., where the sales tax on meals is 10%. She leaves a 15% tip on the price of her meal before the sales tax is added, and the tax is calculated on the pre-tip amount. She spends a total of \$27.50 for dinner. What is the cost of her dinner without tax or tip?

   (A) \$18 **(B)** \$20 **(C)** \$21 **(D)** \$22 **(E)** \$24

6. In order to estimate the value of $x - y$ where x and y are real numbers with $x > y > 0$, Xiaoli rounded x up by a small amount, rounded y down by the same amount, and then subtracted her rounded values. Which of the following statements is necessarily correct?

   (A) Her estimate is larger than $x - y$.

   (B) Her estimate is smaller than $x - y$.

   (C) Her estimate equals $x - y$.

   (D) Her estimate equals $y - x$.

   (E) Her estimate is 0.

7. For a science project, Sammy observed a chipmunk and a squirrel stashing acorns in holes. The chipmunk hid 3 acorns in each of the holes it dug. The squirrel hid 4 acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed 4 fewer holes. How many acorns did the chipmunk hide?

   (A) 30 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 54

8. What is the sum of all integer solutions to $1 < (x-2)^2 < 25$?

   (A) 10 **(B)** 12 **(C)** 15 **(D)** 19 **(E)** 25

9. Two integers have a sum of 26. When two more integers are added to the first two integers the sum is 41. Finally when two more integers are added to the sum of the previous four integers the sum is 57. What is the minimum number of even integers among the 6 integers?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

10. How many ordered pairs of positive integers (M, N) satisfy the equation $\frac{M}{6} = \frac{6}{N}$?

    (A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

11. A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

    (A) 729 **(B)** 972 **(C)** 1024 **(D)** 2187 **(E)** 2304

12. Point B is due east of point A. Point C is due north of point B. The distance between points A and C is $10\sqrt{2}$ meters, and $\angle BAC = 45°$. Point D is 20 meters due north of point C. The distance AD is between which two integers?

(A) 30 and 31 **(B)** 31 and 32 **(C)** 32 and 33 **(D)** 33 and 34 **(E)** 34 and 35

13. It takes Clea 60 seconds to walk down an escalator when it is not operating, and only 24 seconds to walk down the escalator when it is operating. How many seconds does it take Clea to ride down the operating escalator when she just stands on it?

(A) 36 **(B)** 40 **(C)** 42 **(D)** 48 **(E)** 52

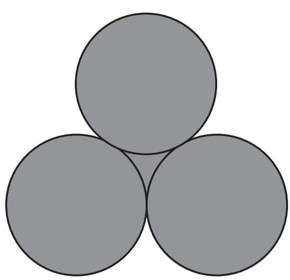
14. Two equilateral triangles are contained in a square whose side length is $2\sqrt{3}$. The bases of these triangles are the opposite sides of the square, and their intersection is a rhombus. What is the area of the rhombus?

(A) $\frac{3}{2}$ **(B)** $\sqrt{3}$ **(C)** $2\sqrt{2}-1$ **(D)** $8\sqrt{3}-12$ **(E)** $\frac{4\sqrt{3}}{3}$

15. In a round-robin tournament with 6 teams, each team plays one game against each other team, and each game results in one team winning and one team losing. At the end of the tournament, the teams are ranked by the number of games won. What is the maximum number of teams that could be tied for the most wins at the end of the tournament?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

16. Three circles with radius 2 are mutually tangent. What is the total area of the circles and the region bounded by them, as shown in the figure?



(A) $10\pi + 4\sqrt{3}$ **(B)** $13\pi - \sqrt{3}$ **(C)** $12\pi + \sqrt{3}$ **(D)** $10\pi + 9$ **(E)** 13π

17. Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger?

(A) $\frac{1}{8}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{\sqrt{10}}{10}$ **(D)** $\frac{\sqrt{5}}{6}$ **(E)** $\frac{\sqrt{10}}{5}$

18. Suppose that one of every 500 people in a certain population has a particular disease, which displays no symptoms. A blood test is available for screening for this disease. For a person who has this disease, the test always turns out positive. For a person who does not have the disease, however, there is a 2% false positive rate—in other words, for such people, 98% of the time the test will turn out negative, but 2% of the time the test will turn out positive and will incorrectly indicate that the person has the disease. Let p be the probability that a person who is chosen at random from this population and gets a positive test result actually has the disease. Which of the following is closest to p?

(A) $\frac{1}{98}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{11}$ **(D)** $\frac{49}{99}$ **(E)** $\frac{98}{99}$

19. In rectangle $ABCD$, $AB = 6$, $AD = 30$, and G is the midpoint of $\overline{AD}$. Segment AB is extended 2 units beyond B to point E, and F is the intersection of $\overline{ED}$ and $\overline{BC}$. What is the area of $BFDG$?

(A) $\frac{133}{2}$ **(B)** 67 **(C)** $\frac{135}{2}$ **(D)** 68 **(E)** $\frac{137}{2}$

20. Bernardo and Silvia play the following game. An integer between 0 and 999, inclusive, is selected and given to Bernardo. Whenever Bernardo receives a number, he doubles it and passes the result to Silvia. Whenever Silvia receives a number, she adds 50 to it and passes the result to Bernardo. The winner is the last person who produces a number less than 1000. Let N be the smallest initial number that results in a win for Bernardo. What is the sum of the digits of N?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

21. Four distinct points are arranged in a plane so that the segments connecting them have lengths a, a, a, a, $2a$, and b. What is the ratio of b to a?

(A) $\sqrt{3}$ **(B)** 2 **(C)** $\sqrt{5}$ **(D)** 3 **(E)** π

22. Let $(a_1, a_2, \ldots, a_{10})$ be a list of the first 10 positive integers such that for each $2 \le i \le 10$ either $a_i + 1$ or $a_i - 1$ or both appear somewhere before a_i in the list. How many such lists are there?

    (A) 120 **(B)** 512 **(C)** 1024 **(D)** 181,440 **(E)** 362,880

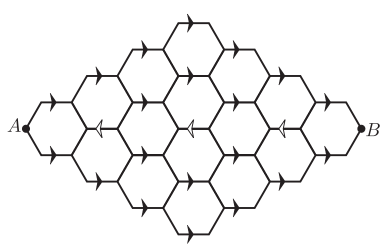
23. A solid tetrahedron is sliced off a solid wooden unit cube by a plane passing through two nonadjacent vertices on one face and one vertex on the opposite face not adjacent to either of the first two vertices. The tetrahedron is discarded and the remaining portion of the cube is placed on a table with the cut surface face down. What is the height of this object?

    (A) $\frac{\sqrt{3}}{3}$ **(B)** $\frac{2\sqrt{2}}{3}$ **(C)** 1 **(D)** $\frac{2\sqrt{3}}{3}$ **(E)** $\sqrt{2}$

24. Amy, Beth, and Jo listen to four different songs and discuss which ones they like. No song is liked by all three. Furthermore, for each of the three pairs of the girls, there is at least one song liked by those two girls but disliked by the third. In how many different ways is this possible?

    (A) 108 **(B)** 132 **(C)** 671 **(D)** 846 **(E)** 1105

25. A bug travels from A to B along the segments in the hexagonal lattice pictured below. The segments marked with an arrow can be traveled only in the direction of the arrow, and the bug never travels the same segment more than once. How many different paths are there?



    (A) 2112 **(B)** 2304 **(C)** 2368 **(D)** 2384 **(E)** 2400

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2013

AMC 10 A

DO NOT OPEN UNTIL TUESDAY, February 5, 2013

****Administration On An Earlier Date Will Disqualify Your School's Results****

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2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

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1. A taxi ride costs \$1.50 plus \$0.25 per mile traveled. How much does a 5-mile taxi ride cost?

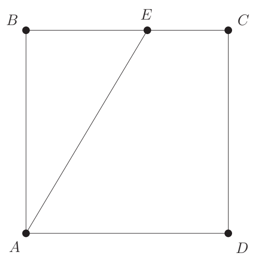
   (A) \$2.25 **(B)** \$2.50 **(C)** \$2.75 **(D)** \$3.00 **(E)** \$3.25

2. Alice is making a batch of cookies and needs $2\frac{1}{2}$ cups of sugar. Unfortunately, her measuring cup holds only $\frac{1}{4}$ cup of sugar. How many times must she fill that cup to get the correct amount of sugar?

   (A) 8 **(B)** 10 **(C)** 12 **(D)** 16 **(E)** 20

3. Square $ABCD$ has side length 10. Point E is on $\overline{BC}$, and the area of $\triangle ABE$ is 40. What is BE?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8



4. A softball team played ten games, scoring 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 runs. They lost by one run in exactly five games. In each of their other games, they scored twice as many runs as their opponent. How many total runs did their opponents score?

   (A) 35 **(B)** 40 **(C)** 45 **(D)** 50 **(E)** 55

5. Tom, Dorothy, and Sammy went on a vacation and agreed to split the costs evenly. During their trip Tom paid \$105, Dorothy paid \$125, and Sammy paid \$175. In order to share the costs equally, Tom gave Sammy t dollars, and Dorothy gave Sammy d dollars. What is $t-d$?

   (A) 15 **(B)** 20 **(C)** 25 **(D)** 30 **(E)** 35

6. Joey and his five brothers are ages 3, 5, 7, 9, 11, and 13. One afternoon two of his brothers whose ages sum to 16 went to the movies, two brothers younger than 10 went to play baseball, and Joey and the 5-year-old stayed home. How old is Joey?

   (A) 3 **(B)** 7 **(C)** 9 **(D)** 11 **(E)** 13

7. A student must choose a program of four courses from a menu of courses consisting of English, Algebra, Geometry, History, Art, and Latin. This program must contain English and at least one mathematics course. In how many ways can this program be chosen?

   (A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 16

8. What is the value of
$$\frac{2^{2014}+2^{2012}}{2^{2014}-2^{2012}}?$$

   (A) -1 **(B)** 1 **(C)** $\frac{5}{3}$ **(D)** 2013 **(E)** 2^{4024}

9. In a recent basketball game, Shenille attempted only three-point shots and two-point shots. She was successful on 20% of her three-point shots and 30% of her two-point shots. Shenille attempted 30 shots. How many points did she score?

   (A) 12 **(B)** 18 **(C)** 24 **(D)** 30 **(E)** 36

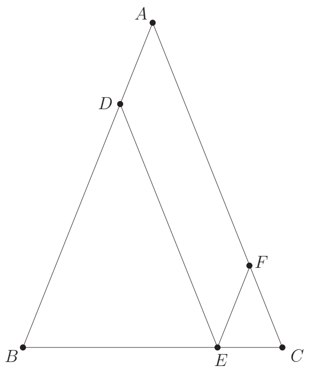
10. A flower bouquet contains pink roses, red roses, pink carnations, and red carnations. One third of the pink flowers are roses, three fourths of the red flowers are carnations, and six tenths of the flowers are pink. What percent of the flowers are carnations?

    (A) 15 **(B)** 30 **(C)** 40 **(D)** 60 **(E)** 70

11. A student council must select a two-person welcoming committee and a three-person planning committee from among its members. There are exactly 10 ways to select a two-person team for the welcoming committee. It is possible for students to serve on both committees. In how many different ways can a three-person planning committee be selected?

    (A) 10 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 25

12. In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points D, E, and F are on sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively, such that $\overline{DE}$ and $\overline{EF}$ are parallel to $\overline{AC}$ and $\overline{AB}$, respectively. What is the perimeter of parallelogram $ADEF$?



(A) 48 **(B)** 52 **(C)** 56 **(D)** 60 **(E)** 72

13. How many three-digit numbers are not divisible by 5, have digits that sum to less than 20, and have the first digit equal to the third digit?

(A) 52 **(B)** 60 **(C)** 66 **(D)** 68 **(E)** 70

14. A solid cube of side length 1 is removed from each corner of a solid cube of side length 3. How many edges does the remaining solid have?

(A) 36 **(B)** 60 **(C)** 72 **(D)** 84 **(E)** 108

15. Two sides of a triangle have lengths 10 and 15. The length of the altitude to the third side is the average of the lengths of the altitudes to the two given sides. How long is the third side?

(A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 18

16. A triangle with vertices $(6, 5)$, $(8, -3)$, and $(9, 1)$ is reflected about the line $x = 8$ to create a second triangle. What is the area of the union of the two triangles?

(A) 9 **(B)** $\frac{28}{3}$ **(C)** 10 **(D)** $\frac{31}{3}$ **(E)** $\frac{32}{3}$

17. Daphne is visited periodically by her three best friends: Alice, Beatrix, and Claire. Alice visits every third day, Beatrix visits every fourth day, and Claire visits every fifth day. All three friends visited Daphne yesterday. How many days of the next 365-day period will exactly two friends visit her?

(A) 48 **(B)** 54 **(C)** 60 **(D)** 66 **(E)** 72

18. Let points $A = (0,0)$, $B = (1,2)$, $C = (3,3)$, and $D = (4,0)$. Quadrilateral $ABCD$ is cut into equal area pieces by a line passing through A. This line intersects $\overline{CD}$ at point $(\frac{p}{q}, \frac{r}{s})$, where these fractions are in lowest terms. What is $p+q+r+s$?

(A) 54 **(B)** 58 **(C)** 62 **(D)** 70 **(E)** 75

19. In base 10, the number 2013 ends in the digit 3. In base 9, on the other hand, the same number is written as $(2676)_9$ and ends in the digit 6. For how many positive integers b does the base-b representation of 2013 end in the digit 3?

(A) 6 **(B)** 9 **(C)** 13 **(D)** 16 **(E)** 18

20. A unit square is rotated 45° about its center. What is the area of the region swept out by the interior of the square?

(A) $1-\frac{\sqrt{2}}{2}+\frac{\pi}{4}$ **(B)** $\frac{1}{2}+\frac{\pi}{4}$ **(C)** $2-\sqrt{2}+\frac{\pi}{4}$

(D) $\frac{\sqrt{2}}{2}+\frac{\pi}{4}$ **(E)** $1+\frac{\sqrt{2}}{4}+\frac{\pi}{8}$

21. A group of 12 pirates agree to divide a treasure chest of gold coins among themselves as follows. The k^{th} pirate to take a share takes $\frac{k}{12}$ of the coins that remain in the chest. The number of coins initially in the chest is the smallest number for which this arrangement will allow each pirate to receive a positive whole number of coins. How many coins does the 12^{th} pirate receive?

(A) 720 **(B)** 1296 **(C)** 1728 **(D)** 1925 **(E)** 3850

22. Six spheres of radius 1 are positioned so that their centers are at the vertices of a regular hexagon of side length 2. The six spheres are internally tangent to a larger sphere whose center is the center of the hexagon. An eighth sphere is externally tangent to the six smaller spheres and internally tangent to the larger sphere. What is the radius of this eighth sphere?

(A) $\sqrt{2}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{5}{3}$ **(D)** $\sqrt{3}$ **(E)** 2

23. In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

(A) 11 **(B)** 28 **(C)** 33 **(D)** 61 **(E)** 72

24. Central High School is competing against Northern High School in a backgammon match. Each school has three players, and the contest rules require that each player play two games against each of the other school's players. The match takes place in six rounds, with three games played simultaneously in each round. In how many different ways can the match be scheduled?

(A) 540 **(B)** 600 **(C)** 720 **(D)** 810 **(E)** 900

25. All 20 diagonals are drawn in a regular octagon. At how many distinct points in the interior of the octagon (not on the boundary) do two or more diagonals intersect?

(A) 49 **(B)** 65 **(C)** 70 **(D)** 96 **(E)** 128

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The problems and solutions for this AMC 10 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 10 Subcommittee Chair:

Dr. Leroy Wenstrom

2013 AIME

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MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

14th Annual

AMC 10 B

American Mathematics Contest 10 B

Wednesday, February 20, 2013

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
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2013

AMC 10 B

DO NOT OPEN UNTIL WEDNESDAY, FEBRUARY 20, 2013

****Administration On An Earlier Date Will Disqualify Your School's Results****

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1. What is $\frac{2+4+6}{1+3+5} - \frac{1+3+5}{2+4+6}$?

   (A) -1 **(B)** $\dfrac{5}{36}$ **(C)** $\dfrac{7}{12}$ **(D)** $\dfrac{49}{20}$ **(E)** $\dfrac{43}{3}$

2. Mr. Green measures his rectangular garden by walking two of the sides and finds that it is 15 steps by 20 steps. Each of Mr. Green's steps is 2 feet long. Mr. Green expects a half a pound of potatoes per square foot from his garden. How many pounds of potatoes does Mr. Green expect from his garden?

   (A) 600 **(B)** 800 **(C)** 1000 **(D)** 1200 **(E)** 1400

3. On a particular January day, the high temperature in Lincoln, Nebraska, was 16 degrees higher than the low temperature, and the average of the high and low temperatures was 3°. In degrees, what was the low temperature in Lincoln that day?

   (A) -13 **(B)** -8 **(C)** -5 **(D)** -3 **(E)** 11

4. When counting from 3 to 201, 53 is the 51^{st} number counted. When counting backwards from 201 to 3, 53 is the n^{th} number counted. What is n ?

   (A) 146 **(B)** 147 **(C)** 148 **(D)** 149 **(E)** 150

5. Positive integers a and b are each less than 6. What is the smallest possible value for $2 \cdot a - a \cdot b$?

   (A) -20 **(B)** -15 **(C)** -10 **(D)** 0 **(E)** 2

6. The average age of 33 fifth-graders is 11. The average age of 55 of their parents is 33. What is the average age of all of these parents and fifth-graders?

   (A) 22 **(B)** 23.25 **(C)** 24.75 **(D)** 26.25 **(E)** 28

7. Six points are equally spaced around a circle of radius 1. Three of these points are the vertices of a triangle that is neither equilateral nor isosceles. What is the area of this triangle?

   (A) $\dfrac{\sqrt{3}}{3}$ **(B)** $\dfrac{\sqrt{3}}{2}$ **(C)** 1 **(D)** $\sqrt{2}$ **(E)** 2

8. Ray's car averages 40 miles per gallon of gasoline, and Tom's car averages 10 miles per gallon of gasoline. Ray and Tom each drive the same number of miles. What is the cars' combined rate of miles per gallon of gasoline?

   (A) 10 **(B)** 16 **(C)** 25 **(D)** 30 **(E)** 40

9. Three positive integers are each greater than 1, have a product of 27,000, and are pairwise relatively prime. What is the sum of these integers?

   (A) 100 **(B)** 137 **(C)** 156 **(D)** 160 **(E)** 165

10. A basketball team's players were successful on 50% of their two-point shots and 40% of their three-point shots, which resulted in 54 points. They attempted 50% more two-point shots than three-point shots. How many three-point shots did they attempt?

    (A) 10 **(B)** 15 **(C)** 20 **(D)** 25 **(E)** 30

11. Real numbers x and y satisfy the equation $x^2 + y^2 = 10x - 6y - 34$. What is $x + y$?

    (A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 8

12. Let S be the set of sides and diagonals of a regular pentagon. A pair of elements of S are selected at random without replacement. What is the probability that the two chosen segments have the same length?

    (A) $\frac{2}{5}$ **(B)** $\frac{4}{9}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{5}{9}$ **(E)** $\frac{4}{5}$

13. Jo and Blair take turns counting from 1 to one more than the last number said by the other person. Jo starts by saying "1", so Blair follows by saying "1, 2". Jo then says "1, 2, 3", and so on. What is the 53^{rd} number said?

    (A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 8

14. Define $a\clubsuit b = a^2b - ab^2$. Which of the following describes the set of points (x, y) for which $x\clubsuit y = y\clubsuit x$?

(A) a finite set of points

(B) one line

(C) two parallel lines

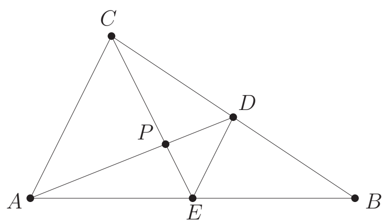
(D) two intersecting lines

(E) three lines

15. A wire is cut into two pieces, one of length a and the other of length b. The piece of length a is bent to form an equilateral triangle, and the piece of length b is bent to form a regular hexagon. The triangle and the hexagon have equal area. What is $\frac{a}{b}$?

(A) 1 **(B)** $\dfrac{\sqrt{6}}{2}$ **(C)** $\sqrt{3}$ **(D)** 2 **(E)** $\dfrac{3\sqrt{2}}{2}$

16. In $\triangle ABC$, medians $\overline{AD}$ and $\overline{CE}$ intersect at P, $PE = 1.5$, $PD = 2$, and $DE = 2.5$. What is the area of $AEDC$?



(A) 13 **(B)** 13.5 **(C)** 14 **(D)** 14.5 **(E)** 15

17. Alex has 75 red tokens and 75 blue tokens. There is a booth where Alex can give two red tokens and receive in return a silver token and a blue token, and another booth where Alex can give three blue tokens and receive in return a silver token and a red token. Alex continues to exchange tokens until no more exchanges are possible. How many silver tokens will Alex have at the end?

(A) 62 **(B)** 82 **(C)** 83 **(D)** 102 **(E)** 103

18. The number 2013 has the property that its units digit is the sum of its other digits, that is $2 + 0 + 1 = 3$. How many integers less than 2013 but greater than 1000 share this property?

(A) 33 **(B)** 34 **(C)** 45 **(D)** 46 **(E)** 58

19. The real numbers c, b, a form an arithmetic sequence with $a \geq b \geq c \geq 0$. The quadratic $ax^2 + bx + c$ has exactly one root. What is this root?

(A) $-7 - 4\sqrt{3}$ **(B)** $-2 - \sqrt{3}$ **(C)** -1 **(D)** $-2 + \sqrt{3}$ **(E)** $-7 + 4\sqrt{3}$

20. The number 2013 is expressed in the form

$$2013 = \frac{a_1!a_2!\cdots a_m!}{b_1!b_2!\cdots b_n!},$$

where $a_1 \geq a_2 \geq \cdots \geq a_m$ and $b_1 \geq b_2 \geq \cdots \geq b_n$ are positive integers and $a_1 + b_1$ is as small as possible. What is $|a_1 - b_1|$?

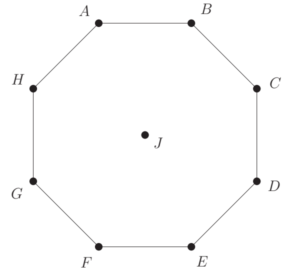
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

21. Two non-decreasing sequences of nonnegative integers have different first terms. Each sequence has the property that each term beginning with the third is the sum of the previous two terms, and the seventh term of each sequence is N. What is the smallest possible value of N?

(A) 55 **(B)** 89 **(C)** 104 **(D)** 144 **(E)** 273

22. The regular octagon $ABCDEFGH$ has its center at J. Each of the vertices and the center are to be associated with one of the digits 1 through 9, with each digit used once, in such a way that the sums of the numbers on the lines AJE, BJF, CJG, and DJH are equal. In how many ways can this be done?

(A) 384 **(B)** 576 **(C)** 1152 **(D)** 1680 **(E)** 3456



23. In triangle ABC, $AB = 13$, $BC = 14$, and $CA = 15$. Distinct points D, E, and F lie on segments $\overline{BC}$, $\overline{CA}$, and $\overline{DE}$, respectively, such that $\overline{AD} \perp \overline{BC}$, $\overline{DE} \perp \overline{AC}$, and $\overline{AF} \perp \overline{BF}$. The length of segment $\overline{DF}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 18 **(B)** 21 **(C)** 24 **(D)** 27 **(E)** 30

24. A positive integer n is *nice* if there is a positive integer m with exactly four positive divisors (including 1 and m) such that the sum of the four divisors is equal to n. How many numbers in the set $\{2010, 2011, 2012, \ldots, 2019\}$ are nice?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

25. Bernardo chooses a three-digit positive integer N and writes both its base-5 and base-6 representations on a blackboard. Later LeRoy sees the two numbers Bernardo has written. Treating the two numbers as base-10 integers, he adds them to obtain an integer S. For example, if $N = 749$, Bernardo writes the numbers 10,444 and 3,245, and LeRoy obtains the sum $S = 13{,}689$. For how many choices of N are the two rightmost digits of S, in order, the same as those of $2N$?

(A) 5 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 25

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American Mathematics Competitions

15th Annual

AMC 10 A

American Mathematics Contest 10 A

Tuesday, February 4, 2014

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2014

AMC 10 A

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American Mathematics Competitions

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Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

American Mathematics Competitions
1740 Vine Street
Lincoln, NE 68508
Phone 402-472-2257 | Fax 402-472-6087 | amcinfo@maa.org

The problems and solutions for this AMC 10 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 10 Subcommittee Chair:

Dr. Leroy Wenstrom

2014 AIME

The 32nd annual AIME will be held on Thursday, March 13, with the alternate on Wednesday, March 26. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 43rd Annual USA Mathematical Olympiad (USAMO) on April 29-30, 2014. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: maa.org/math-competitions

1. What is $10 \cdot (\frac{1}{2} + \frac{1}{5} + \frac{1}{10})^{-1}$?

   (A) 3 **(B)** 8 **(C)** $\dfrac{25}{2}$ **(D)** $\dfrac{170}{3}$ **(E)** 170

2. Roy's cat eats $\frac{1}{3}$ of a can of cat food every morning and $\frac{1}{4}$ of a can of cat food every evening. Before feeding his cat on Monday morning, Roy opened a box containing 6 cans of cat food. On what day of the week did the cat finish eating all the cat food in the box?

   (A) Tuesday **(B)** Wednesday **(C)** Thursday **(D)** Friday

   (E) Saturday

3. Bridget bakes 48 loaves of bread for her bakery. She sells half of them in the morning for \$2.50 each. In the afternoon she sells two thirds of what she has left, and because they are not fresh, she charges only half price. In the late afternoon she sells the remaining loaves at a dollar each. Each loaf costs \$0.75 for her to make. In dollars, what is her profit for the day?

   (A) 24 **(B)** 36 **(C)** 44 **(D)** 48 **(E)** 52

4. Walking down Jane Street, Ralph passed four houses in a row, each painted a different color. He passed the orange house before the red house, and he passed the blue house before the yellow house. The blue house was not next to the yellow house. How many orderings of the colored houses are possible?

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

5. On an algebra quiz, 10% of the students scored 70 points, 35% scored 80 points, 30% scored 90 points, and the rest scored 100 points. What is the difference between the mean and the median of the students' scores on this quiz?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

6. Suppose that a cows give b gallons of milk in c days. At this rate, how many gallons of milk will d cows give in e days?

   (A) $\dfrac{bde}{ac}$ **(B)** $\dfrac{ac}{bde}$ **(C)** $\dfrac{abde}{c}$ **(D)** $\dfrac{bcde}{a}$ **(E)** $\dfrac{abc}{de}$

7. Nonzero real numbers x, y, a, and b satisfy $x < a$ and $y < b$. How many of the following inequalities must be true?

   (I) $x + y < a + b$

   (II) $x - y < a - b$

   (III) $xy < ab$

   (IV) $\frac{x}{y} < \frac{a}{b}$

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

8. Which of the following numbers is a perfect square?

   (A) $\dfrac{14!15!}{2}$ **(B)** $\dfrac{15!16!}{2}$ **(C)** $\dfrac{16!17!}{2}$ **(D)** $\dfrac{17!18!}{2}$ **(E)** $\dfrac{18!19!}{2}$

9. The two legs of a right triangle, which are altitudes, have lengths $2\sqrt{3}$ and 6. How long is the third altitude of the triangle?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

10. Five positive consecutive integers starting with a have average b. What is the average of 5 consecutive integers that start with b?

    (A) $a + 3$ **(B)** $a + 4$ **(C)** $a + 5$ **(D)** $a + 6$ **(E)** $a + 7$

11. A customer who intends to purchase an appliance has three coupons, only one of which may be used:

    Coupon 1: 10% off the listed price if the listed price is at least \$50

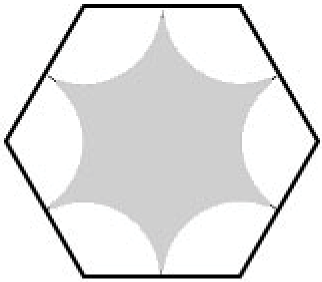
    Coupon 2: \$20 off the listed price if the listed price is at least \$100

    Coupon 3: 18% off the amount by which the listed price exceeds \$100

    For which of the following listed prices will coupon 1 offer a greater price reduction than either coupon 2 or coupon 3?

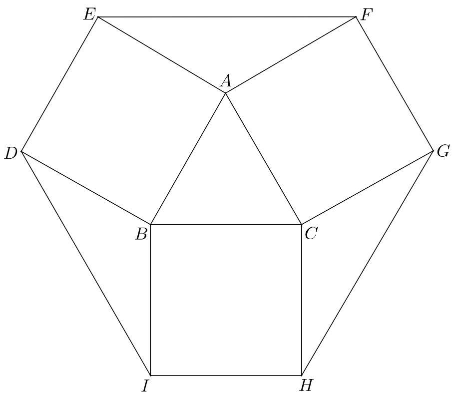
    (A) \$179.95 **(B)** \$199.95 **(C)** \$219.95 **(D)** \$239.95 **(E)** \$259.95

12. A regular hexagon has side length 6. Congruent arcs with radius 3 are drawn with the center at each of the vertices, creating circular sectors as shown. The region inside the hexagon but outside the sectors is shaded as shown. What is the area of the shaded region?



(A) $27\sqrt{3} - 9\pi$ **(B)** $27\sqrt{3} - 6\pi$ **(C)** $54\sqrt{3} - 18\pi$
(D) $54\sqrt{3} - 12\pi$ **(E)** $108\sqrt{3} - 9\pi$

13. Equilateral $\triangle ABC$ has side length 1, and squares $ABDE$, $BCHI$, and $CAFG$ lie outside the triangle. What is the area of hexagon $DEFGHI$?



(A) $\dfrac{12+3\sqrt{3}}{4}$ **(B)** $\dfrac{9}{2}$ **(C)** $3+\sqrt{3}$ **(D)** $\dfrac{6+3\sqrt{3}}{2}$ **(E)** 6

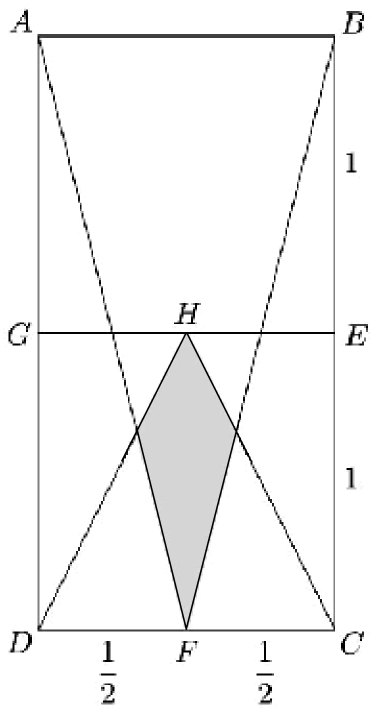
14. The y-intercepts, P and Q, of two perpendicular lines intersecting at the point $A(6, 8)$ have a sum of zero. What is the area of $\triangle APQ$?

(A) 45 **(B)** 48 **(C)** 54 **(D)** 60 **(E)** 72

15. David drives from his home to the airport to catch a flight. He drives 35 miles in the first hour, but realizes that he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way to the airport and arrives 30 minutes early. How many miles is the airport from his home?

(A) 140 **(B)** 175 **(C)** 210 **(D)** 245 **(E)** 280

16. In rectangle $ABCD$, $AB = 1$, $BC = 2$, and points E, F, and G are midpoints of $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$, respectively. Point H is the midpoint of $\overline{GE}$. What is the area of the shaded region?



(A) $\frac{1}{12}$ **(B)** $\frac{\sqrt{3}}{18}$ **(C)** $\frac{\sqrt{2}}{12}$ **(D)** $\frac{\sqrt{3}}{12}$ **(E)** $\frac{1}{6}$

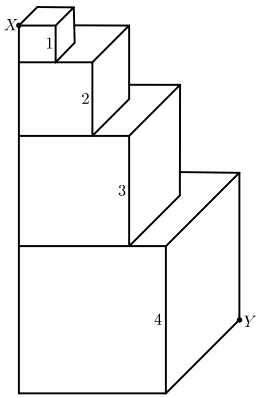
17. Three fair six-sided dice are rolled. What is the probability that the values shown on two of the dice sum to the value shown on the remaining die?

(A) $\frac{1}{6}$ **(B)** $\frac{13}{72}$ **(C)** $\frac{7}{36}$ **(D)** $\frac{5}{24}$ **(E)** $\frac{2}{9}$

18. A square in the coordinate plane has vertices whose y-coordinates are 0, 1, 4, and 5. What is the area of the square?

(A) 16 **(B)** 17 **(C)** 25 **(D)** 26 **(E)** 27

19. Four cubes with edge lengths 1, 2, 3, and 4 are stacked as shown. What is the length of the portion of $\overline{XY}$ contained in the cube with edge length 3?



(A) $\frac{3\sqrt{33}}{5}$ **(B)** $2\sqrt{3}$ **(C)** $\frac{2\sqrt{33}}{3}$ **(D)** 4 **(E)** $3\sqrt{2}$

20. The product $(8)(888\ldots8)$, where the second factor has k digits, is an integer whose digits have a sum of 1000. What is k?

(A) 901 **(B)** 911 **(C)** 919 **(D)** 991 **(E)** 999

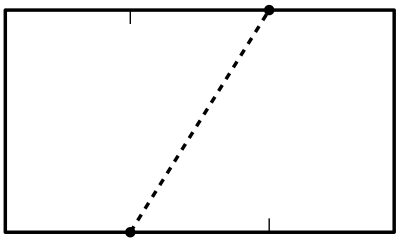
21. Positive integers a and b are such that the graphs of $y = ax + 5$ and $y = 3x + b$ intersect the x-axis at the same point. What is the sum of all possible x-coordinates of these points of intersection?

(A) -20 **(B)** -18 **(C)** -15 **(D)** -12 **(E)** -8

22. In rectangle $ABCD$, $AB = 20$ and $BC = 10$. Let E be a point on $\overline{CD}$ such that $\angle CBE = 15^\circ$. What is AE?

(A) $\frac{20\sqrt{3}}{3}$ **(B)** $10\sqrt{3}$ **(C)** 18 **(D)** $11\sqrt{3}$ **(E)** 20

23. A rectangular piece of paper whose length is $\sqrt{3}$ times the width has area A. The paper is divided into three equal sections along the opposite lengths, and then a dotted line is drawn from the first divider to the second divider on the opposite side as shown. The paper is then folded flat along this dotted line to create a new shape with area B. What is the ratio $B : A$?



(A) $1:2$ **(B)** $3:5$ **(C)** $2:3$ **(D)** $3:4$ **(E)** $4:5$

24. A sequence of natural numbers is constructed by listing the first 4, then skipping one, listing the next 5, skipping 2, listing 6, skipping 3, and, on the nth iteration, listing $n+3$ and skipping n. The sequence begins $1, 2, 3, 4, 6, 7, 8, 9, 10, 13$. What is the 500,000th number in the sequence?

(A) 996,506 **(B)** 996,507 **(C)** 996,508 **(D)** 996,509 **(E)** 996,510

25. The number 5^{867} is between 2^{2013} and 2^{2014}. How many pairs of integers (m, n) are there such that $1 \le m \le 2012$ and

$$5^n < 2^m < 2^{m+2} < 5^{n+1}\,?$$

(A) 278 **(B)** 279 **(C)** 280 **(D)** 281 **(E)** 282

MAA

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

15th Annual

AMC 10 B

American Mathematics Contest 10 B

Wednesday February 19, 2014

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
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5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 120 or above or finish in the top 2.5% on this AMC 10 will be invited to take the 32nd annual American Invitational Mathematics Examination (AIME) on Thursday, March 13, 2014 or Wednesday, March 26, 2014. More details about the AIME and other information are on the back page of this test booklet.

2014

AMC 10 B

DO NOT OPEN UNTIL WEDNESDAY, February 19, 2014

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 19, 2014. Nothing is needed from inside this package until February 19.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

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1. Leah has 13 coins, all of which are pennies and nickels. If she had one more nickel than she has now, then she would have the same number of pennies and nickels. In cents, how much are Leah's coins worth?

   (A) 33 **(B)** 35 **(C)** 37 **(D)** 39 **(E)** 41

2. What is $\dfrac{2^3 + 2^3}{2^{-3} + 2^{-3}}$?

   (A) 16 **(B)** 24 **(C)** 32 **(D)** 48 **(E)** 64

3. Randy drove the first third of his trip on a gravel road, the next 20 miles on pavement, and the remaining one-fifth on a dirt road. In miles, how long was Randy's trip?

   (A) 30 **(B)** $\dfrac{400}{11}$ **(C)** $\dfrac{75}{2}$ **(D)** 40 **(E)** $\dfrac{300}{7}$

4. Susie pays for 4 muffins and 3 bananas. Calvin spends twice as much paying for 2 muffins and 16 bananas. A muffin is how many times as expensive as a banana?

   (A) $\dfrac{3}{2}$ **(B)** $\dfrac{5}{3}$ **(C)** $\dfrac{7}{4}$ **(D)** 2 **(E)** $\dfrac{13}{4}$

5. Doug constructs a square window using 8 equal-size panes of glass, as shown. The ratio of the height to width for each pane is $5 : 2$, and the borders around and between the panes are 2 inches wide. In inches, what is the side length of the square window?

   (A) 26 **(B)** 28 **(C)** 30 **(D)** 32 **(E)** 34

6. Orvin went to the store with just enough money to buy 30 balloons. When he arrived he discovered that the store had a special sale on balloons: buy 1 balloon at the regular price and get a second at $\frac{1}{3}$ off the regular price. What is the greatest number of balloons Orvin could buy?

   (A) 33 **(B)** 34 **(C)** 36 **(D)** 38 **(E)** 39

7. Suppose $A > B > 0$ and A is $x\%$ greater than B. What is x?

   (A) $100\left(\frac{A-B}{B}\right)$ **(B)** $100\left(\frac{A+B}{B}\right)$ **(C)** $100\left(\frac{A+B}{A}\right)$

   (D) $100\left(\frac{A-B}{A}\right)$ **(E)** $100\left(\frac{A}{B}\right)$

8. A truck travels $\frac{b}{6}$ feet every t seconds. There are 3 feet in a yard. How many yards does the truck travel in 3 minutes?

   (A) $\frac{b}{1080t}$ **(B)** $\frac{30t}{b}$ **(C)** $\frac{30b}{t}$ **(D)** $\frac{10t}{b}$ **(E)** $\frac{10b}{t}$

9. For real numbers w and z,

$$\frac{\frac{1}{w}+\frac{1}{z}}{\frac{1}{w}-\frac{1}{z}} = 2014.$$

   What is $\frac{w+z}{w-z}$?

   (A) -2014 **(B)** $\frac{-1}{2014}$ **(C)** $\frac{1}{2014}$ **(D)** 1 **(E)** 2014

10. In the addition shown below A, B, C, and D are distinct digits. How many different values are possible for D?

$$\begin{array}{cr} & ABBCB \\ + & BCADA \\ \hline & DBDDD \end{array}$$

   (A) 2 **(B)** 4 **(C)** 7 **(D)** 8 **(E)** 9

11. For the consumer, a single discount of $n\%$ is more advantageous than any of the following discounts:

    (1) two successive 15% discounts

    (2) three successive 10% discounts

    (3) a 25% discount followed by a 5% discount

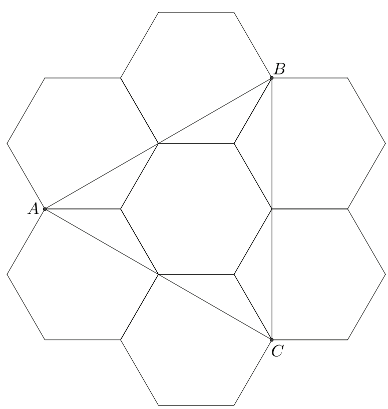
    What is the smallest possible positive integer value of n?

    (A) 27 **(B)** 28 **(C)** 29 **(D)** 31 **(E)** 33

12. The largest divisor of 2,014,000,000 is itself. What is its fifth largest divisor?

    (A) 125,875,000 **(B)** 201,400,000 **(C)** 251,750,000 **(D)** 402,800,000
    (E) 503,500,000

13. Six regular hexagons surround a regular hexagon of side length 1 as shown. What is the area of $\triangle ABC$?

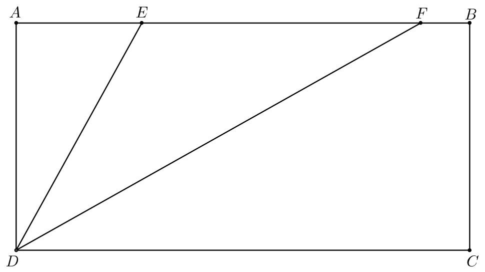


    (A) $2\sqrt{3}$ **(B)** $3\sqrt{3}$ **(C)** $1+3\sqrt{2}$ **(D)** $2+2\sqrt{3}$ **(E)** $3+2\sqrt{3}$

14. Danica drove her new car on a trip for a whole number of hours, averaging 55 miles per hour. At the beginning of the trip, abc miles was displayed on the odometer, where abc is a 3-digit number with $a \geq 1$ and $a+b+c \leq 7$. At the end of the trip, the odometer showed cba miles. What is $a^2+b^2+c^2$?

    (A) 26 **(B)** 27 **(C)** 36 **(D)** 37 **(E)** 41

15. In rectangle $ABCD$, $DC = 2CB$ and points E and F lie on $\overline{AB}$ so that $\overline{ED}$ and $\overline{FD}$ trisect $\angle ADC$ as shown. What is the ratio of the area of $\triangle DEF$ to the area of rectangle $ABCD$?



(A) $\frac{\sqrt{3}}{6}$ **(B)** $\frac{\sqrt{6}}{8}$ **(C)** $\frac{3\sqrt{3}}{16}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{\sqrt{2}}{4}$

16. Four fair six-sided dice are rolled. What is the probability that at least three of the four dice show the same value?

(A) $\frac{1}{36}$ **(B)** $\frac{7}{72}$ **(C)** $\frac{1}{9}$ **(D)** $\frac{5}{36}$ **(E)** $\frac{1}{6}$

17. What is the greatest power of 2 that is a factor of $10^{1002} - 4^{501}$?

(A) 2^{1002} **(B)** 2^{1003} **(C)** 2^{1004} **(D)** 2^{1005} **(E)** 2^{1006}

18. A list of 11 positive integers has a mean of 10, a median of 9, and a unique mode of 8. What is the largest possible value of an integer in the list?

(A) 24 **(B)** 30 **(C)** 31 **(D)** 33 **(E)** 35

19. Two concentric circles have radii 1 and 2. Two points on the outer circle are chosen independently and uniformly at random. What is the probability that the chord joining the two points intersects the inner circle?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{2-\sqrt{2}}{2}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

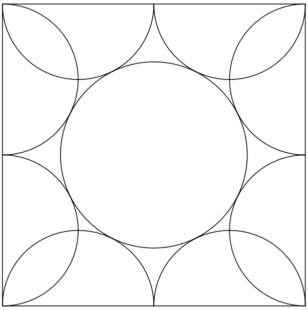
20. For how many integers x is the number $x^4 - 51x^2 + 50$ negative?

    (A) 8 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 16

21. Trapezoid $ABCD$ has parallel sides $\overline{AB}$ of length 33 and $\overline{CD}$ of length 21. The other two sides are of lengths 10 and 14. The angles at A and B are acute. What is the length of the shorter diagonal of $ABCD$?

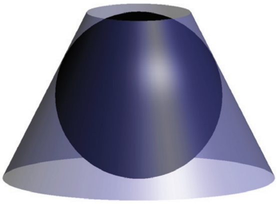
    (A) $10\sqrt{6}$ **(B)** 25 **(C)** $8\sqrt{10}$ **(D)** $18\sqrt{2}$ **(E)** 26

22. Eight semicircles line the inside of a square with side length 2 as shown. What is the radius of the circle tangent to all of these semicircles?



    (A) $\dfrac{1+\sqrt{2}}{4}$ **(B)** $\dfrac{\sqrt{5}-1}{2}$ **(C)** $\dfrac{\sqrt{3}+1}{4}$ **(D)** $\dfrac{2\sqrt{3}}{5}$ **(E)** $\dfrac{\sqrt{5}}{3}$

23. A sphere is inscribed in a truncated right circular cone as shown. The volume of the truncated cone is twice that of the sphere. What is the ratio of the radius of the bottom base of the truncated cone to the radius of the top base of the truncated cone?



    (A) $\dfrac{3}{2}$ **(B)** $\dfrac{1+\sqrt{5}}{2}$ **(C)** $\sqrt{3}$ **(D)** 2 **(E)** $\dfrac{3+\sqrt{5}}{2}$

24. The numbers 1, 2, 3, 4, 5 are to be arranged in a circle. An arrangement is *bad* if it is not true that for every n from 1 to 15 one can find a subset of the numbers that appear consecutively on the circle that sum to n. Arrangements that differ only by a rotation or a reflection are considered the same. How many different bad arrangements are there?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

25. In a small pond there are eleven lily pads in a row labeled 0 through 10. A frog is sitting on pad 1. When the frog is on pad N, $0 < N < 10$, it will jump to pad $N-1$ with probability $\frac{N}{10}$ and to pad $N+1$ with probability $1-\frac{N}{10}$. Each jump is independent of the previous jumps. If the frog reaches pad 0 it will be eaten by a patiently waiting snake. If the frog reaches pad 10 it will exit the pond, never to return. What is the probability that the frog will escape being eaten by the snake?

(A) $\frac{32}{79}$ **(B)** $\frac{161}{384}$ **(C)** $\frac{63}{146}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{1}{2}$

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PUBLICATIONS

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MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

16th Annual

AMC 10 A

American Mathematics Contest 10 A

Tuesday, February 3, 2015

INSTRUCTIONS

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2015

AMC 10 A

DO NOT OPEN UNTIL TUESDAY, February 3, 2015

****Administration On An Earlier Date Will Disqualify Your School's Results****

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2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

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MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

WRITE TO US!

Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

MAA American Mathematics Competitions
PO Box 471
Annapolis Junction, MD 20701
Phone 800.527.3690 | Fax 240.396.5647 | amcinfo@maa.org

The problems and solutions for this AMC 10 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 10 Subcommittee Chair:

Silvia Fernandez

2015 AIME

The 33rd annual AIME will be held on Thursday, March 19, with the alternate on Wednesday, March 25. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 44th Annual USA Mathematical Olympiad (USAMO) on April 28–29, 2015. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: maa.org/math-competitions

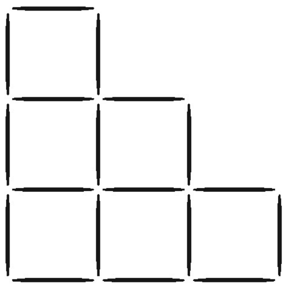
1. What is the value of $(2^0 - 1 + 5^2 + 0)^{-1} \times 5$?

   (A) -125 **(B)** -120 **(C)** $\frac{1}{5}$ **(D)** $\frac{5}{24}$ **(E)** 25

2. A box contains a collection of triangular and square tiles. There are 25 tiles in the box, containing 84 edges total. How many square tiles are there in the box?

   (A) 3 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 11

3. Ann made a 3-step staircase using 18 toothpicks as shown in the figure. How many toothpicks does she need to add to complete a 5-step staircase?



   (A) 9 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24

4. Pablo, Sofia, and Mia got some candy eggs at a party. Pablo had three times as many eggs as Sofia, and Sofia had twice as many eggs as Mia. Pablo decides to give some of his eggs to Sofia and Mia so that all three will have the same number of eggs. What fraction of his eggs should Pablo give to Sofia?

   (A) $\frac{1}{12}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

5. Mr. Patrick teaches math to 15 students. He was grading tests and found that when he graded everyone's test except Payton's, the average grade for the class was 80. After he graded Payton's test, the class average became 81. What was Payton's score on the test?

   (A) 81 **(B)** 85 **(C)** 91 **(D)** 94 **(E)** 95

6. The sum of two positive numbers is 5 times their difference. What is the ratio of the larger number to the smaller?

(A) $\frac{5}{4}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{9}{5}$ **(D)** 2 **(E)** $\frac{5}{2}$

7. How many terms are there in the arithmetic sequence $13, 16, 19, \ldots, 70, 73$?

(A) 20 **(B)** 21 **(C)** 24 **(D)** 60 **(E)** 61

8. Two years ago Pete was three times as old as his cousin Claire. Two years before that, Pete was four times as old as Claire. In how many years will the ratio of their ages be $2 : 1$?

(A) 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

9. Two right circular cylinders have the same volume. The radius of the second cylinder is 10% more than the radius of the first. What is the relationship between the heights of the two cylinders?

(A) The second height is 10% less than the first.

(B) The first height is 10% more than the second.

(C) The second height is 21% less than the first.

(D) The first height is 21% more than the second.

(E) The second height is 80% of the first.

10. How many rearrangements of $abcd$ are there in which no two adjacent letters are also adjacent letters in the alphabet? For example, no such rearrangements could include either ab or ba.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

11. The ratio of the length to the width of a rectangle is $4 : 3$. If the rectangle has diagonal of length d, then the area may be expressed as kd^2 for some constant k. What is k?

(A) $\frac{2}{7}$ **(B)** $\frac{3}{7}$ **(C)** $\frac{12}{25}$ **(D)** $\frac{16}{25}$ **(E)** $\frac{3}{4}$

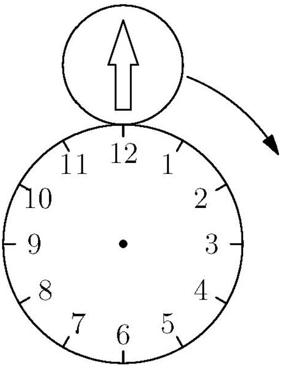
12. Points $(\sqrt{\pi}, a)$ and $(\sqrt{\pi}, b)$ are distinct points on the graph of $y^2 + x^4 = 2x^2y + 1$. What is $|a - b|$?

(A) 1 **(B)** $\frac{\pi}{2}$ **(C)** 2 **(D)** $\sqrt{1+\pi}$ **(E)** $1+\sqrt{\pi}$

13. Claudia has 12 coins, each of which is a 5-cent coin or a 10-cent coin. There are exactly 17 different values that can be obtained as combinations of one or more of her coins. How many 10-cent coins does Claudia have?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

14. The diagram below shows the circular face of a clock with radius 20 cm and a circular disk with radius 10 cm externally tangent to the clock face at 12 o'clock The disk has an arrow painted on it, initially pointing in the upward vertical direction. Let the disk roll clockwise around the clock face. At what point on the clock face will the disk be tangent when the arrow is next pointing in the upward vertical direction?



(A) 2 o'clock **(B)** 3 o'clock **(C)** 4 o'clock **(D)** 6 o'clock **(E)** 8 o'clock

15. Consider the set of all fractions $\frac{x}{y}$, where x and y are relatively prime positive integers. How many of these fractions have the property that if both numerator and denominator are increased by 1, the value of the fraction is increased by 10%?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinitely many

16. If $y + 4 = (x - 2)^2$, $x + 4 = (y - 2)^2$, and $x \neq y$, what is the value of $x^2 + y^2$?

(A) 10 **(B)** 15 **(C)** 20 **(D)** 25 **(E)** 30

17. A line that passes through the origin intersects both the line $x = 1$ and the line $y = 1 + \frac{\sqrt{3}}{3}x$. The three lines create an equilateral triangle. What is the perimeter of the triangle?

(A) $2\sqrt{6}$ **(B)** $2 + 2\sqrt{3}$ **(C)** 6 **(D)** $3 + 2\sqrt{3}$ **(E)** $6 + \dfrac{\sqrt{3}}{3}$

18. Hexadecimal (base-16) numbers are written using the numeric digits 0 through 9 as well as the letters A through F to represent 10 through 15. Among the first 1000 positive integers, there are n whose hexadecimal representation contains only numeric digits. What is the sum of the digits of n?

(A) 17 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** 21

19. The isosceles right triangle ABC has right angle at C and area 12.5. The rays trisecting $\angle ACB$ intersect AB at D and E. What is the area of $\triangle CDE$?

(A) $\dfrac{5\sqrt{2}}{3}$ **(B)** $\dfrac{50\sqrt{3} - 75}{4}$ **(C)** $\dfrac{15\sqrt{3}}{8}$ **(D)** $\dfrac{50 - 25\sqrt{3}}{2}$ **(E)** $\dfrac{25}{6}$

20. A rectangle has area A cm^2 and perimeter P cm, where A and P are positive integers. Which of the following numbers cannot equal $A + P$?

(A) 100 **(B)** 102 **(C)** 104 **(D)** 106 **(E)** 108

21. Tetrahedron $ABCD$ has $AB = 5$, $AC = 3$, $BC = 4$, $BD = 4$, $AD = 3$, and $CD = \frac{12}{5}\sqrt{2}$. What is the volume of the tetrahedron?

(A) $3\sqrt{2}$ **(B)** $2\sqrt{5}$ **(C)** $\dfrac{24}{5}$ **(D)** $3\sqrt{3}$ **(E)** $\dfrac{24}{5}\sqrt{2}$

22. Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?

(A) $\dfrac{47}{256}$ **(B)** $\dfrac{3}{16}$ **(C)** $\dfrac{49}{256}$ **(D)** $\dfrac{25}{128}$ **(E)** $\dfrac{51}{256}$

23. The zeros of the function $f(x) = x^2 - ax + 2a$ are integers. What is the sum of the possible values of a?

(A) 7 **(B)** 8 **(C)** 16 **(D)** 17 **(E)** 18

24. For some positive integers p, quadrilateral $ABCD$ with positive integer side lengths has perimeter p, right angles at B and C, $AB = 2$, and $CD = AD$. How many different values of $p < 2015$ are possible?

(A) 30 **(B)** 31 **(C)** 61 **(D)** 62 **(E)** 63

25. Let S be a square of side length 1. Two points are chosen independently at random on the sides of S. The probability that the straight-line distance between the points is at least $\frac{1}{2}$ is $\frac{a-b\pi}{c}$, where a, b, and c are positive integers and $\gcd(a, b, c) = 1$. What is $a + b + c$?

(A) 59 **(B)** 60 **(C)** 61 **(D)** 62 **(E)** 63

MAA100
MATHEMATICAL ASSOCIATION OF AMERICA
CELEBRATING A CENTURY OF ADVANCING MATHEMATICS

American Mathematics Competitions

16th Annual

AMC 10 B

American Mathematics Contest 10 B

Wednesday February 25, 2015

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 120 or above or finish in the top 2.5% on this AMC 10 will be invited to take the 33rd annual American Invitational Mathematics Examination (AIME) on Thursday, March 19, 2015 or Wednesday, March 25, 2015. More details about the AIME are on the back page of this test booklet.

2015

AMC 10 B

DO NOT OPEN UNTIL WEDNESDAY, February 25, 2015

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PUBLICATIONS

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1. What is the value of $2 - (-2)^{-2}$?

   (A) -2 **(B)** $\dfrac{1}{16}$ **(C)** $\dfrac{7}{4}$ **(D)** $\dfrac{9}{4}$ **(E)** 6

2. Marie does three equally time-consuming tasks in a row without taking breaks. She begins the first task at 1:00 PM and finishes the second task at 2:40 PM. When does she finish the third task?

   (A) 3:10 PM **(B)** 3:30 PM **(C)** 4:00 PM **(D)** 4:10 PM **(E)** 4:30 PM

3. Isaac has written down one integer two times and another integer three times. The sum of the five numbers is 100, and one of the numbers is 28. What is the other number?

   (A) 8 **(B)** 11 **(C)** 14 **(D)** 15 **(E)** 18

4. Four siblings ordered an extra large pizza. Alex ate $\frac{1}{5}$, Beth $\frac{1}{3}$, and Cyril $\frac{1}{4}$ of the pizza. Dan got the leftovers. What is the sequence of the siblings in decreasing order of the part of the pizza they consumed?

   (A) Alex, Beth, Cyril, Dan

   (B) Beth, Cyril, Alex, Dan

   (C) Beth, Cyril, Dan, Alex

   (D) Beth, Dan, Cyril, Alex

   (E) Dan, Beth, Cyril, Alex

5. David, Hikmet, Jack, Marta, Rand, and Todd were in a 12-person race with 6 other people. Rand finished 6 places ahead of Hikmet. Marta finished 1 place behind Jack. David finished 2 places behind Hikmet. Jack finished 2 places behind Todd. Todd finished 1 place behind Rand. Marta finished in 6th place. Who finished in 8th place?

   (A) David **(B)** Hikmet **(C)** Jack **(D)** Rand **(E)** Todd

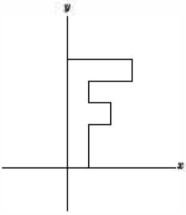
6. Marley practices exactly one sport each day of the week. She runs three days a week but never on two consecutive days. On Monday she plays basketball and two days later golf. She swims and plays tennis, but she never plays tennis the day after running or swimming. Which day of the week does Marley swim?

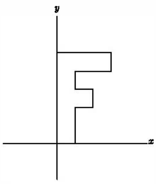
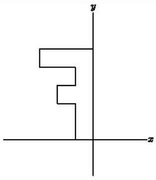
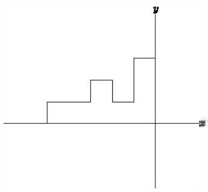
   (A) Sunday **(B)** Tuesday **(C)** Thursday **(D)** Friday **(E)** Saturday

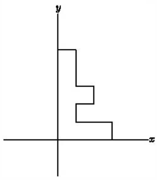
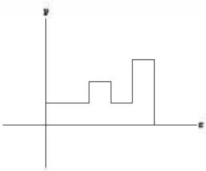
7. Consider the operation "minus the reciprocal of," defined by $a \diamond b = a - \frac{1}{b}$. What is $\big((1 \diamond 2) \diamond 3\big) - \big(1 \diamond (2 \diamond 3)\big)$?

(A) $-\dfrac{7}{30}$ **(B)** $-\dfrac{1}{6}$ **(C)** 0 **(D)** $\dfrac{1}{6}$ **(E)** $\dfrac{7}{30}$

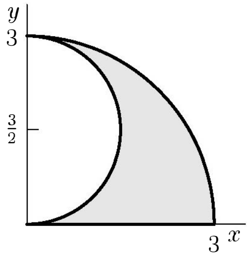
8. The letter F shown below is rotated 90° clockwise around the origin, then reflected in the y-axis, and then rotated a half turn around the origin. What is the final image?



(A) 
(B) 
(C) 

(D) 
(E) 

9. The shaded region below is called a shark's fin falcata, a figure studied by Leonardo da Vinci. It is bounded by the portion of the circle of radius 3 and center $(0,0)$ that lies in the first quadrant, the portion of the circle of radius $\frac{3}{2}$ and center $(0,\frac{3}{2})$ that lies in the first quadrant, and the line segment from $(0,0)$ to $(3,0)$. What is the area of the shark's fin falcata?



(A) $\frac{4\pi}{5}$ **(B)** $\frac{9\pi}{8}$ **(C)** $\frac{4\pi}{3}$ **(D)** $\frac{7\pi}{5}$ **(E)** $\frac{3\pi}{2}$

10. What are the sign and units digit of the product of all the odd negative integers strictly greater than -2015?

(A) It is a negative number ending with a 1

(B) It is a positive number ending with a 1

(C) It is a negative number ending with a 5

(D) It is a positive number ending with a 5

(E) It is a negative number ending with a 0

11. Among the positive integers less than 100, each of whose digits is a prime number, one is selected at random. What is the probability that the selected number is prime?

(A) $\frac{8}{99}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{9}{20}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{9}{16}$

12. For how many integers x is the point $(x,-x)$ inside or on the circle of radius 10 centered at $(5,5)$?

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

13. The line $12x + 5y = 60$ forms a triangle with the coordinate axes. What is the sum of the lengths of the altitudes of this triangle?

(A) 20 **(B)** $\frac{360}{17}$ **(C)** $\frac{107}{5}$ **(D)** $\frac{43}{2}$ **(E)** $\frac{281}{13}$

14. Let a, b, and c be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation $(x-a)(x-b)+(x-b)(x-c)=0$?

(A) 15 **(B)** 15.5 **(C)** 16 **(D)** 16.5 **(E)** 17

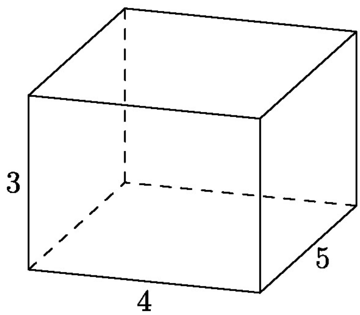
15. The town of Hamlet has 3 people for each horse, 4 sheep for each cow, and 3 ducks for each person. Which of the following could not possibly be the total number of people, horses, sheep, cows, and ducks in Hamlet?

(A) 41 **(B)** 47 **(C)** 59 **(D)** 61 **(E)** 66

16. Al, Bill, and Cal will each randomly be assigned a whole number from 1 to 10, inclusive, with no two of them getting the same number. What is the probability that Al's number will be a whole number multiple of Bill's and Bill's number will be a whole number multiple of Cal's?

(A) $\frac{9}{1000}$ **(B)** $\frac{1}{90}$ **(C)** $\frac{1}{80}$ **(D)** $\frac{1}{72}$ **(E)** $\frac{2}{121}$

17. The centers of the faces of the right rectangular prism shown below are joined to create an octahedron. What is the volume of the octahedron?



(A) $\frac{75}{12}$ **(B)** 10 **(C)** 12 **(D)** $10\sqrt{2}$ **(E)** 15

18. Johann has 64 fair coins. He flips all the coins. Any coin that lands on tails is tossed again. Coins that land on tails on the second toss are tossed a third time. What is the expected number of coins that are now heads?

(A) 32 **(B)** 40 **(C)** 48 **(D)** 56 **(E)** 64

19. In $\triangle ABC$, $\angle C = 90^\circ$ and $AB = 12$. Squares $ABXY$ and $ACWZ$ are constructed outside of the triangle. The points X, Y, Z, and W lie on a circle. What is the perimeter of the triangle?

(A) $12 + 9\sqrt{3}$ **(B)** $18 + 6\sqrt{3}$ **(C)** $12 + 12\sqrt{2}$ **(D)** 30 **(E)** 32

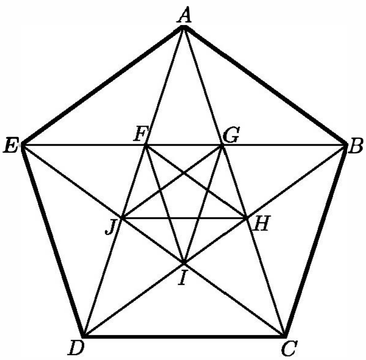
20. Erin the ant starts at a given corner of a cube and crawls along exactly 7 edges in such a way that she visits every corner exactly once and then finds that she is unable to return along an edge to her starting point. How many paths are there meeting these conditions?

(A) 6 **(B)** 9 **(C)** 12 **(D)** 18 **(E)** 24

21. Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. However, instead of walking up the steps one at a time, both Cozy and Dash jump. Cozy goes two steps up with each jump (though if necessary, he will just jump the last step). Dash goes five steps up with each jump (though if necessary, he will just jump the last steps if there are fewer than 5 steps left). Suppose that Dash takes 19 fewer jumps than Cozy to reach the top of the staircase. Let s denote the sum of all possible numbers of steps this staircase can have. What is the sum of the digits of s?

(A) 9 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 15

22. In the figure shown below, $ABCDE$ is a regular pentagon and $AG = 1$. What is $FG + JH + CD$?



(A) 3 **(B)** $12 - 4\sqrt{5}$ **(C)** $\frac{5 + 2\sqrt{5}}{3}$ **(D)** $1 + \sqrt{5}$ **(E)** $\frac{11 + 11\sqrt{5}}{10}$

23. Let n be a positive integer greater than 4 such that the decimal representation of $n!$ ends in k zeros and the decimal representation of $(2n)!$ ends in $3k$ zeros Let s denote the sum of the four least possible values of n. What is the sum of the digits of s?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

24. Aaron the ant walks on the coordinate plane according to the following rules He starts at the origin $p_0 = (0, 0)$ facing to the east and walks one unit, arriving at $p_1 = (1, 0)$. For $n = 1, 2, 3, \ldots$, right after arriving at the point p_n, if Aaron can turn 90° left and walk one unit to an unvisited point p_{n+1}, he does that Otherwise, he walks one unit straight ahead to reach p_{n+1}. Thus the sequence of points continues $p_2 = (1, 1)$, $p_3 = (0, 1)$, $p_4 = (-1, 1)$, $p_5 = (-1, 0)$, and so on in a counterclockwise spiral pattern. What is p_{2015}?

(A) $(-22, -13)$ **(B)** $(-13, -22)$ **(C)** $(-13, 22)$ **(D)** $(13, -22)$

(E) $(22, -13)$

25. A rectangular box measures $a \times b \times c$, where a, b, and c are integers and $1 \leq a \leq b \leq c$. The volume and the surface area of the box are numerically equal How many ordered triples (a, b, c) are possible?

(A) 4 **(B)** 10 **(C)** 12 **(D)** 21 **(E)** 26

MATHEMATICAL ASSOCIATION OF AMERICA
CELEBRATING A CENTURY OF ADVANCING MATHEMATICS

American Mathematics Competitions

17th Annual

AMC 10A

American Mathematics Contest 10A
Tuesday, February 2, 2016

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
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2016

AMC 10A

DO NOT OPEN UNTIL TUESDAY, February 2, 2016

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL. PLEASE READ THE MANUAL BEFORE FEBRUARY 2, 2016.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
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American Mathematics Competitions

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Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

MAA American Mathematics Competitions
PO Box 471
Annapolis Junction, MD 20701
Phone 800.527.3690 | Fax 240.396.5647 | amcinfo@maa.org

The problems and solutions for this AMC 10 were prepared by MAA's Subcommittee on the AMC10/AMC12 Exams, under the direction of the co-chairs Jerrold W. Grossman and Silvia Fernandez.

2016 AIME

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PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
www.maa.org/amc

1. What is the value of $\frac{11!-10!}{9!}$?

   (A) 99 **(B)** 100 **(C)** 110 **(D)** 121 **(E)** 132

2. For what value of x does $10^x \cdot 100^{2x} = 1000^5$?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

3. For every dollar Ben spent on bagels, David spent 25 cents less. Ben paid \$12.50 more than David. How much did they spend in the bagel store together?

   (A) \$37.50 **(B)** \$50.00 **(C)** \$87.50 **(D)** \$90.00 **(E)** \$92.50

4. The remainder function can be defined for all real numbers x and y with $y \neq 0$ by
$$\text{rem}(x, y) = x - y\left\lfloor \frac{x}{y} \right\rfloor,$$
   where $\left\lfloor \frac{x}{y} \right\rfloor$ denotes the greatest integer less than or equal to $\frac{x}{y}$. What is the value of $\text{rem}\left(\frac{3}{8}, -\frac{2}{5}\right)$?

   (A) $-\frac{3}{8}$ **(B)** $-\frac{1}{40}$ **(C)** 0 **(D)** $\frac{3}{8}$ **(E)** $\frac{31}{40}$

5. A rectangular box has integer side lengths in the ratio $1 : 3 : 4$. Which of the following could be the volume of the box?

   (A) 48 **(B)** 56 **(C)** 64 **(D)** 96 **(E)** 144

6. Ximena lists the whole numbers 1 through 30 once. Emilio copies Ximena's numbers, replacing each occurrence of the digit 2 by the digit 1. Ximena adds her numbers and Emilio adds his numbers. How much larger is Ximena's sum than Emilio's?

   (A) 13 **(B)** 26 **(C)** 102 **(D)** 103 **(E)** 110

7. The mean, median, and mode of the 7 data values $60, 100, x, 40, 50, 200, 90$ are all equal to x. What is the value of x?

   (A) 50 **(B)** 60 **(C)** 75 **(D)** 90 **(E)** 100

8. Trickster Rabbit agrees with Foolish Fox to double Fox's money every time Fox crosses the bridge by Rabbit's house, as long as Fox pays 40 coins in toll to Rabbit after each crossing. The payment is made after the doubling. Fox is

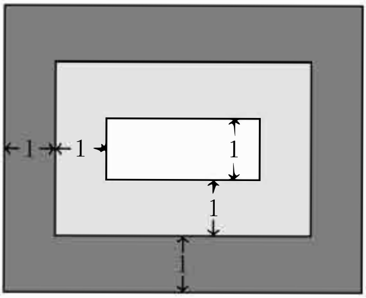
excited about his good fortune until he discovers that all his money is gone after crossing the bridge three times. How many coins did Fox have at the beginning?

(A) 20 **(B)** 30 **(C)** 35 **(D)** 40 **(E)** 45

9. A triangular array of 2016 coins has 1 coin in the first row, 2 coins in the second row, 3 coins in the third row, and so on up to N coins in the Nth row. What is the sum of the digits of N?

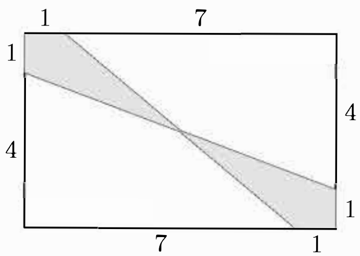
(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

10. A rug is made with three different colors as shown. The areas of the three differently colored regions form an arithmetic progression. The inner rectangle is one foot wide, and each of the two shaded regions is 1 foot wide on all four sides. What is the length in feet of the inner rectangle?



(A) 1 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

11. What is the area of the shaded region of the given 8×5 rectangle?



(A) $4\frac{3}{4}$ **(B)** 5 **(C)** $5\frac{1}{4}$ **(D)** $6\frac{1}{2}$ **(E)** 8

12. Three distinct integers are selected at random between 1 and 2016, inclusive. Which of the following is a correct statement about the probability p that the product of the three integers is odd?

(A) $p < \frac{1}{8}$ **(B)** $p = \frac{1}{8}$ **(C)** $\frac{1}{8} < p < \frac{1}{3}$ **(D)** $p = \frac{1}{3}$ **(E)** $p > \frac{1}{3}$

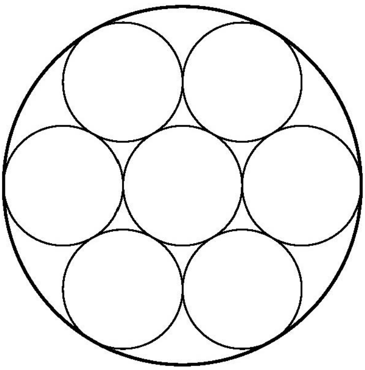
13. Five friends sat in a movie theater in a row containing 5 seats, numbered 1 to 5 from left to right. (The directions "left" and "right" are from the point of view of the people as they sit in the seats.) During the movie Ada went to the lobby to get some popcorn. When she returned, she found that Bea had moved two seats to the right, Ceci had moved one seat to the left, and Dee and Edie had switched seats, leaving an end seat for Ada. In which seat had Ada been sitting before she got up?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

14. How many ways are there to write 2016 as the sum of twos and threes, ignoring order? (For example, $1008 \cdot 2 + 0 \cdot 3$ and $402 \cdot 2 + 404 \cdot 3$ are two such ways.)

(A) 236 **(B)** 336 **(C)** 337 **(D)** 403 **(E)** 672

15. Seven cookies of radius 1 inch are cut from a circle of cookie dough, as shown. Neighboring cookies are tangent, and all except the center cookie are tangent to the edge of the dough. The leftover scrap is reshaped to form another cookie of the same thickness. What is the radius in inches of the scrap cookie?



(A) $\sqrt{2}$ **(B)** 1.5 **(C)** $\sqrt{\pi}$ **(D)** $\sqrt{2\pi}$ **(E)** π

16. A triangle with vertices $A(0, 2)$, $B(-3, 2)$, and $C(-3, 0)$ is reflected about the x-axis; then the image $\triangle A'B'C'$ is rotated counterclockwise around the origin by 90° to produce $\triangle A''B''C''$. Which of the following transformations will return $\triangle A''B''C''$ to $\triangle ABC$?

(A) counterclockwise rotation around the origin by 90°

(B) clockwise rotation around the origin by 90°

(C) reflection about the x-axis

(D) reflection about the line $y = x$

(E) reflection about the y-axis

17. Let N be a positive multiple of 5. One red ball and N green balls are arranged in a line in random order. Let $P(N)$ be the probability that at least $\frac{3}{5}$ of the green balls are on the same side of the red ball. Observe that $P(5) = 1$ and that $P(N)$ approaches $\frac{4}{5}$ as N grows large. What is the sum of the digits of the least value of N such that $P(N) < \frac{321}{400}$?

(A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

18. Each vertex of a cube is to be labeled with an integer from 1 through 8, with each integer being used once, in such a way that the sum of the four numbers on the vertices of a face is the same for each face. Arrangements that can be obtained from each other through rotations of the cube are considered to be the same. How many different arrangements are possible?

(A) 1 **(B)** 3 **(C)** 6 **(D)** 12 **(E)** 24

19. In rectangle $ABCD$, $AB = 6$ and $BC = 3$. Point E between B and C, and point F between E and C are such that $BE = EF = FC$. Segments $\overline{AE}$ and $\overline{AF}$ intersect $\overline{BD}$ at P and Q, respectively. The ratio $BP : PQ : QD$ can be written as $r : s : t$, where the greatest common factor of r, s, and t is 1. What is $r + s + t$?

(A) 7 **(B)** 9 **(C)** 12 **(D)** 15 **(E)** 20

20. For some particular value of N, when $(a + b + c + d + 1)^N$ is expanded and like terms are combined, the resulting expression contains exactly 1001 terms that include all four variables, a, b, c, and d, each to some positive power. What is N?

(A) 9 **(B)** 14 **(C)** 16 **(D)** 17 **(E)** 19

21. Circles with centers P, Q, and R, having radii 1, 2, and 3, respectively, lie on the same side of line l and are tangent to l at P', Q', and R', respectively, with Q' between P' and R'. The circle with center Q is externally tangent to each of the other two circles. What is the area of $\triangle PQR$?

    (A) 0 **(B)** $\sqrt{\frac{2}{3}}$ **(C)** 1 **(D)** $\sqrt{6} - \sqrt{2}$ **(E)** $\sqrt{\frac{3}{2}}$

22. For some positive integer n, the number $110n^3$ has 110 positive integer divisors, including 1 and the number $110n^3$. How many positive integer divisors does the number $81n^4$ have?

    (A) 110 **(B)** 191 **(C)** 261 **(D)** 325 **(E)** 425

23. A binary operation $\diamondsuit$ has the properties that $a \diamondsuit (b \diamondsuit c) = (a \diamondsuit b) \cdot c$ and that $a \diamondsuit a = 1$ for all nonzero real numbers a, b, and c. (Here the dot $\cdot$ represents the usual multiplication operation.) The solution to the equation $2016 \diamondsuit (6 \diamondsuit x) = 100$ can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers What is $p + q$?

    (A) 109 **(B)** 201 **(C)** 301 **(D)** 3049 **(E)** 33,601

24. A quadrilateral is inscribed in a circle of radius $200\sqrt{2}$. Three of the sides of this quadrilateral have length 200. What is the length of its fourth side?

    (A) 200 **(B)** $200\sqrt{2}$ **(C)** $200\sqrt{3}$ **(D)** $300\sqrt{2}$ **(E)** 500

25. How many ordered triples (x, y, z) of positive integers satisfy $\text{lcm}(x, y) = 72$, $\text{lcm}(x, z) = 600$, and $\text{lcm}(y, z) = 900$?

    (A) 15 **(B)** 16 **(C)** 24 **(D)** 27 **(E)** 64

MATHEMATICAL ASSOCIATION OF AMERICA
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American Mathematics Competitions

17th Annual

AMC 10B

American Mathematics Contest 10B
Wednesday February 17, 2016

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded. **No copies.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
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2016

AMC 10B

DO NOT OPEN UNTIL WEDNESDAY, February 17, 2016

****Administration On An Earlier Date Will Disqualify Your School's Results****

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2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.*

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2016 AIME

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PUBLICATIONS

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www.maa.org/amc

1. What is the value of
$$\frac{2a^{-1}+\frac{a^{-1}}{2}}{a}$$
when $a=\frac{1}{2}$?

(A) 1 **(B)** 2 **(C)** $\frac{5}{2}$ **(D)** 10 **(E)** 20

2. If $n\heartsuit m = n^3 m^2$, what is $\frac{2\heartsuit 4}{4\heartsuit 2}$?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** 4

3. Let $x=-2016$. What is the value of $\Bigg\vert\Big\vert |x|-x\Big\vert-|x|\Bigg\vert-x$?

(A) -2016 **(B)** 0 **(C)** 2016 **(D)** 4032 **(E)** 6048

4. Zoey read 15 books, one at a time. The first book took her 1 day to read, the second book took her 2 days to read, the third book took her 3 days to read, and so on, with each book taking her 1 more day to read than the previous book. Zoey finished the first book on a Monday and the second on a Wednesday. On what day of the week did she finish her 15th book?

(A) Sunday **(B)** Monday **(C)** Wednesday **(D)** Friday **(E)** Saturday

5. The mean age of Amanda's 4 cousins is 8, and their median age is 5. What is the sum of the ages of Amanda's youngest and oldest cousins?

(A) 13 **(B)** 16 **(C)** 19 **(D)** 22 **(E)** 25

6. Laura added two three-digit positive integers. All six digits in these numbers are different. Laura's sum is a three-digit number S. What is the smallest possible value for the sum of the digits of S?

(A) 1 **(B)** 4 **(C)** 5 **(D)** 15 **(E)** 21

7. The ratio of the measures of two acute angles is $5:4$, and the complement of one of these two angles is twice as large as the complement of the other. What is the sum of the degree measures of the two angles?

(A) 75 **(B)** 90 **(C)** 135 **(D)** 150 **(E)** 270

8. What is the tens digit of $2015^{2016} - 2017$?

(A) 0 **(B)** 1 **(C)** 3 **(D)** 5 **(E)** 8

9. All three vertices of $\triangle ABC$ lie on the parabola defined by $y = x^2$, with A at the origin and $\overline{BC}$ parallel to the x-axis. The area of the triangle is 64. What is the length BC?

(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 16

10. A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 ounces. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length 5 inches. Which of the following is closest to the weight, in ounces, of the second piece?

(A) 14.0 **(B)** 16.0 **(C)** 20.0 **(D)** 33.3 **(E)** 55.6

11. Carl decided to fence in his rectangular garden. He bought 20 fence posts, placed one on each of the four corners, and spaced out the rest evenly along the edges of the garden, leaving exactly 4 yards between neighboring posts. The longer side of his garden, including the corners, has twice as many posts as the shorter side, including the corners. What is the area, in square yards, of Carl's garden?

(A) 256 **(B)** 336 **(C)** 384 **(D)** 448 **(E)** 512

12. Two different numbers are selected at random from $\{1, 2, 3, 4, 5\}$ and multiplied together. What is the probability that the product is even?

(A) 0.2 **(B)** 0.4 **(C)** 0.5 **(D)** 0.7 **(E)** 0.8

13. At Megapolis Hospital one year, multiple-birth statistics were as follows: Sets of twins, triplets, and quadruplets accounted for 1000 of the babies born. There were four times as many sets of triplets as sets of quadruplets, and three times as many sets of twins as sets of triplets. How many of these 1000 babies were in sets of quadruplets?

(A) 25 **(B)** 40 **(C)** 64 **(D)** 100 **(E)** 160

14. How many squares whose sides are parallel to the axes and whose vertices have coordinates that are integers lie entirely within the region bounded by the line $y = \pi x$, the line $y = -0.1$, and the line $x = 5.1$?

(A) 30 **(B)** 41 **(C)** 45 **(D)** 50 **(E)** 57

15. All the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ are written in a 3×3 array of squares, one number in each square, in such a way that if two numbers are consecutive then they occupy squares that share an edge. The numbers in the four corners add up to 18. What number is in the center?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

16. The sum of an infinite geometric series is a positive number S, and the second term in the series is 1. What is the smallest possible value of S?

(A) $\frac{1+\sqrt{5}}{2}$ **(B)** 2 **(C)** $\sqrt{5}$ **(D)** 3 **(E)** 4

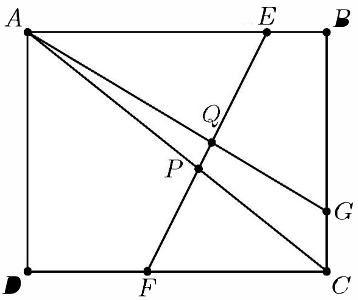
17. All the numbers $2, 3, 4, 5, 6, 7$ are assigned to the six faces of a cube, one number to each face. For each of the eight vertices of the cube, a product of three numbers is computed, where the three numbers are the numbers assigned to the three faces that include that vertex. What is the greatest possible value of the sum of these eight products?

(A) 312 **(B)** 343 **(C)** 625 **(D)** 729 **(E)** 1680

18. In how many ways can 345 be written as the sum of an increasing sequence of two or more consecutive positive integers?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 7

19. Rectangle $ABCD$ has $AB = 5$ and $BC = 4$. Point E lies on $\overline{AB}$ so that $EB = 1$, point G lies on $\overline{BC}$ so that $CG = 1$, and point F lies on $\overline{CD}$ so that $DF = 2$. Segments $\overline{AG}$ and $\overline{AC}$ intersect $\overline{EF}$ at Q and P, respectively. What is the value of $\frac{PQ}{EF}$?



(A) $\frac{\sqrt{3}}{16}$ **(B)** $\frac{\sqrt{2}}{13}$ **(C)** $\frac{9}{82}$ **(D)** $\frac{10}{91}$ **(E)** $\frac{1}{9}$

20. A dilation of the plane—that is, a size transformation with a positive scale factor—sends the circle of radius 2 centered at $A(2,2)$ to the circle of radius 3 centered at $A'(5,6)$. What distance does the origin $O(0,0)$ move under this transformation?

(A) 0 **(B)** 3 **(C)** $\sqrt{13}$ **(D)** 4 **(E)** 5

21. What is the area of the region enclosed by the graph of the equation $x^2+y^2=|x|+|y|$?

(A) $\pi+\sqrt{2}$ **(B)** $\pi+2$ **(C)** $\pi+2\sqrt{2}$ **(D)** $2\pi+\sqrt{2}$ **(E)** $2\pi+2\sqrt{2}$

22. A set of teams held a round-robin tournament in which every team played every other team exactly once. Every team won 10 games and lost 10 games; there were no ties. How many sets of three teams $\{A,B,C\}$ were there in which A beat B, B beat C, and C beat A?

(A) 385 **(B)** 665 **(C)** 945 **(D)** 1140 **(E)** 1330

23. In regular hexagon $ABCDEF$, points W, X, Y, and Z are chosen on sides $\overline{BC}$, $\overline{CD}$, $\overline{EF}$, and $\overline{FA}$, respectively, so that lines AB, ZW, YX, and ED are parallel and equally spaced. What is the ratio of the area of hexagon $WCXYFZ$ to the area of hexagon $ABCDEF$?

(A) $\frac{1}{3}$ **(B)** $\frac{10}{27}$ **(C)** $\frac{11}{27}$ **(D)** $\frac{4}{9}$ **(E)** $\frac{13}{27}$

24. How many four-digit positive integers $abcd$, with $a\neq 0$, have the property that the three two-digit integers $ab<bc<cd$ form an increasing arithmetic sequence? One such number is 4692, where $a=4$, $b=6$, $c=9$, and $d=2$.

(A) 9 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 20

25. Let $f(x)=\sum_{k=2}^{10}(\lfloor kx\rfloor-k\lfloor x\rfloor)$, where $\lfloor r\rfloor$ denotes the greatest integer less than or equal to r. How many distinct values does $f(x)$ assume for $x\geq 0$?

(A) 32 **(B)** 36 **(C)** 45 **(D)** 46 **(E)** infinitely many

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

18th Annual

AMC 10A

American Mathematics Competition 10A
Tuesday, February 7, 2017

INSTRUCTIONS

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2017

AMC 10A

DO NOT OPEN UNTIL TUESDAY, February 7, 2017

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MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

WRITE TO US!

Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

MAA American Mathematics Competitions
PO Box 471
Annapolis Junction, MD 20701
Phone 800.527.3690 | Fax 240.396.5647 | amcinfo@maa.org

The problems and solutions for this AMC 10 were prepared by MAA's Subcommittee on the AMC10/AMC12 Exams, under the direction of the co-chairs Jerrold W. Grossman and Carl Yerger.

2017 AIME

The 35th annual AIME will be held on Thursday, March 7, 2017 with the alternate on Wednesday, March 22, 2017. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate if you achieve a high score on this competition. Top-scoring students on the AMC 10/12/AIME will be selected to take the 46th Annual USA Mathematical Olympiad (USAMO) on April 19–20, 2017. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
www.maa.org/amc

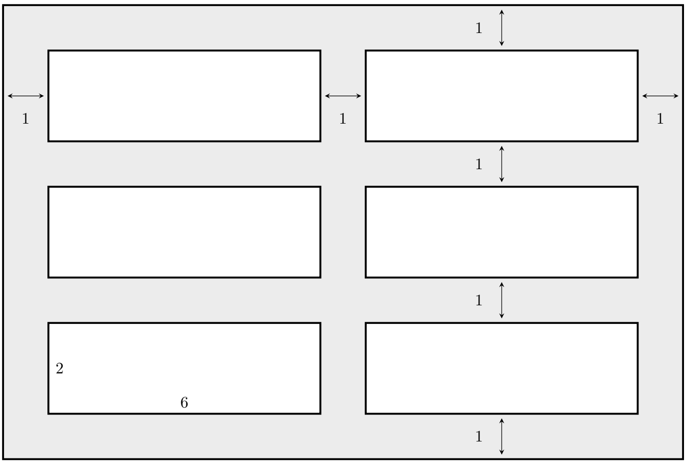
1. What is the value of $(2(2(2(2(2(2+1)+1)+1)+1)+1)+1)$?

   (A) 70 **(B)** 97 **(C)** 127 **(D)** 159 **(E)** 729

2. Pablo buys popsicles for his friends. The store sells single popsicles for \$1 each, 3-popsicle boxes for \$2, and 5-popsicle boxes for \$3. What is the greatest number of popsicles that Pablo can buy with \$8?

   (A) 8 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 15

3. Tamara has three rows of two 6-feet by 2-feet flower beds in her garden. The beds are separated and also surrounded by 1-foot-wide walkways, as shown on the diagram. What is the total area of the walkways, in square feet?



   (A) 72 **(B)** 78 **(C)** 90 **(D)** 120 **(E)** 150

4. Mia is "helping" her mom pick up 30 toys that are strewn on the floor. Mia's mom manages to put 3 toys into the toy box every 30 seconds, but each time immediately after those 30 seconds have elapsed, Mia takes 2 toys out of the box. How much time, in minutes, will it take Mia and her mom to put all 30 toys into the box for the first time?

   (A) 13.5 **(B)** 14 **(C)** 14.5 **(D)** 15 **(E)** 15.5

5. The sum of two nonzero real numbers is 4 times their product. What is the sum of the reciprocals of the two numbers?

   (A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 12

6. Ms. Carroll promised that anyone who got all the multiple choice questions right on the upcoming exam would receive an A on the exam. Which one of these statements necessarily follows logically?

   (A) If Lewis did not receive an A, then he got all of the multiple choice questions wrong.

   (B) If Lewis did not receive an A, then he got at least one of the multiple choice questions wrong.

   (C) If Lewis got at least one of the multiple choice questions wrong, then he did not receive an A.

   (D) If Lewis received an A, then he got all of the multiple choice questions right.

   (E) If Lewis received an A, then he got at least one of the multiple choice questions right.

7. Jerry and Silvia wanted to go from the southwest corner of a square field to the northeast corner. Jerry walked due east and then due north to reach the goal, but Silvia headed northeast and reached the goal walking in a straight line. Which of the following is closest to how much shorter Silvia's trip was, compared to Jerry's trip?

   (A) 30% **(B)** 40% **(C)** 50% **(D)** 60% **(E)** 70%

8. At a gathering of 30 people, there are 20 people who all know each other and 10 people who know no one. People who know each other hug, and people who do not know each other shake hands. How many handshakes occur?

   (A) 240 **(B)** 245 **(C)** 290 **(D)** 480 **(E)** 490

9. Minnie rides on a flat road at 20 kilometers per hour (kph), downhill at 30 kph, and uphill at 5 kph. Penny rides on a flat road at 30 kph, downhill at 40 kph, and uphill at 10 kph. Minnie goes from town A to town B, a distance of 10 km all uphill, then from town B to town C, a distance of 15 km all downhill, and then back to town A, a distance of 20 km on the flat. Penny goes the other way around using the same route. How many more minutes does it take Minnie to complete the 45-km ride than it takes Penny?

   (A) 45 **(B)** 60 **(C)** 65 **(D)** 90 **(E)** 95

10. Joy has 30 thin rods, one each of every integer length from 1 cm through 30 cm. She places the rods with lengths 3 cm, 7 cm, and 15 cm on a table. She then wants to choose a fourth rod that she can put with these three to form a quadrilateral with positive area. How many of the remaining rods can she choose as the fourth rod?

    (A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

11. The region consisting of all points in three-dimensional space within 3 units of line segment $\overline{AB}$ has volume 216π. What is the length AB?

    (A) 6 **(B)** 12 **(C)** 18 **(D)** 20 **(E)** 24

12. Let S be the set of points (x, y) in the coordinate plane such that two of the three quantities 3, $x+2$, and $y-4$ are equal and the third of the three quantities is no greater than this common value. Which of the following is a correct description of S?

    (A) a single point **(B)** two intersecting lines

    (C) three lines whose pairwise intersections are three distinct points

    (D) a triangle **(E)** three rays with a common endpoint

13. Define a sequence recursively by $F_0 = 0$, $F_1 = 1$, and $F_n =$ the remainder when $F_{n-1} + F_{n-2}$ is divided by 3, for all $n \geq 2$. Thus the sequence starts $0, 1, 1, 2, 0, 2, \ldots$. What is $F_{2017} + F_{2018} + F_{2019} + F_{2020} + F_{2021} + F_{2022} + F_{2023} + F_{2024}$?

    (A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

14. Every week Roger pays for a movie ticket and a soda out of his allowance. Last week, Roger's allowance was A dollars. The cost of his movie ticket was 20% of the difference between A and the cost of his soda, while the cost of his soda was 5% of the difference between A and the cost of his movie ticket. To the nearest whole percent, what fraction of A did Roger pay for his movie ticket and soda?

    (A) 9% **(B)** 19% **(C)** 22% **(D)** 23% **(E)** 25%

15. Chloé chooses a real number uniformly at random from the interval $[0, 2017]$. Independently, Laurent chooses a real number uniformly at random from the interval $[0, 4034]$. What is the probability that Laurent's number is greater than Chloé's number?

    (A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{5}{6}$ **(E)** $\frac{7}{8}$

16. There are 10 horses, named Horse 1, Horse 2, ..., Horse 10. They get their names from how many minutes it takes them to run one lap around a circular race track: Horse k runs one lap in exactly k minutes. At time 0 all the horses are together at the starting point on the track. The horses start running in the same direction, and they keep running around the circular track at their constant speeds. The least time $S > 0$, in minutes, at which all 10 horses will again simultaneously be at the starting point is $S = 2520$. Let $T > 0$ be the least time, in minutes, such that at least 5 of the horses are again at the starting point. What is the sum of the digits of T?

    (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

17. Distinct points P, Q, R, and S lie on the circle $x^2 + y^2 = 25$ and have integer coordinates. The distances PQ and RS are irrational numbers. What is the greatest possible value of the ratio $\frac{PQ}{RS}$?

    (A) 3 **(B)** 5 **(C)** $3\sqrt{5}$ **(D)** 7 **(E)** $5\sqrt{2}$

18. Amelia has a coin that lands on heads with probability $\frac{1}{3}$, and Blaine has a coin that lands on heads with probability $\frac{2}{5}$. Amelia and Blaine alternately toss their coins until someone gets a head; the first one to get a head wins. All coin tosses are independent. Amelia goes first. The probability that Amelia wins is $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $q - p$?

    (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

19. Alice refuses to sit next to either Bob or Carla. Derek refuses to sit next to Eric. How many ways are there for the five of them to sit in a row of 5 chairs under these conditions?

    (A) 12 **(B)** 16 **(C)** 28 **(D)** 32 **(E)** 40

20. Let $S(n)$ equal the sum of the digits of positive integer n. For example, $S(1507) = 13$. For a particular positive integer n, $S(n) = 1274$. Which of the following could be the value of $S(n+1)$?

    (A) 1 **(B)** 3 **(C)** 12 **(D)** 1239 **(E)** 1265

21. A square with side length x is inscribed in a right triangle with sides of length 3, 4, and 5 so that one vertex of the square coincides with the right-angle vertex of the triangle. A square with side length y is inscribed in another right triangle with sides of length 3, 4, and 5 so that one side of the square lies on the hypotenuse of the triangle. What is $\frac{x}{y}$?

    (A) $\dfrac{12}{13}$ **(B)** $\dfrac{35}{37}$ **(C)** 1 **(D)** $\dfrac{37}{35}$ **(E)** $\dfrac{13}{12}$

22. Sides $\overline{AB}$ and $\overline{AC}$ of equilateral triangle ABC are tangent to a circle at points B and C, respectively. What fraction of the area of $\triangle ABC$ lies outside the circle?

    (A) $\dfrac{4\sqrt{3}\pi}{27} - \dfrac{1}{3}$ **(B)** $\dfrac{\sqrt{3}}{2} - \dfrac{\pi}{8}$ **(C)** $\dfrac{1}{2}$ **(D)** $\sqrt{3} - \dfrac{2\sqrt{3}\pi}{9}$
    (E) $\dfrac{4}{3} - \dfrac{4\sqrt{3}\pi}{27}$

23. How many triangles with positive area have all their vertices at points (i, j) in the coordinate plane, where i and j are integers between 1 and 5, inclusive?

    (A) 2128 **(B)** 2148 **(C)** 2160 **(D)** 2200 **(E)** 2300

24. For certain real numbers a, b, and c, the polynomial

    $$g(x) = x^3 + ax^2 + x + 10$$

    has three distinct roots, and each root of $g(x)$ is also a root of the polynomial

    $$f(x) = x^4 + x^3 + bx^2 + 100x + c.$$

    What is $f(1)$?

    (A) -9009 **(B)** -8008 **(C)** -7007 **(D)** -6006 **(E)** -5005

25. How many integers between 100 and 999, inclusive, have the property that some permutation of its digits is a multiple of 11 between 100 and 999 ? For example, both 121 and 211 have this property.

    (A) 226 **(B)** 243 **(C)** 270 **(D)** 469 **(E)** 486

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

18th Annual

AMC 10B

American Mathematics Competition 10B
Wednesday, February 15, 2017

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded. **No copies.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score well on this AMC 10 will be invited to take the 35th annual American Invitational Mathematics Examination (AIME) on Thursday, March 7, 2017 or Wednesday, March 22, 2017. More details about the AIME are on the back page of this test booklet.

2017

AMC 10B

DO NOT OPEN UNTIL WEDNESDAY, February 15, 2017

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the Teachers' Manual. PLEASE READ THE MANUAL BEFORE FEBRUARY 15, 2017.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found on amc.maa.org under 'AMC 10/12') that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.*

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MAA American Mathematics Competitions

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PUBLICATIONS

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www.maa.org/amc

1. Mary thought of a positive two-digit number. She multiplied it by 3 and added 11. Then she switched the digits of the result, obtaining a number between 71 and 75, inclusive. What was Mary's number?

   (A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

2. Sofia ran 5 laps around the 400-meter track at her school. For each lap, she ran the first 100 meters at an average speed of 4 meters per second and the remaining 300 meters at an average speed of 5 meters per second. How much time did Sofia take running the 5 laps?

   (A) 5 minutes and 35 seconds **(B)** 6 minutes and 40 seconds

   (C) 7 minutes and 5 seconds **(D)** 7 minutes and 25 seconds

   (E) 8 minutes and 10 seconds

3. Real numbers x, y, and z satisfy the inequalities

   $$0 < x < 1, \qquad -1 < y < 0, \qquad \text{and} \qquad 1 < z < 2.$$

   Which of the following numbers is necessarily positive?

   (A) $y + x^2$ **(B)** $y + xz$ **(C)** $y + y^2$ **(D)** $y + 2y^2$

   (E) $y + z$

4. Suppose that x and y are nonzero real numbers such that

   $$\frac{3x + y}{x - 3y} = -2.$$

   What is the value of

   $$\frac{x + 3y}{3x - y}\,?$$

   (A) -3 **(B)** -1 **(C)** 1 **(D)** 2 **(E)** 3

5. Camilla had twice as many blueberry jelly beans as cherry jelly beans. After eating 10 pieces of each kind, she now has three times as many blueberry jelly beans as cherry jelly beans. How many blueberry jelly beans did she originally have?

   (A) 10 **(B)** 20 **(C)** 30 **(D)** 40 **(E)** 50

6. What is the largest number of solid 2-in $\times$ 2-in $\times$ 1-in blocks that can fit in a 3-in $\times$ 2-in $\times$ 3-in box?

   (A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

7. Samia set off on her bicycle to visit her friend, traveling at an average speed of 17 kilometers per hour. When she had gone half the distance to her friend's house, a tire went flat, and she walked the rest of the way at 5 kilometers per hour. In all it took her 44 minutes to reach her friend's house. In kilometers rounded to the nearest tenth, how far did Samia walk?

   (A) 2.0 **(B)** 2.2 **(C)** 2.8 **(D)** 3.4 **(E)** 4.4

8. Points $A(11, 9)$ and $B(2, -3)$ are vertices of $\triangle ABC$ with $AB = AC$. The altitude from A meets the opposite side at $D(-1, 3)$. What are the coordinates of point C?

   (A) $(-8, 9)$ **(B)** $(-4, 8)$ **(C)** $(-4, 9)$ **(D)** $(-2, 3)$ **(E)** $(-1, 0)$

9. A radio program has a quiz consisting of 3 multiple-choice questions, each with 3 choices. A contestant wins if he or she gets 2 or more of the questions right. The contestant answers randomly to each question. What is the probability of winning?

   (A) $\frac{1}{27}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{2}{9}$ **(D)** $\frac{7}{27}$ **(E)** $\frac{1}{2}$

10. The lines with equations $ax - 2y = c$ and $2x + by = -c$ are perpendicular and intersect at $(1, -5)$. What is c?

    (A) -13 **(B)** -8 **(C)** 2 **(D)** 8 **(E)** 13

11. At Typico High School, 60% of the students like dancing, and the rest dislike it. Of those who like dancing, 80% say that they like it, and the rest say that they dislike it. Of those who dislike dancing, 90% say that they dislike it, and the rest say that they like it. What fraction of students who say they dislike dancing actually like it?

    (A) 10% **(B)** 12% **(C)** 20% **(D)** 25% **(E)** $33\frac{1}{3}\%$

12. Elmer's new car gets 50% better fuel efficiency, measured in kilometers per liter, than his old car. However, his new car uses diesel fuel, which is 20% more expensive per liter than the gasoline his old car uses. By what percent will Elmer save money if he uses his new car instead of his old car for a long trip?

(A) 20% **(B)** $26\frac{2}{3}\%$ **(C)** $27\frac{7}{9}\%$ **(D)** $33\frac{1}{3}\%$ **(E)** $41\frac{2}{3}\%$

13. There are 20 students participating in an after-school program offering classes in yoga, bridge, and painting. Each student must take at least one of these three classes, but may take two or all three. There are 10 students taking yoga, 13 taking bridge, and 9 taking painting. There are 9 students taking at least two classes. How many students are taking all three classes?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

14. An integer N is selected at random in the range $1 \leq N \leq 2020$. What is the probability that the remainder when N^{16} is divided by 5 is 1?

(A) $\frac{1}{5}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{3}{5}$ **(D)** $\frac{4}{5}$ **(E)** 1

15. Rectangle $ABCD$ has $AB = 3$ and $BC = 4$. Point E is the foot of the perpendicular from B to diagonal $\overline{AC}$. What is the area of $\triangle ADE$?

(A) 1 **(B)** $\frac{42}{25}$ **(C)** $\frac{28}{15}$ **(D)** 2 **(E)** $\frac{54}{25}$

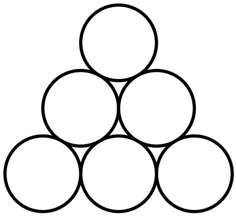
16. How many of the base-ten numerals for the positive integers less than or equal to 2017 contain the digit 0?

(A) 469 **(B)** 471 **(C)** 475 **(D)** 478 **(E)** 481

17. Call a positive integer *monotonous* if it is a one-digit number or its digits, when read from left to right, form either a strictly increasing or a strictly decreasing sequence. For example, 3, 23578, and 987620 are monotonous, but 88, 7434, and 23557 are not. How many monotonous positive integers are there?

(A) 1024 **(B)** 1524 **(C)** 1533 **(D)** 1536 **(E)** 2048

18. In the figure below, 3 of the 6 disks are to be painted blue, 2 are to be painted red, and 1 is to be painted green. Two paintings that can be obtained from one another by a rotation or a reflection of the entire figure are considered the same. How many different paintings are possible?



(A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 15

19. Let ABC be an equilateral triangle. Extend side $\overline{AB}$ beyond B to a point B' so that $BB' = 3AB$. Similarly, extend side $\overline{BC}$ beyond C to a point C' so that $CC' = 3BC$, and extend side $\overline{CA}$ beyond A to a point A' so that $AA' = 3CA$. What is the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$?

(A) $9:1$ **(B)** $16:1$ **(C)** $25:1$ **(D)** $36:1$ **(E)** $37:1$

20. The number $21! = 51{,}090{,}942{,}171{,}709{,}440{,}000$ has over 60,000 positive integer divisors. One of them is chosen at random. What is the probability that it is odd?

(A) $\frac{1}{21}$ **(B)** $\frac{1}{19}$ **(C)** $\frac{1}{18}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{11}{21}$

21. In $\triangle ABC$, $AB = 6$, $AC = 8$, $BC = 10$, and D is the midpoint of $\overline{BC}$. What is the sum of the radii of the circles inscribed in $\triangle ADB$ and $\triangle ADC$?

(A) $\sqrt{5}$ **(B)** $\frac{11}{4}$ **(C)** $2\sqrt{2}$ **(D)** $\frac{17}{6}$ **(E)** 3

22. The diameter $\overline{AB}$ of a circle of radius 2 is extended to a point D outside the circle so that $BD = 3$. Point E is chosen so that $ED = 5$ and line ED is perpendicular to line AD. Segment $\overline{AE}$ intersects the circle at a point C between A and E. What is the area of $\triangle ABC$?

(A) $\frac{120}{37}$ **(B)** $\frac{140}{39}$ **(C)** $\frac{145}{39}$ **(D)** $\frac{140}{37}$ **(E)** $\frac{120}{31}$

23. Let $N = 123456789101112\dots4344$ be the 79-digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when N is divided by 45?

(A) 1 **(B)** 4 **(C)** 9 **(D)** 18 **(E)** 44

24. The vertices of an equilateral triangle lie on the hyperbola $xy = 1$, and a vertex of this hyperbola is the centroid of the triangle. What is the square of the area of the triangle?

(A) 48 **(B)** 60 **(C)** 108 **(D)** 120 **(E)** 169

25. Last year Isabella took 7 math tests and received 7 different scores, each an integer between 91 and 100, inclusive. After each test she noticed that the average of her test scores was an integer. Her score on the seventh test was 95. What was her score on the sixth test?

(A) 92 **(B)** 94 **(C)** 96 **(D)** 98 **(E)** 100

MAA AMC

American Mathematics Competitions

American Mathematics Competitions

19th Annual

AMC 10A

American Mathematics Competition 10A

Wednesday, February 7, 2018

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR COMPETITION MANAGER TELLS YOU.
2. This is a 25-question multiple-choice exam. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
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4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only scratch paper, graph paper, rulers, compasses, protractors, and erasers are allowed as aids. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the exam will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
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Students who score well on this AMC 10 will be invited to take the 36th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 6, 2018 or Wednesday, March 21, 2018. More details about the AIME are on the back page of this test booklet.

1. What is the value of

$$\left(\left((2+1)^{-1}+1\right)^{-1}+1\right)^{-1}+1\,?$$

(A) $\frac{5}{8}$ **(B)** $\frac{11}{7}$ **(C)** $\frac{8}{5}$ **(D)** $\frac{18}{11}$ **(E)** $\frac{15}{8}$

2. Liliane has 50% more soda than Jacqueline, and Alice has 25% more soda than Jacqueline. What is the relationship between the amounts of soda that Liliane and Alice have?

(A) Liliane has 20% more soda than Alice.

(B) Liliane has 25% more soda than Alice.

(C) Liliane has 45% more soda than Alice.

(D) Liliane has 75% more soda than Alice.

(E) Liliane has 100% more soda than Alice.

3. A unit of blood expires after $10! = 10 \cdot 9 \cdot 8 \cdots 1$ seconds. Yasin donates a unit of blood at noon on January 1. On what day does his unit of blood expire?

(A) January 2 **(B)** January 12 **(C)** January 22

(D) February 11 **(E)** February 12

4. How many ways can a student schedule 3 mathematics courses—algebra, geometry, and number theory—in a 6-period day if no two mathematics courses can be taken in consecutive periods? (What courses the student takes during the other 3 periods is of no concern here.)

(A) 3 **(B)** 6 **(C)** 12 **(D)** 18 **(E)** 24

5. Alice, Bob, and Charlie were on a hike and were wondering how far away the nearest town was. When Alice said, "We are at least 6 miles away," Bob replied, "We are at most 5 miles away." Charlie then remarked, "Actually the nearest town is at most 4 miles away." It turned out that none of the three statements was true. Let d be the distance in miles to the nearest town. Which of the following intervals is the set of all possible values of d?

(A) $(0,4)$ **(B)** $(4,5)$ **(C)** $(4,6)$ **(D)** $(5,6)$ **(E)** $(5,\infty)$

6. Sangho uploaded a video to a website where viewers can vote that they like or dislike a video. Each video begins with a score of 0, and the score increases by 1 for each like vote and decreases by 1 for each dislike vote. At one point Sangho saw that his video had a score of 90, and that 65% of the votes cast on his video were like votes. How many votes had been cast on Sangho's video at that point?

   (A) 200 **(B)** 300 **(C)** 400 **(D)** 500 **(E)** 600

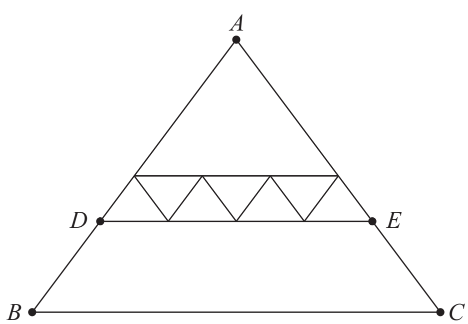
7. For how many (not necessarily positive) integer values of n is the value of $4000 \cdot \left(\frac{2}{5}\right)^n$ an integer?

   (A) 3 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 9

8. Joe has a collection of 23 coins, consisting of 5-cent coins, 10-cent coins, and 25-cent coins. He has 3 more 10-cent coins than 5-cent coins, and the total value of his collection is 320 cents. How many more 25-cent coins does Joe have than 5-cent coins?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

9. All of the triangles in the diagram below are similar to isosceles triangle ABC, in which $AB = AC$. Each of the 7 smallest triangles has area 1, and $\triangle ABC$ has area 40. What is the area of trapezoid $DBCE$?



   (A) 16 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24

10. Suppose that real number x satisfies

$$\sqrt{49 - x^2} - \sqrt{25 - x^2} = 3.$$

   What is the value of $\sqrt{49 - x^2} + \sqrt{25 - x^2}$?

   (A) 8 **(B)** $\sqrt{33} + 3$ **(C)** 9 **(D)** $2\sqrt{10} + 4$ **(E)** 12

11. When 7 fair standard 6-sided dice are thrown, the probability that the sum of the numbers on the top faces is 10 can be written as

$$\frac{n}{6^7},$$

where n is a positive integer. What is n?

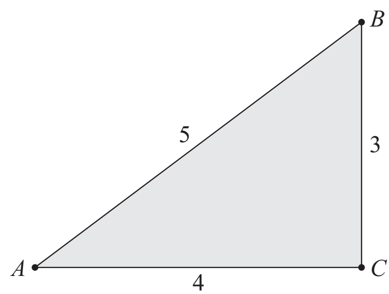
(A) 42 **(B)** 49 **(C)** 56 **(D)** 63 **(E)** 84

12. How many ordered pairs of real numbers (x, y) satisfy the following system of equations?

$$x + 3y = 3$$
$$\big||x| - |y|\big| = 1$$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 8

13. A paper triangle with sides of lengths 3, 4, and 5 inches, as shown, is folded so that point A falls on point B. What is the length in inches of the crease?



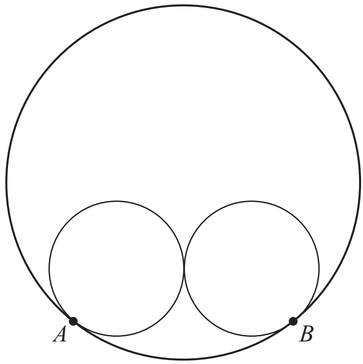
(A) $1 + \frac{1}{2}\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** $\frac{7}{4}$ **(D)** $\frac{15}{8}$ **(E)** 2

14. What is the greatest integer less than or equal to

$$\frac{3^{100} + 2^{100}}{3^{96} + 2^{96}}?$$

(A) 80 **(B)** 81 **(C)** 96 **(D)** 97 **(E)** 625

15. Two circles of radius 5 are externally tangent to each other and are internally tangent to a circle of radius 13 at points A and B, as shown in the diagram. The distance AB can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?



(A) 21 **(B)** 29 **(C)** 58 **(D)** 69 **(E)** 93

16. Right triangle ABC has leg lengths $AB = 20$ and $BC = 21$. Including $\overline{AB}$ and $\overline{BC}$, how many line segments with integer length can be drawn from vertex B to a point on hypotenuse $\overline{AC}$?

(A) 5 **(B)** 8 **(C)** 12 **(D)** 13 **(E)** 15

17. Let S be a set of 6 integers taken from $\{1, 2, \ldots, 12\}$ with the property that if a and b are elements of S with $a < b$, then b is not a multiple of a. What is the least possible value of an element of S?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 7

18. How many nonnegative integers can be written in the form

$$a_7 \cdot 3^7 + a_6 \cdot 3^6 + a_5 \cdot 3^5 + a_4 \cdot 3^4 + a_3 \cdot 3^3 + a_2 \cdot 3^2 + a_1 \cdot 3^1 + a_0 \cdot 3^0,$$

where $a_i \in \{-1, 0, 1\}$ for $0 \le i \le 7$?

(A) 512 **(B)** 729 **(C)** 1094 **(D)** 3281 **(E)** 59,048

19. A number m is randomly selected from the set $\{11, 13, 15, 17, 19\}$, and a number n is randomly selected from $\{1999, 2000, 2001, \ldots, 2018\}$. What is the probability that m^n has a units digit of 1?

(A) $\frac{1}{5}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{3}{10}$ **(D)** $\frac{7}{20}$ **(E)** $\frac{2}{5}$

20. A scanning code consists of a 7×7 grid of squares, with some of its squares colored black and the rest colored white. There must be at least one square of each color in this grid of 49 squares. A scanning code is called *symmetric* if its look does not change when the entire square is rotated by a multiple of $90°$ counterclockwise around its center, nor when it is reflected across a line joining opposite corners or a line joining midpoints of opposite sides. What is the total number of possible symmetric scanning codes?

(A) 510 **(B)** 1022 **(C)** 8190 **(D)** 8192 **(E)** 65,534

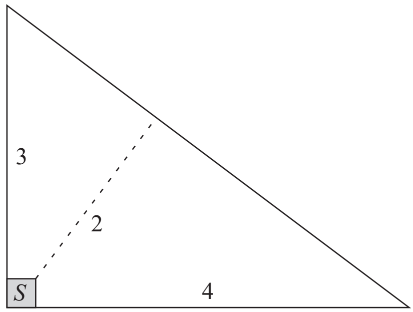
21. Which of the following describes the set of values of a for which the curves $x^2 + y^2 = a^2$ and $y = x^2 - a$ in the real xy-plane intersect at exactly 3 points?

(A) $a = \frac{1}{4}$ **(B)** $\frac{1}{4} < a < \frac{1}{2}$ **(C)** $a > \frac{1}{4}$ **(D)** $a = \frac{1}{2}$ **(E)** $a > \frac{1}{2}$

22. Let a, b, c, and d be positive integers such that $\gcd(a, b) = 24$, $\gcd(b, c) = 36$, $\gcd(c, d) = 54$, and $70 < \gcd(d, a) < 100$. Which of the following must be a divisor of a?

(A) 5 **(B)** 7 **(C)** 11 **(D)** 13 **(E)** 17

23. Farmer Pythagoras has a field in the shape of a right triangle. The right triangle's legs have lengths of 3 and 4 units. In the corner where those sides meet at a right angle, he leaves a small unplanted square S so that from the air it looks like the right angle symbol. The rest of the field is planted. The shortest distance from S to the hypotenuse is 2 units. What fraction of the field is planted?



(A) $\frac{25}{27}$ **(B)** $\frac{26}{27}$ **(C)** $\frac{73}{75}$ **(D)** $\frac{145}{147}$ **(E)** $\frac{74}{75}$

24. Triangle ABC with $AB = 50$ and $AC = 10$ has area 120. Let D be the midpoint of $\overline{AB}$, and let E be the midpoint of $\overline{AC}$. The angle bisector of $\angle BAC$ intersects $\overline{DE}$ and $\overline{BC}$ at F and G, respectively. What is the area of quadrilateral $FDBG$?

    (A) 60 **(B)** 65 **(C)** 70 **(D)** 75 **(E)** 80

25. For a positive integer n and nonzero digits a, b, and c, let A_n be the n-digit integer each of whose digits is equal to a; let B_n be the n-digit integer each of whose digits is equal to b; and let C_n be the $2n$-digit (not n-digit) integer each of whose digits is equal to c. What is the greatest possible value of $a + b + c$ for which there are at least two values of n such that $C_n - B_n = A_n^2$?

    (A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

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2018 AIME

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2018

AMC 10A

DO NOT OPEN UNTIL WEDNESDAY, February 7, 2018

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2. Your PRINCIPAL or VICE PRINCIPAL must verify on the AMC 10/12 COMPETITION CERTIFICATION FORM (found on maa.org/amc under 'AMC 10/12') that you followed all rules associated with the administration of the exam.
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19th Annual

AMC 10B

American Mathematics Competition 10B

Thursday, February 15, 2018

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR COMPETITION MANAGER TELLS YOU.
2. This is a 25-question multiple-choice exam. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
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6. Figures are not necessarily drawn to scale.
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8. When your competition manager gives the signal, begin working on the problems. You will have **75 minutes** to complete the exam.
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1. Kate bakes a 20-inch by 18-inch pan of cornbread. The cornbread is cut into pieces that measure 2 inches by 2 inches. How many pieces of cornbread does the pan contain?

   (A) 90 **(B)** 100 **(C)** 180 **(D)** 200 **(E)** 360

2. Sam drove 96 miles in 90 minutes. His average speed during the first 30 minutes was 60 mph (miles per hour), and his average speed during the second 30 minutes was 65 mph. What was his average speed, in mph, during the last 30 minutes?

   (A) 64 **(B)** 65 **(C)** 66 **(D)** 67 **(E)** 68

3. In the expression $(\_\_\_ \times \_\_\_) + (\_\_\_ \times \_\_\_)$ each blank is to be filled in with one of the digits 1, 2, 3, or 4, with each digit being used once. How many different values can be obtained?

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** 24

4. A three-dimensional rectangular box with dimensions X, Y, and Z has faces whose surface areas are 24, 24, 48, 48, 72, and 72 square units. What is $X + Y + Z$?

   (A) 18 **(B)** 22 **(C)** 24 **(D)** 30 **(E)** 36

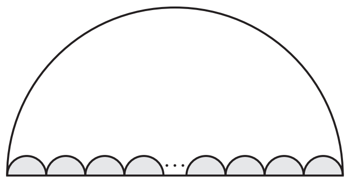
5. How many subsets of $\{2, 3, 4, 5, 6, 7, 8, 9\}$ contain at least one prime number?

   (A) 128 **(B)** 192 **(C)** 224 **(D)** 240 **(E)** 256

6. A box contains 5 chips, numbered 1, 2, 3, 4, and 5. Chips are drawn randomly one at a time without replacement until the sum of the values drawn exceeds 4. What is the probability that 3 draws are required?

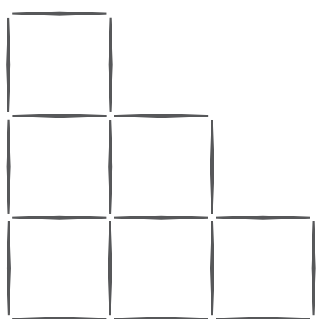
   (A) $\frac{1}{15}$ **(B)** $\frac{1}{10}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{5}$ **(E)** $\frac{1}{4}$

7. In the figure below, N congruent semicircles are drawn along a diameter of a large semicircle, with their diameters covering the diameter of the large semicircle with no overlap. Let A be the combined area of the small semicircles and B be the area of the region inside the large semicircle but outside the small semicircles. The ratio $A : B$ is $1 : 18$. What is N?



(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 36

8. Sara makes a staircase out of toothpicks as shown:



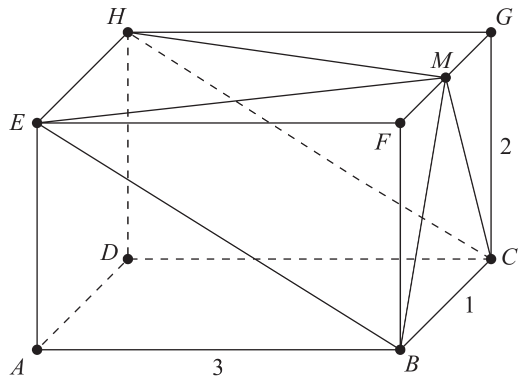
This is a 3-step staircase and uses 18 toothpicks. How many steps would be in a staircase that used 180 toothpicks?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 24 **(E)** 30

9. The faces of each of 7 standard dice are labeled with the integers from 1 to 6. Let p be the probability that when all 7 dice are rolled, the sum of the numbers on the top faces is 10. What other sum occurs with the same probability p?

(A) 13 **(B)** 26 **(C)** 32 **(D)** 39 **(E)** 42

10. In the rectangular parallelepiped shown, $AB = 3$, $BC = 1$, and $CG = 2$. Point M is the midpoint of $\overline{FG}$. What is the volume of the rectangular pyramid with base $BCHE$ and apex M?



(A) 1 **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{5}{3}$ **(E)** 2

11. Which of the following expressions is never a prime number when p is a prime number?

(A) $p^2 + 16$ **(B)** $p^2 + 24$ **(C)** $p^2 + 26$ **(D)** $p^2 + 46$
(E) $p^2 + 96$

12. Line segment $\overline{AB}$ is a diameter of a circle with $AB = 24$. Point C, not equal to A or B, lies on the circle. As point C moves around the circle, the centroid (center of mass) of $\triangle ABC$ traces out a closed curve missing two points. To the nearest positive integer, what is the area of the region bounded by this curve?

(A) 25 **(B)** 38 **(C)** 50 **(D)** 63 **(E)** 75

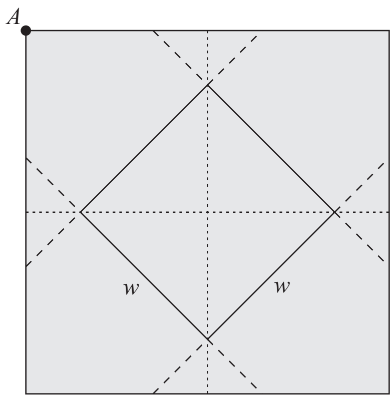
13. How many of the first 2018 numbers in the sequence 101, 1001, 10001, 100001, ... are divisible by 101?

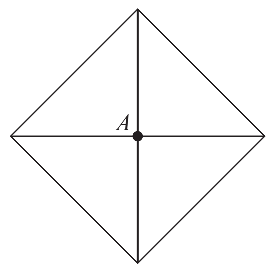
(A) 253 **(B)** 504 **(C)** 505 **(D)** 506 **(E)** 1009

14. A list of 2018 positive integers has a unique mode, which occurs exactly 10 times. What is the least number of distinct values that can occur in the list?

(A) 202 **(B)** 223 **(C)** 224 **(D)** 225 **(E)** 234

15. A closed box with a square base is to be wrapped with a square sheet of wrapping paper. The box is centered on the wrapping paper with the vertices of the base lying on the midlines of the square sheet of paper, as shown in the figure on the left. The four corners of the wrapping paper are to be folded up over the sides and brought together to meet at the center of the top of the box, point A in the figure on the right. The box has base length w and height h. What is the area of the sheet of wrapping paper?





(A) $2(w+h)^2$ **(B)** $\dfrac{(w+h)^2}{2}$ **(C)** $2w^2+4wh$ **(D)** $2w^2$

(E) w^2h

16. Let $a_1, a_2, \ldots, a_{2018}$ be a strictly increasing sequence of positive integers such that

$$a_1 + a_2 + \cdots + a_{2018} = 2018^{2018}.$$

What is the remainder when $a_1^3 + a_2^3 + \cdots + a_{2018}^3$ is divided by 6?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

17. In rectangle $PQRS$, $PQ = 8$ and $QR = 6$. Points A and B lie on $\overline{PQ}$, points C and D lie on $\overline{QR}$, points E and F lie on $\overline{RS}$, and points G and H lie on $\overline{SP}$ so that $AP = BQ < 4$ and the convex octagon $ABCDEFGH$ is equilateral. The length of a side of this octagon can be expressed in the form $k + m\sqrt{n}$, where k, m, and n are integers and n is not divisible by the square of any prime. What is $k+m+n$?

(A) 1 **(B)** 7 **(C)** 21 **(D)** 92 **(E)** 106

18. Three young brother-sister pairs from different families need to take a trip in a van. These six children will occupy the second and third rows in the van, each of which has three seats. To avoid disruptions, siblings may not sit right next to each other in the same row, and no child may sit directly in front of his or her sibling. How many seating arrangements are possible for this trip?

(A) 60 **(B)** 72 **(C)** 92 **(D)** 96 **(E)** 120

19. Joey and Chloe and their daughter Zoe all have the same birthday. Joey is 1 year older than Chloe, and Zoe is exactly 1 year old today. Today is the first of the 9 birthdays on which Chloe's age will be an integral multiple of Zoe's age. What will be the sum of the two digits of Joey's age the next time his age is a multiple of Zoe's age?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

20. A function f is defined recursively by $f(1) = f(2) = 1$ and

$$f(n) = f(n-1) - f(n-2) + n$$

for all integers $n \geq 3$. What is $f(2018)$?

(A) 2016 **(B)** 2017 **(C)** 2018 **(D)** 2019 **(E)** 2020

21. Mary chose an even 4-digit number n. She wrote down all the divisors of n in increasing order from left to right: $1, 2, \ldots, \frac{n}{2}, n$. At some moment Mary wrote 323 as a divisor of n. What is the smallest possible value of the next divisor written to the right of 323?

(A) 324 **(B)** 330 **(C)** 340 **(D)** 361 **(E)** 646

22. Real numbers x and y are chosen independently and uniformly at random from the interval $[0, 1]$. Which of the following numbers is closest to the probability that x, y, and 1 are the side lengths of an obtuse triangle?

(A) 0.21 **(B)** 0.25 **(C)** 0.29 **(D)** 0.50 **(E)** 0.79

23. How many ordered pairs (a, b) of positive integers satisfy the equation

$$a \cdot b + 63 = 20 \cdot \text{lcm}(a, b) + 12 \cdot \gcd(a, b),$$

where $\gcd(a, b)$ denotes the greatest common divisor of a and b, and $\text{lcm}(a, b)$ denotes their least common multiple?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

24. Let $ABCDEF$ be a regular hexagon with side length 1. Denote by X, Y, and Z the midpoints of sides $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$, respectively. What is the area of the convex hexagon whose interior is the intersection of the interiors of $\triangle ACE$ and $\triangle XYZ$?

(A) $\frac{3}{8}\sqrt{3}$ **(B)** $\frac{7}{16}\sqrt{3}$ **(C)** $\frac{15}{32}\sqrt{3}$ **(D)** $\frac{1}{2}\sqrt{3}$ **(E)** $\frac{9}{16}\sqrt{3}$

25. Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. How many real numbers x satisfy the equation $x^2 + 10{,}000\lfloor x \rfloor = 10{,}000x$?

(A) 197 **(B)** 198 **(C)** 199 **(D)** 200 **(E)** 201

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2018 AIME

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2018

AMC 10B

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2019 AIME

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AMC 10A

American Mathematics Competition 10A

Thursday, February 7, 2019

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2019

AMC 10A

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Conference Board of the Mathematical Sciences
Mu Alpha Theta
Society for Industrial and Applied Mathematics

1. What is the value of

$$2^{\left(0^{\left(1^9\right)}\right)} + \left(\left(2^0\right)^1\right)^9 ?$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

2. What is the hundreds digit of $(20! - 15!)$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 5

3. Ana and Bonita were born on the same date in different years, n years apart. Last year Ana was 5 times as old as Bonita. This year Ana's age is the square of Bonita's age. What is n?

(A) 3 **(B)** 5 **(C)** 9 **(D)** 12 **(E)** 15

4. A box contains 28 red balls, 20 green balls, 19 yellow balls, 13 blue balls, 11 white balls, and 9 black balls. What is the minimum number of balls that must be drawn from the box without replacement to guarantee that at least 15 balls of a single color will be drawn?

(A) 75 **(B)** 76 **(C)** 79 **(D)** 84 **(E)** 91

5. What is the greatest number of consecutive integers whose sum is 45 ?

(A) 9 **(B)** 25 **(C)** 45 **(D)** 90 **(E)** 120

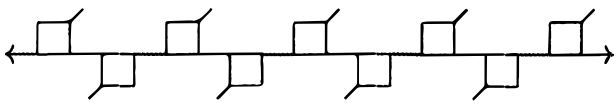
6. For how many of the following types of quadrilaterals does there exist a point in the plane of the quadrilateral that is equidistant from all four vertices of the quadrilateral?
   - a square
   - a rectangle that is not a square
   - a rhombus that is not a square
   - a parallelogram that is not a rectangle or a rhombus
   - an isosceles trapezoid that is not a parallelogram

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

7. Two lines with slopes $\frac{1}{2}$ and 2 intersect at $(2, 2)$. What is the area of the triangle enclosed by these two lines and the line $x + y = 10$?

(A) 4 **(B)** $4\sqrt{2}$ **(C)** 6 **(D)** 8 **(E)** $6\sqrt{2}$

8. The figure below shows line ℓ with a regular, infinite, recurring pattern of squares and line segments.



How many of the following four kinds of rigid motion transformations of the plane in which this figure is drawn, other than the identity transformation, will transform this figure into itself?

- some rotation around a point on line ℓ
- some translation in the direction parallel to line ℓ
- the reflection across line ℓ
- some reflection across a line perpendicular to line ℓ

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

9. What is the greatest three-digit positive integer n for which the sum of the first n positive integers is <u>*not*</u> a divisor of the product of the first n positive integers?

(A) 995 **(B)** 996 **(C)** 997 **(D)** 998 **(E)** 999

10. A rectangular floor that is 10 feet wide and 17 feet long is tiled with 170 one-foot square tiles. A bug walks from one corner to the opposite corner in a straight line. Including the first and last tile, how many tiles does the bug visit?

(A) 17 **(B)** 25 **(C)** 26 **(D)** 27 **(E)** 28

11. How many positive integer divisors of 201^9 are perfect squares or perfect cubes (or both)?

(A) 32 **(B)** 36 **(C)** 37 **(D)** 39 **(E)** 41

12. Melanie computes the mean μ, the median M, and modes of the 365 values that are the dates in the months of 2019. Thus her data consist of 12 1s, 12 2s, ..., 12 28s, 11 29s, 11 30s, and 7 31s. Let d be the median of the modes. Which of the following statements is true?

(A) $\mu < d < M$ **(B)** $M < d < \mu$ **(C)** $d = M = \mu$
(D) $d < M < \mu$ **(E)** $d < \mu < M$

13. Let $\triangle ABC$ be an isosceles triangle with $BC = AC$ and $\angle ACB = 40°$. Construct the circle with diameter $\overline{BC}$, and let D and E be the other intersection points of the circle with the sides $\overline{AC}$ and $\overline{AB}$, respectively. Let F be the intersection of the diagonals of the quadrilateral $BCDE$. What is the degree measure of $\angle BFC$?

(A) 90 **(B)** 100 **(C)** 105 **(D)** 110 **(E)** 120

14. For a set of four distinct lines in a plane, there are exactly N distinct points that lie on two or more of the lines. What is the sum of all possible values of N?

(A) 14 **(B)** 16 **(C)** 18 **(D)** 19 **(E)** 21

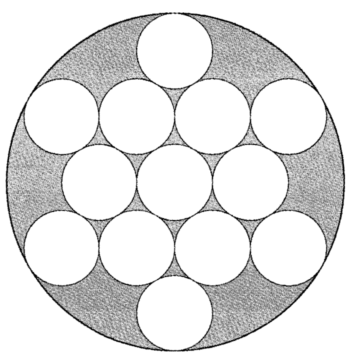
15. A sequence of numbers is defined recursively by $a_1 = 1$, $a_2 = \frac{3}{7}$, and

$$a_n = \frac{a_{n-2} \cdot a_{n-1}}{2a_{n-2} - a_{n-1}}$$

for all $n \geq 3$. Then a_{2019} can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p + q$?

(A) 2020 **(B)** 4039 **(C)** 6057 **(D)** 6061 **(E)** 8078

16. The figure below shows 13 circles of radius 1 within a larger circle. All the intersections occur at points of tangency. What is the area of the region, shaded in the figure, inside the larger circle but outside all of the circles of radius 1?



(A) $4\pi\sqrt{3}$ **(B)** 7π **(C)** $\pi(3\sqrt{3} + 2)$ **(D)** $10\pi(\sqrt{3} - 1)$

(E) $\pi(\sqrt{3} + 6)$

17. A child builds towers using identically shaped cubes of different colors. How many different towers with a height of 8 cubes can the child build with 2 red cubes, 3 blue cubes, and 4 green cubes? (One cube will be left out.)

(A) 24 **(B)** 288 **(C)** 312 **(D)** 1,260 **(E)** 40,320

18. For some positive integer k, the repeating base-k representation of the (base-ten) fraction $\frac{7}{51}$ is $0.\overline{23}_k = 0.23232323\ldots_k$. What is k?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

19. What is the least possible value of

$$(x+1)(x+2)(x+3)(x+4)+2019,$$

where x is a real number?

(A) 2017 **(B)** 2018 **(C)** 2019 **(D)** 2020 **(E)** 2021

20. The numbers $1, 2, \ldots, 9$ are randomly placed into the 9 squares of a 3×3 grid. Each square gets one number, and each of the numbers is used once. What is the probability that the sum of the numbers in each row and each column is odd?

(A) $\dfrac{1}{21}$ **(B)** $\dfrac{1}{14}$ **(C)** $\dfrac{5}{63}$ **(D)** $\dfrac{2}{21}$ **(E)** $\dfrac{1}{7}$

21. A sphere with center O has radius 6. A triangle with sides of length 15, 15, and 24 is situated in space so that each of its sides is tangent to the sphere. What is the distance between O and the plane determined by the triangle?

(A) $2\sqrt{3}$ **(B)** 4 **(C)** $3\sqrt{2}$ **(D)** $2\sqrt{5}$ **(E)** 5

22. Real numbers between 0 and 1, inclusive, are chosen in the following manner. A fair coin is flipped. If it lands heads, then it is flipped again and the chosen number is 0 if the second flip is heads, and 1 if the second flip is tails. On the other hand, if the first coin flip is tails, then the number is chosen uniformly at random from the closed interval $[0, 1]$. Two random numbers x and y are chosen independently in this manner. What is the probability that $|x - y| > \frac{1}{2}$?

(A) $\dfrac{1}{3}$ **(B)** $\dfrac{7}{16}$ **(C)** $\dfrac{1}{2}$ **(D)** $\dfrac{9}{16}$ **(E)** $\dfrac{2}{3}$

23. Travis has to babysit the terrible Thompson triplets. Knowing that they love big numbers, Travis devises a counting game for them. First Tadd will say the number 1, then Todd must say the next two numbers (2 and 3), then Tucker must say the next three numbers (4, 5, 6), then Tadd must say the next four numbers (7, 8, 9, 10), and the process continues to rotate through the three children in order, each saying one more number than the previous child did, until the number 10,000 is reached. What is the 2019th number said by Tadd?

(A) 5743 **(B)** 5885 **(C)** 5979 **(D)** 6001 **(E)** 6011

24. Let p, q, and r be the distinct roots of the polynomial $x^3 - 22x^2 + 80x - 67$. There exist real numbers A, B, and C such that

$$\frac{1}{s^3 - 22s^2 + 80s - 67} = \frac{A}{s-p} + \frac{B}{s-q} + \frac{C}{s-r}$$

for all real numbers s with $s \notin \{p, q, r\}$. What is $\frac{1}{A} + \frac{1}{B} + \frac{1}{C}$?

(A) 243 **(B)** 244 **(C)** 245 **(D)** 246 **(E)** 247

25. For how many integers n between 1 and 50, inclusive, is

$$\frac{(n^2-1)!}{(n!)^n}$$

an integer? (Recall that $0! = 1$.)

(A) 31 **(B)** 32 **(C)** 33 **(D)** 34 **(E)** 35

MAA AMC

American Mathematics Competitions

American Mathematics Competitions

20th Annual

AMC 10B

American Mathematics Competition 10B

Wednesday, February 13, 2019

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR COMPETITION MANAGER TELLS YOU.
2. This is a 25-question multiple-choice exam. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Sheet with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer sheet will be graded. **You must use and submit the original answer sheets provided by the MAA AMC. Photocopies will not be scored.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only scratch paper, graph paper, rulers, compasses, protractors, and erasers are allowed as aids. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the exam will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the exam, your competition manager will ask you to record certain information on the answer form.
8. When your competition manager gives the signal, begin working on the problems. You will have **75 minutes** to complete the exam.
9. When you finish the exam, *sign your name* in the space provided at the top of the Answer Sheet.

The Committee on the American Mathematics Competitions reserves the right to disqualify scores from a school if it determines that the required security procedures were not followed.

Students who score well on this AMC 10 will be invited to take the 37th annual American Invitational Mathematics Examination (AIME) on Wednesday, March 13, 2019, or Thursday, March 21, 2019. More details about the AIME are on the back page of this exam booklet.

2019

AMC 10B

DO NOT OPEN UNTIL WEDNESDAY, February 13, 2019

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All the information needed to administer this exam is contained in the AMC 10/12 Teacher's Manual. PLEASE READ THE MANUAL BEFORE FEBRUARY 13, 2019.
2. Your PRINCIPAL or VICE PRINCIPAL must verify on the AMC 10/12 COMPETITION CERTIFICATION FORM (found on **maa.org/amc** under 'AMC 10/12') that you followed all rules associated with the administration of the exam.
3. Answer sheets must be returned to the MAA AMC office the day after the competition. Ship with appropriate postage using a trackable method. FedEx or UPS is strongly recommended.
4. The publication, reproduction, or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

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We acknowledge the generosity of the following organizations in supporting the MAA AMC and Invitational Competitions:

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1. Alicia had two containers. The first was $\frac{5}{6}$ full of water and the second was empty. She poured all the water from the first container into the second container, at which point the second container was $\frac{3}{4}$ full of water. What is the ratio of the volume of the smaller container to the volume of the larger container?

   (A) $\frac{5}{8}$ **(B)** $\frac{4}{5}$ **(C)** $\frac{7}{8}$ **(D)** $\frac{9}{10}$ **(E)** $\frac{11}{12}$

2. Consider the statement, "If n is not prime, then $n-2$ is prime." Which of the following values of n is a counterexample to this statement?

   (A) 11 **(B)** 15 **(C)** 19 **(D)** 21 **(E)** 27

3. In a high school with 500 students, 40% of the seniors play a musical instrument, while 30% of the non-seniors do not play a musical instrument. In all, 46.8% of the students do not play a musical instrument. How many non-seniors play a musical instrument?

   (A) 66 **(B)** 154 **(C)** 186 **(D)** 220 **(E)** 266

4. All lines with equation $ax+by=c$ such that a, b, c form an arithmetic progression pass through a common point. What are the coordinates of that point?

   (A) $(-1,2)$ **(B)** $(0,1)$ **(C)** $(1,-2)$ **(D)** $(1,0)$
   (E) $(1,2)$

5. Triangle ABC lies in the first quadrant. Points A, B, and C are reflected across the line $y=x$ to points A', B', and C', respectively. Assume that none of the vertices of the triangle lie on the line $y=x$. Which of the following statements is *not* always true?

   (A) Triangle $A'B'C'$ lies in the first quadrant.

   (B) Triangles ABC and $A'B'C'$ have the same area.

   (C) The slope of line AA' is -1.

   (D) The slopes of lines AA' and CC' are the same.

   (E) Lines AB and $A'B'$ are perpendicular to each other.

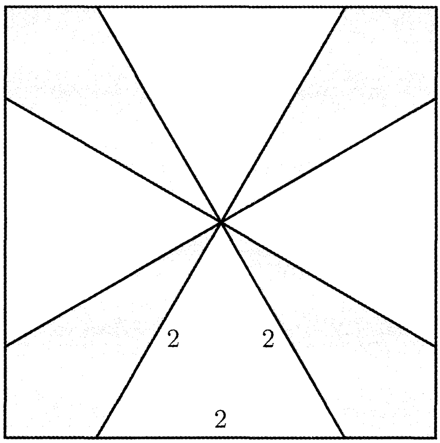
6. A positive integer n satisfies the equation $(n+1)!+(n+2)!=440\cdot n!$. What is the sum of the digits of n?

   (A) 2 **(B)** 5 **(C)** 10 **(D)** 12 **(E)** 15

7. Each piece of candy in a shop costs a whole number of cents. Casper has exactly enough money to buy either 12 pieces of red candy, 14 pieces of green candy, 15 pieces of blue candy, or n pieces of purple candy. A piece of purple candy costs 20 cents. What is the least possible value of n?

   (A) 18 **(B)** 21 **(C)** 24 **(D)** 25 **(E)** 28

8. The figure below shows a square and four equilateral triangles, with each triangle having a side lying on a side of the square, such that each triangle has side length 2 and the third vertices of the triangles meet at the center of the square. The region inside the square but outside the triangles is shaded. What is the area of the shaded region?



   (A) 4 **(B)** $12 - 4\sqrt{3}$ **(C)** $3\sqrt{3}$ **(D)** $4\sqrt{3}$ **(E)** $16 - 4\sqrt{3}$

9. The function f is defined by

$$f(x) = \lfloor |x| \rfloor - |\lfloor x \rfloor|$$

   for all real numbers x, where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to the real number r. What is the range of f?

   (A) $\{-1, 0\}$ **(B)** the set of nonpositive integers **(C)** $\{-1, 0, 1\}$

   (D) $\{0\}$ **(E)** the set of nonnegative integers

10. In a given plane, points A and B are 10 units apart. How many points C are there in the plane such that the perimeter of $\triangle ABC$ is 50 units and the area of $\triangle ABC$ is 100 square units?

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** infinitely many

11. Two jars each contain the same number of marbles, and every marble is either blue or green. In Jar 1 the ratio of blue to green marbles is $9:1$, and the ratio of blue to green marbles in Jar 2 is $8:1$. There are 95 green marbles in all. How many more blue marbles are in Jar 1 than in Jar 2?

(A) 5 **(B)** 10 **(C)** 25 **(D)** 45 **(E)** 50

12. What is the greatest possible sum of the digits in the base-seven representation of a positive integer less than 2019?

(A) 11 **(B)** 14 **(C)** 22 **(D)** 23 **(E)** 27

13. What is the sum of all real numbers x for which the median of the numbers 4, 6, 8, 17, and x is equal to the mean of those five numbers?

(A) -5 **(B)** 0 **(C)** 5 **(D)** $\frac{15}{4}$ **(E)** $\frac{35}{4}$

14. The base-ten representation for 19! is $121{,}6T5{,}100{,}40M{,}832{,}H00$, where T, M, and H denote digits that are not given. What is $T+M+H$?

(A) 3 **(B)** 8 **(C)** 12 **(D)** 14 **(E)** 17

15. Right triangles T_1 and T_2 have areas 1 and 2, respectively. A side of T_1 is congruent to a side of T_2, and a different side of T_1 is congruent to a different side of T_2. What is the square of the product of the lengths of the other (third) sides of T_1 and T_2?

(A) $\frac{28}{3}$ **(B)** 10 **(C)** $\frac{21}{2}$ **(D)** $\frac{32}{3}$ **(E)** 12

16. In $\triangle ABC$ with a right angle at C, point D lies in the interior of $\overline{AB}$ and point E lies in the interior of $\overline{BC}$ so that $AC = CD$, $DE = EB$, and the ratio $AC:DE = 4:3$. What is the ratio $AD:DB$?

(A) $2:3$ **(B)** $2:\sqrt{5}$ **(C)** $1:1$ **(D)** $3:\sqrt{5}$ **(E)** $3:2$

17. A red ball and a green ball are randomly and independently tossed into bins numbered with the positive integers so that for each ball, the probability that it is tossed into bin k is 2^{-k} for $k = 1, 2, 3, \ldots$. What is the probability that the red ball is tossed into a higher-numbered bin than the green ball?

(A) $\frac{1}{4}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{3}{7}$

18. Henry decides one morning to do a workout, and he walks $\frac{3}{4}$ of the way from his home to his gym. The gym is 2 kilometers away from Henry's home. At that point, he changes his mind and walks $\frac{3}{4}$ of the way from where he is back toward home. When he reaches that point, he changes his mind again and walks $\frac{3}{4}$ of the distance from there back toward the gym. If Henry keeps changing his mind when he has walked $\frac{3}{4}$ of the distance toward either the gym or home from the point where he last changed his mind, he will get very close to walking back and forth between a point A kilometers from home and a point B kilometers from home. What is $|A - B|$?

    (A) $\frac{2}{3}$ **(B)** 1 **(C)** $1\frac{1}{5}$ **(D)** $1\frac{1}{4}$ **(E)** $1\frac{1}{2}$

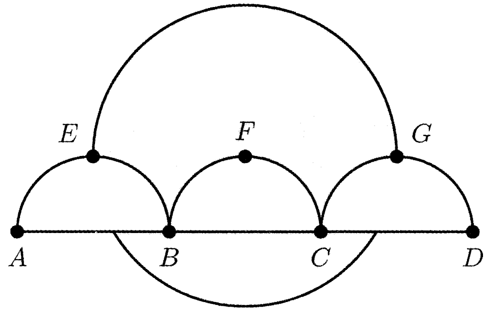
19. Let S be the set of all positive integer divisors of 100,000. How many numbers are the product of two distinct elements of S?

    (A) 98 **(B)** 100 **(C)** 117 **(D)** 119 **(E)** 121

20. As shown in the figure, line segment $\overline{AD}$ is trisected by points B and C so that $AB = BC = CD = 2$. Three semicircles of radius 1, $\overparen{AEB}$, $\overparen{BFC}$, and $\overparen{CGD}$, have their diameters on $\overline{AD}$, lie in the same halfplane determined by line AD, and are tangent to line EG at E, F, and G, respectively. A circle of radius 2 has its center at F. The area of the region inside the circle but outside the three semicircles, shaded in the figure, can be expressed in the form

$$\frac{a}{b} \cdot \pi - \sqrt{c} + d,$$

where a, b, c, and d are positive integers and a and b are relatively prime. What is $a + b + c + d$?



(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

21. Debra flips a fair coin repeatedly, keeping track of how many heads and how many tails she has seen in total, until she gets either two heads in a row or two tails in a row, at which point she stops flipping. What is the probability that she gets two heads in a row but she sees a second tail before she sees a second head?

(A) $\frac{1}{36}$ **(B)** $\frac{1}{24}$ **(C)** $\frac{1}{18}$ **(D)** $\frac{1}{12}$ **(E)** $\frac{1}{6}$

22. Raashan, Sylvia, and Ted play the following game. Each starts with \$1. A bell rings every 15 seconds, at which time each of the players who currently has money simultaneously chooses one of the other two players independently and at random and gives \$1 to that player. What is the probability that after the bell has rung 2019 times, each player will have \$1? (For example, Raashan and Ted may each decide to give \$1 to Sylvia, and Sylvia may decide to give her dollar to Ted, at which point Raashan will have \$0, Sylvia will have \$2, and Ted will have \$1, and that is the end of the first round of play. In the second round Raashan has no money to give, but Sylvia and Ted might choose each other to give their \$1 to, and the holdings will be the same at the end of the second round.)

(A) $\frac{1}{7}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

23. Points $A(6,13)$ and $B(12,11)$ lie on a circle ω in the plane. Suppose that the tangent lines to ω at A and B intersect at a point on the x-axis. What is the area of ω?

(A) $\frac{83\pi}{8}$ **(B)** $\frac{21\pi}{2}$ **(C)** $\frac{85\pi}{8}$ **(D)** $\frac{43\pi}{4}$ **(E)** $\frac{87\pi}{8}$

24. Define a sequence recursively by $x_0 = 5$ and

$$x_{n+1} = \frac{x_n^2 + 5x_n + 4}{x_n + 6}$$

for all nonnegative integers n. Let m be the least positive integer such that

$$x_m \leq 4 + \frac{1}{2^{20}}.$$

In which of the following intervals does m lie?

(A) $[9,26]$ **(B)** $[27,80]$ **(C)** $[81,242]$ **(D)** $[243,728]$

(E) $[729,\infty)$

25. How many sequences of 0s and 1s of length 19 are there that begin with a 0, end with a 0, contain no two consecutive 0s, and contain no three consecutive 1s?

(A) 55 **(B)** 60 **(C)** 65 **(D)** 70 **(E)** 75

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Questions and comments about problems
and solutions for this exam should be sent to:

amchq@maa.org

Send questions and comments about administrative arrangements to:

amcinfo@maa.org

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The problems and solutions for this AMC 10 were prepared by MAA's Subcommittee on the AMC 10/AMC 12 Exams, under the direction of the co-chairs Jerrold W. Grossman and Carl Yerger.

2019 AIME

The 37th annual AIME will be held on Wednesday, March 13, 2019, with the alternate on Thursday, March 21, 2019. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate if you achieve a high score on this competition. Top-scoring students on the AMC 10/12/AIME will be selected to take the 48th Annual USA Mathematical Olympiad (USAMO) on April 17–18, 2019.